УДК 629.5
DOI https://doi.org/10.15589/znp2024.4(497).20

ARBITRARILY CONVERGING TIME CONTROLLER FOR TRAJECTORY TRACKING OF DYNAMICALLY POSITIONED VESSEL

ДОВІЛЬНО ЗБІЖНИЙ ЧАСОВИЙ КОНТРОЛЕР ДЛЯ ВІДСТЕЖЕННЯ ТРАЕКТОРІЇ СУДНА З ДИНАМІЧНИМ ПОЗИЦІЮВАННЯМ

Benwei Zhang[1]
bwzhang@ccs.org.cn
Zhaodi Yang[2]
yangzhaodi@jou.edu.cn
ORCID: 0009-0008-1368-1315
Xiaoming Xia[3]
xiaxiaoming@jou.edu.cn
ORCID: 0000-0002-3993-5028
Heorhii Babkin[4]
gvbabkin@gmail.com
ORCID: 0000-0001-6303-0993
Pei Xu[3]
xupei_0824@jou.edu.cn

Бенвей Чжан[1],
д-р філософії
Чжаоді Ян[2],
магістрант
Сяомін Ся[3],
д-р філософії
Георгій Бабкін[4],
канд. техн. наук, доцент
Пей Сю[3],
д-р філософії

[1]China Classification Society, Pekin
[2]Makarov College of Marine Engineering, Jiangsu Ocean University, Tsziansu
[3]School of Ocean Engineering, Jiangsu Ocean University, Tsziansu
[4]Admiral Makarov National University of Shipbuilding, Mykolaiv
[1]Китайське класифікаційне товариство, м. Пекін
[2]Макарівський коледж морської інженерії, Університет океану, м. Цзянсу
[3]Школа морської інженерії, Університет океану, м. Цзянсу
[4]Національний університет суднобудування імені адмірала Макарова, м. Миколаїв

Abstract. In this paper, the trajectory tracking problem of dynamic positioning vessel with model uncertainty and external disturbances is investigated. The task is formulated to design a state feedback controller for a vessel with model uncertainty and unknown disturbances and to make the vessel track a virtual vessel. Different from previous strategies, the concept of arbitrary convergence time is introduced and applied to the design process of observer and controller. A new observer with arbitrary convergence time is designed to estimate unknown dynamics induced by model uncertainty and external disturbances, and a controller with arbitrary convergence time is designed base on the observer. We consider a system that contains an equilibrium point, which is a continuous positive definite function. Consider a model in which the observation error converges to the equilibrium point in the desired time. A ship model with known dynamics is considered and the proposed controller can guarantee that all signals in the closed system are bounded and coincide. A positive equilibrium point is assumed to exist, meaning that the velocity cannot maintain a steady state. The stability of the closed system is analyzed. A controller that could implement arbitrary temporal stabilization and a controller coupled with an observer that ensured arbitrary temporal convergence of the estimation error were presented. An observer with an arbitrary convergence time is proposed and a controller is developed based on it. The controller is designed in such a way that it can make the ship follow the virtual ship at a predetermined time, and this time is independent of system parameters and initial conditions. Then, the feasibility of the controller is illustrated through simulation. In the course of simulation, results were obtained on the basis of which it is possible to conclude that the proposed controller provides not only convergence in a fixed time, but also the possibility of choosing the time of convergence. It is shown that a ship can successfully track a virtual ship over time, accurate estimates of unknown dynamics can be quickly obtained using the proposed observer.
Key words: trajectory tracking; DP vessel; arbitrary convergence time.

Анотація. У цій статті досліджується задача відстеження траєкторії, судна з динамічним позиціюванням з невизначеністю моделі та зовнішніми збуреннями. Сформульовано завдання розробити контролер зі зворотним зв'язком стану для судна з невизначеністю моделі та невідомими збуреннями та змусити судно відстежувати віртуальне судно. На відміну від попередніх стратегій, концепція довільного часу конвергенції введена та застосована до процесу проектування спостерігача та контролера. Новий спостерігач із довільним часом збіжності призначений для оцінки невідомої динаміки, викликаної невизначеністю моделі та зовнішніми збуреннями, а контролер із довільним часом збіжності розроблений на основі спостерігача. Розглянуто систему, яка містить точку рівноваги, що є неперервною позитивно визначеною функцією. Розглянемо модель в якої помилка спостереження збігається до точки рівноваги за бажаний час. Розглянуто модель корабля з відомою динамікою та запропонований контролер може гарантувати, що всі сигнали в замкнутій системі обмежені та збігаються. Припущено, що існує позитивна точка рівноваги, це означає, що швидкість не може підтримувати стаціонарний стан. Аналізується стійкість замкнутої системи. Представлено контролер, який міг би реалізувати довільну часову стабілізацію, та контролер, поєднаний із спостерігачем, який забезпечував довільну часову збіжність похибки оцінки. Запропоновано спостерігач із довільним часом збіжності та розроблено на його основі контролер. Контролер сконструйований таким чином, що він може змусити судно стежити за віртуальним кораблем у заздалегідь визначений час, і цей час не залежить від параметрів системи та початкових умов. Потім здійсненність контролера ілюструється за допомогою моделювання. В ході моделювання отримані результати на підставі яких можливо зробити висновок що запропонований контролер забезпечує не тільки збіжність у фіксованому часі, але й можливість вибору часу збіжності. Показано що судно може успішно відстежувати віртуальне судно протягом часу, точні оцінки невідомої динаміки можна швидко отримати за допомогою запропонованого спостерігача.
Ключові слова: відстеження траєкторії; судно з динамічним позиціюванням; довільний час збіжності.

SETTING OBJECTIVES

As the paper [33] mentioned, the definition: Arbitrary time stabilization is introduced to highlight exciting property of the proposed class systems.

Consider the nonlinear system

$$\dot{x} = f(t, x, \omega),\ x(t_0) = x_0 \tag{1}$$

where x is the system state, ω is constant parameter. f is a nonlinear function satisfied $f(t, 0, \omega) = 0$, and it means that equilibrium point is 0, t_0 is the initial time.

Definition 1: (Global Finite Time Stability). The equilibrium point of the system is globally asymptotically stable, and any solution $x(t, t_0, x_0)$ of (1) converges to the equilibrium point at fixed time. Then the equilibrium point is called global finite time stability.

Definition 2: (Fixed Time Stability). The equilibrium point of the system is global finite-time stability, and the settling time is bounded. Then the equilibrium point is said to possess the property of fixed-time stability.

Definition 3: (Arbitrary time stabilization). The equilibrium point of the system is named arbitrary time stabilization if

– it is fixed time stable;

– there is a convergence time $T_a > 0$, which depends on the known ω and which can be evaluated in advance for some given ω;

– T_a can be adjusted arbitrarily;

– for any given ω, $T_a > T_{tf}$ (Weak arbitrary time stable), $T_a = T_{tf}$ (Strong arbitrary time stable), T_{tf} is the true fixed time.

Theorem 1. Consider the system the works in Pal et al,(1) define D as a domain which containing the equilibrium point, $\alpha_1(x)$ and $\alpha_2(x)$ are continuous positive definite functions on D. Suppose that there exists a function V which is real-valued continuously differentiable, and it is expressed as $V : D_t \times D \to \mathbb{R}$ with $D_t = [t_0, t_f]$, such that

$$\alpha_1(x) \le V(t, x) \le \alpha_2(x),\ \forall t \in D_t, \forall x \in D \setminus \{0\} \tag{2}$$

$$\dot{V} \le \frac{-k_1\left(e^V - k_2\right)}{e^V\left(t_f - t\right)},\ \forall t \in D_t \tag{3}$$

where k_1 and k_2 are positive constants.

On a similar method to the works in Pal et al, [33] this paper raises another differential equation as follows

$$\dot{\varepsilon} = \begin{cases} \dfrac{-k_1\left(e^{\varepsilon} - k_2\right)}{e^{\varepsilon}\left(t_f - t\right)}, & t_0 \le t < t_f \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

where ε is the state variable, and t_f is the settling time moment independent of any system parameters or initial conditions. The solution to (4) is

$$\varepsilon = \ln\left(C_1\left(t_f - t\right)^{k_1} + k_2\right) \tag{5}$$

where $C_1 = \dfrac{e^{\varepsilon(t_0)} - k_2}{\left(t_f - t_0\right)^{k_1}}$ is integration constant.

From (5), we have

$$\dot{\varepsilon} = \frac{-k_1 C_1\left(t_f - t\right)^{k_1 - 1}}{C_1\left(t_f - t\right)^{k_1} + k_2} \tag{6}$$

From (6), it can see that when $t = t_f$, $\dot{\varepsilon} = 0$. Further, from (5), $t = t_f$, ε= ln(k_2). Therefore, $\forall t > t_f$, ε = ln(k_2) is maintained. The same result is valid for $V(t, x)$ for the free-will strong arbitrary time.

ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

Trajectory tracking of dynamically positioned (DP) vessel has attracted increasing attention from the control community for developing marine resources [1–6]. DP has two controls, namely surge force and yaw moment, but without sway force control, the feature that makes the control problem both interesting and challenging [7–10]. Much research has been done on the trajectory tracking problem of DP.

Model uncertainty and disturbances are challenges for designing controllers [11–13]. Generally speaking, there are two ways to reduce the impact of unknown dynamics on the system. One way is to design a suitable controller independent of observer, such as proportional integral differential (PID) controller [14], adaptive controller [15] and sliding mode controller [16]. Another way is to design a controller based on observer used to estimate unknown dynamics. A modified barrier Lyapunov function is used to achieve the distributed formation tracking in works of Xia et al, [17] where extended state observer is employed to estimate unknown dynamics consisting of model uncertainty and environmental disturbances. In works of Xia et al, [18] an output feedback controller is designed using a neural network based observer, and the influence of input and output constraints are taken into account. An event-triggered controller is designed for vessels in works of Xia et al, [19–22] where extended state observer is designed to estimate yaw rate and disturbances. A finite-time tracking controller is proposed in works of Wang et al, [23] where a finite-time disturbance observer is incorporated into controller design. A cooperative control strategy is employed for multiple surface vessels in works of Xia et al [24], where finite-time disturbance observer is used to estimate unknown disturbances Another challenge for the trajectory tracking controller of vessels is to solve underactuated problem. Different from the previous control methods [25; 26; 27], many controllers are used to improve system performance. A finite-time path-following controller is designed in works of Ghommam et al [28], where parametric uncertainties, unknown disturbances and input saturation are considered. However, the settling time depends on initial value of system state. An adaptive trajectory tracking controller for vessel is proposed in [29], where transient performance is guaranteed by an error transformation function. A controller combined with the critic function and reinforcement learning is proposed in works of Zheng et al, [30] where the tracking errors remain within the predefined constraint boundaries. Li et al [31] present a nonlinear sliding mode control strategy to deal with formation control of the vessel, and the controller guarantees that the given formation pattern is achieved within a finite time.

In all of the above works, the existing finite time controllers ensure finite-time stabilization but in terminal time that depends on the initial condition. In the light of the question, the trajectory tracking control problem is investigated in works of Zheng et al, [32] where a controller with specified accuracy and settling time is proposed. A controller ensuring the arbitrary time of convergence is designed in works of Pal et al, [33] and the sufficient condition of the arbitrary time stable system is given.

Motivated by the aforementioned works, this paper addresses the controller with arbitrary convergence time for the trajectory tracking of DP subject to model uncertainty and disturbances. First, a controller with arbitrary convergence time is designed under model uncertainty and disturbances are both known. The dynamics consisting of model uncertainty and disturbances are further assumed to be unknown. The important application of the finite-time convergence is observer design, allowing the designer to be sure that after some time moment a controller is using precise estimation. To this setup, an observer with arbitrary convergence time is developed to estimate the dynamics. In combination with the observer, a controller is proposed to achieve convergence within desired time. Finally, the stability of the closed-loop system is analyzed. Simulation results are provided to substantiate the proposed controller.

Compared with the existing results, the features of the proposed design method for the trajectory tracking of DP are as follows. First, compared with the works [28; 31] and [32], the proposed controller allows the designer to adjust the settling time. Second, in contrast to the previous observer, an observer with arbitrary convergence time is first proposed.

THE PURPOSE OF THE STUDY

Synthesis of a DP vessel trajectory tracking controller with arbitrary convergence time based on an observer

THE MAIN MATERIAL

As shown in Figure l, the three degrees of freedom horizontal motion mathematical model for the vessel is given as [34–35]

$$\begin{aligned} \dot{\eta} &= J(\psi)\nu, \\ M\dot{\nu} &= -C(\nu)\nu - D(\nu)\nu + d + \tau, \end{aligned} \tag{7}$$

where $\eta = [x, y, \psi]^T \in \mathbb{R}^3$ is a vector expressed in the earth-fixed frame, (x, y) and ψ represent position and yaw angle, respectively, $\nu = [u, v, r]^T \in \mathbb{R}^3$ is the vector denoting surge, sway, and yaw velocities in the body-fixed frame. $d = [d_1, d_2, d_3]^T \in \mathbb{R}^3$ is the vector representing unmodeled environmental disturbances induced by waves, wind, and ocean currents. $\tau = [\tau_u, 0, \tau_r]^T \in \mathbb{R}^3$ is the control vector which consists of the surge force τ_u and yaw momentτ_r. The matrices $J(\psi)$ is given by

$$J(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

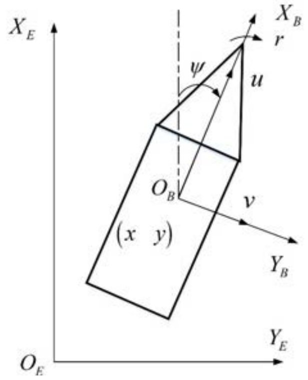

Fig. 1. Earth-fixed frame and body-fixed frame

$M = M^T \in \mathbb{R}^{3\times 3}$ is the vessel inertia matrix. $C(v) = -C^T(v) \in \mathbb{R}^{3\times 3}$ represents a skew symmetric matrix of Coriolis and centripetal term. $D(v) \in \mathbb{R}^{3\times 3}$ is a nonlinear damping matrix.

Problem formulation

The control objective is to design a state feedback controller for a vessel with model uncertainties and unknown disturbances, and force the vessel to track a virtual ship. The position and head angle $\eta_d = [x_d, y_d, \psi_d]^T$ of the virtual ship is generated by

$$\dot{x}_d = u_d \cos(\psi_d) - v_d \sin(\psi_d), \tag{8}$$

$$\dot{y}_d = u_d \sin(\psi_d) + v_d \cos(\psi_d), \tag{9}$$

$$\dot{\psi}_d = r_d, \tag{10}$$

where u_d, v_d and r_d are surge, sway, and yaw velocities of virtual ship respectively. Two objectives to be achieved are

(1) $\lim_{t\to t_f} \|\eta - \eta_d\| \le c_1$

where $c_1 > 0$ is a small constant.

(2) The controller is designed such that it can force the vessel to track the virtual ship at a predetermined time, and the time is independent of the system parameters and initial conditions.

Observer design

In this section, an arbitrary convergence time observer is used for estimating total disturbances consisting of the unknown term of the system matrix $C(v)$, $D(v)$ and environmental disturbances d. The vessel dynamic (7) is rewritten as

$$\begin{aligned} \dot{\eta} &= J(\psi)v, \\ \dot{v} &= M^{-1}\tau + \zeta, \end{aligned} \tag{11}$$

where ζ $[\zeta_1, \zeta_2, \zeta_3, =]^T \in \mathbb{R}^3$ is the unknown dynamics, it is a state vector expressed as $\zeta_1 = M^{-1}(-C(v)v - D(v)v + d)$.

Assumption 1: $\|\zeta\| \le \zeta^*$ with ζ^* is a positive constant.

ζ is a vector of function which contains velocity v, ship parameters C, D, and environmental disturbances d. It is natural to assume that its derivative is bounded because ship parameters and environmental disturbances are bounded. Besides, the control inputs to drive the vessel are limited, so Assumption 1 is reasonable.

Define $\tilde{v} = [\tilde{v}_1, \tilde{v}_2, \tilde{v}_3]^T = v - \tilde{v}$, and inspired by the works in Gao et al, [36] the input signal τ and velocity data v are used to reconstruct the unknown function ζ. An observer with arbitrary time convergence is designed as follows:

$$\begin{aligned} \dot{v} &= M^{-1}\tau + \xi_o, \\ \hat{\zeta} &= \xi_o, \end{aligned} \tag{12}$$

where $\xi_o = \left[\frac{k_{o1}\left(e^{\tilde{v}_1 - 1}\right)}{e^{\tilde{v}_1}(t_f - t)}, \frac{k_{o2}\left(e^{\tilde{v}_2 - 1}\right)}{e^{\tilde{v}_2}(t_f - t)}, \frac{k_{o3}\left(e^{\tilde{v}_3 - 1}\right)}{e^{\tilde{v}_3}(t_f - t)}\right]^T$

with k_{01}, k_{02} and k_{03} are positive constants.

Theorem 2: Consider the model (11) with assumption 1, the observer is designed as (12), the observation error converges to the equilibrium point in desired time.

Proof: The Lyapunov function is chosen as

$$V_o = \tilde{v}^T \tilde{v}. \tag{13}$$

Along with (12), the time derivative of V0 is given

$$\begin{aligned} \dot{V}_o &= 2\tilde{v}^T(-\xi_o + \zeta) \\ &\le 2\sum_{h=1}^{3} |\tilde{v}_h| \frac{-\left(k_{oh} - \bar{t}\zeta^*\right)\left(e^{\tilde{v}_h} - \bar{k}_{th}\right)}{e^{\tilde{v}_h}(t_f - t)} \\ &\le 2\sum_{h=1}^{3} |\tilde{v}_h| \frac{-\left(k_{oh} - \bar{t}\zeta^*\right)\left(e^{|\tilde{v}_h|} - \bar{k}_{th}\right)}{e^{|\tilde{v}_h|}(t_f - t)} \end{aligned}. \tag{14}$$

where $\bar{t} = t_f - t$, $\bar{k}_{th} = \frac{1}{k_{oh} - \bar{t}\zeta^*}$, h=1, 2, 3.

Because of $V_o = \tilde{v}^T\tilde{v} = \tilde{v}_1^2 + \tilde{v}_2^2 + \tilde{v}_3^2$, then $\sqrt{V_o} \ge |\tilde{v}_1|$, $\sqrt{V_o} \ge |\tilde{v}_2|$ and $\sqrt{V_o} \ge |\tilde{v}_3|$ (14) is rewritten as

$$\dot{V}_o \le \frac{-2\bar{k}_o\sqrt{V_o}\left(e^{\sqrt{V_o}} - \bar{k}_t\right)}{e^{\sqrt{V_o}}(t_f - t)}. \tag{15}$$

where $\bar{k}_o = k_{o1} + k_{o2} + k_{o3} - 3\bar{t}\zeta^*$, $\bar{k}_t = \max\{\bar{k}_{t1}, \bar{k}_{t2}, \bar{k}_{t3}\}$. Let $\varepsilon_o = \sqrt{V_o}$, its time derivative is

$$\dot{\varepsilon}_o = \frac{\dot{V}_0}{2\sqrt{V_o}} \le \frac{-2\bar{k}_o\left(e^{\sqrt{V_o}} - \bar{k}_t\right)}{e^{\sqrt{V_o}}(t_f - t)}.$$

According to Theorem 1, we can obtain V_o is a constant for all $t \ge t_f$. Then, it yields $\dot{V}_0 = 0$, for $t \ge t_f$. At this time, it can be obtained that

$$\dot{\tilde{v}} = 0. \tag{16}$$

Then, the estimation error $\tilde{\zeta}$ satisfies

$$\tilde{\zeta} = \hat{\zeta} - \zeta = \xi_o - \zeta = -\dot{\tilde{v}}. \tag{17}$$

It hence yields from (16) that

$$\tilde{\zeta} = 0, t \ge t_f. \tag{18}$$

The proof is end.

Controller design

The section presents a controller which could realize arbitrary time stabilization, and a controller combined with observer which ensuring the arbitrary time convergence of estimation error is designed.

A controller design process is presented under the assumption that the dynamics of vessels are known. To solve the underactuated problem, a auxiliary variable is induced to into the controller design.

Define the first error vector

$$z_1 = [z_{11}, z_{12}, z_{13}]^T = J^T(\psi)(\eta - \eta_d), \tag{19}$$

where $\eta_d = [x_d, y_d, \psi_a]^T$, ψ_a is a form of line-of-sight guidance law similar to the works of [37], and it is chosen as

$$\psi_a = \psi_d - \operatorname{atan}(v/u_d) - \operatorname{atan}(z_{12}/\Delta), \tag{20}$$

where Δ is a constant representing the look-ahead distance.

Taking the time derivative of (19),

$$\dot{z}_1 = -rSz_1 + v - J^T(\psi)\dot{\eta}_d, \tag{21}$$

where

$$S = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

To stabilize z1, the desired value Vd is set as

$$\nu_d = -\xi_1 + J^T(\psi)\dot{\eta}_d, \tag{22}$$

where

$$\xi_1 = \operatorname{diag}\left\{ \frac{e^{z_{11}} - 1}{e^{z_{11}}(t_f - t)}, \frac{e^{z_{12}} - 1}{e^{z_{12}}(t_f - t)}, \frac{e^{z_{13}} - 1}{e^{z_{13}}(t_f - t)} \right\} K_1,$$

$K_1 = [k_{11}, k_{12}, k_{13}]^T$ with k_{11}, k_{12} and k_{13} are positive constants.

Using backstepping method, the second error vector is defined as

$$z_2 = [z_{21}, z_{22}, z_{23}]^T = v - v_d - \bar{\alpha}, \tag{23}$$

where $\bar{\alpha} = [0, \tanh(\alpha), 0]^T$, α is the auxiliary variable used to deal with the underactuated problem.

Taking the time derivative of (23), we have

$$\dot{z}_2 = M^{-1}\tau + \zeta - \dot{v}_d - \dot{\bar{\alpha}}. \tag{24}$$

The stabilizing function is found to be

$$M^{-1}\tau - \dot{\bar{\alpha}} = \begin{cases} -z_1 - \zeta + \dot{\nu}_d - \xi_2, & t_0 \le t \le t_f \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

where

$$\xi_2 = \operatorname{diag}\left\{ \frac{e^{z_{21}} - 1}{e^{z_{21}}(t_f - t)}, \frac{e^{z_{22}} - 1}{e^{z_{22}}(t_f - t)}, \frac{e^{z_{23}} - 1}{e^{z_{23}}(t_f - t)} \right\} K_2$$

with $K_2 = [k_{21}, k_{22}, k_{23}]^T$, k_{21}, k_{22} and k_{23} are positive constants.

Theorem 3: Consider the ship model (7) with the known dynamics ζ, the proposed controller (25) could guarantees all signals in the closed-loop system are bounded and converges at $t \le tf$.

Proof: Consider the Lyapunov function

$$V_1 = z_1^T z_1, \tag{26}$$

along with (21), the time derivative of V_1 is

$$\dot{V}_1 = 2z_1^T\left(\nu - J^T(\psi)\dot{\eta}_d\right), \tag{27}$$

using (22), $\dot{V}_1$ is given as

$$\dot{V}_1 = 2z_1^T(v - v_d - \xi_1). \tag{28}$$

The composite Lyapunov function is chosen as

$$V_2 = V_1 + z_2^T z_2. \tag{29}$$

Using (23), (24) and (28), the time derivative of V_2 is given

$$\begin{aligned} \dot{V}_2 &= 2z_1^T(v - v_d - \xi_1) + 2z_2^T\left(M^{-1}\tau + \zeta - \dot{v}_d - \dot{\bar{\alpha}}\right) = \\ &= 2z_1^T(\nu_2 - \xi_1 - \bar{\alpha}) + 2z_2^T\left(M^{-1}\tau - \dot{\bar{\alpha}} + \zeta - \dot{v}_d\right) \end{aligned}. \tag{30}$$

Substituting (25) into (30), we have

$$\dot{V}_2 = -2z_1^T\xi_1 + 2z_1^T\bar{\alpha} - 2z_2^T\xi_2. \tag{31}$$

Then,

$$\begin{aligned} \dot{V}_2 &= -2z_1^T\xi_1 + 2z_1^T\bar{\alpha} - 2z_2^T\xi_2 = \\ &= -\frac{2k_{11}z_{11}\left(e^{z_{11}} - 1\right)}{e^{z_{11}}(t_f - t)} - \frac{2k_{12}z_{12}\left(e^{z_{12}} - 1\right)}{e^{z_{12}}(t_f - t)} - \frac{2k_{13}z_{13}\left(e^{z_{13}} - 1\right)}{e^{z_{13}}(t_f - t)} + 2z_{12}\tanh(\alpha) \\ &\quad - \frac{2k_{21}z_{21}\left(e^{z_{21}} - 1\right)}{e^{z_{21}}(t_f - t)} - \frac{2k_{22}z_{22}\left(e^{z_{22}} - 1\right)}{e^{z_{22}}(t_f - t)} - \frac{2k_{23}z_{23}\left(e^{z_{23}} - 1\right)}{e^{z_{23}}(t_f - t)} \\ &\le -\frac{2k_{11}|z_{11}|\left(e^{|z_{11}|} - 1\right)}{e^{|z_{11}|}(t_f - t)} - \frac{2k_{12}|z_{12}|\left(e^{|z_{12}|} - 1\right)}{e^{|z_{12}|}(t_f - t)} - \frac{2k_{13}|z_{13}|\left(e^{|z_{13}|} - 1\right)}{e^{|z_{13}|}(t_f - t)} \\ &\quad + 2|z_{12}| - \frac{2k_{21}|z_{21}|\left(e^{|z_{21}|} - 1\right)}{e^{|z_{21}|}(t_f - t)} - \frac{2k_{22}|z_{22}|\left(e^{|z_{22}|} - 1\right)}{e^{|z_{22}|}(t_f - t)} - \frac{2k_{23}|z_{23}|\left(e^{|z_{23}|} - 1\right)}{e^{|z_{23}|}(t_f - t)} \\ &\le -\frac{2k_{11}|z_{11}|\left(e^{|z_{11}|} - 1\right)}{e^{|z_{11}|}(t_f - t)} - \frac{2k_{13}|z_{13}|\left(e^{|z_{13}|} - 1\right)}{e^{|z_{13}|}(t_f - t)} \\ &\quad - \frac{2k_{12}|z_{12}|(k_{12} - \bar{t})\left(e^{|z_{12}|} - \frac{k_{12}}{k_{12} - \bar{t}}\right)}{e^{|z_{12}|}(t_f - t)} - \frac{2k_{21}|z_{21}|\left(e^{|z_{21}|} - 1\right)}{e^{|z_{21}|}(t_f - t)} \\ &\quad - \frac{2k_{22}|z_{22}|\left(e^{|z_{22}|} - 1\right)}{e^{|z_{22}|}(t_f - t)} - \frac{2k_{23}|z_{23}|\left(e^{|z_{23}|} - 1\right)}{e^{|z_{23}|}(t_f - t)} \end{aligned}. \tag{32}$$

From (29), it can see that

$$V_2 \ge z_1^T z_1 \Rightarrow \sqrt{V_2} \ge |z_{11}|, \sqrt{V_2} \ge |z_{12}|, \sqrt{V_2} \ge |z_{13}|, \tag{33}$$

$$V_2 \ge z_2^T z_2 \Rightarrow \sqrt{V_2} \ge |z_{21}|, \sqrt{V_2} \ge |z_{22}|, \sqrt{V_2} \ge |z_{23}|. \tag{34}$$

From (33) and (34), it is concluded that

$$\dot{V}_2 \le -\frac{2\bar{k}\sqrt{V_2}\left(e^{\sqrt{V_2}} - \bar{k}_{12}\right)}{e^{\sqrt{V_2}}(t_f - t)}, \tag{35}$$

where $\bar{k} = k_{11} + k_{12} + k_{13} - \bar{t} + k_{21} + k_{22} + k_{23}$, $\bar{k}_{12} = \frac{k_{12}}{k_{12} - \bar{t}}$

Here, chose a positive constant k_{12} satisfies $k_{12} > \bar{t}$ then $\frac{k_{12}}{k_{12} - \bar{t}} > 1$. Therefore (35) is reasonable.

From (35), we have $\frac{\dot{V}_2}{2\sqrt{V_2}} \le -\frac{\bar{k}\left(e^{\sqrt{V_2}} - \bar{k}_{12}\right)}{e^{\sqrt{V_2}}(t_f - t)}$. Define $\varepsilon = \sqrt{V_2}$,

then $\dot{\varepsilon} = \frac{\dot{V}_2}{2\sqrt{V_2}} \le -\frac{\bar{k}\left(e^{\sqrt{V_2}} - \bar{k}_{12}\right)}{e^{\sqrt{V_2}}(t_f - t)}$, $\dot{\varepsilon} \le -\frac{\bar{k}\left(e^{\varepsilon} - \bar{k}_{12}\right)}{e^{\varepsilon}(t_f - t)}$ can be given. Follow the conclusion of theorem 1, one obtains $\varepsilon = 0$, $\forall t > t_f$. Further, since $\varepsilon = \dot{V}_2$, V_2 is a constant, $\forall t > t_f$. Thus, from (29), we have z_1 and z_2 are constants, $\forall t > t_f$.

Controller design based on Observer

The controller has been designed completely, however, the term ζ is unknown. observer (12) is used to estimate the unknown term, and the estimate $\hat{\zeta}$ is combined with controller design. The controller based on the observer is designed as

$$M^{-1}\tau - \dot{\bar{\alpha}} = \begin{cases} -z_1 - \hat{\zeta} + \dot{v}_d - \xi_2 - \frac{1}{2}z_2, & t_0 \le t \le t_f \\ 0, & \text{otherwise} \end{cases}, \tag{36}$$

Theorem 3: Consider the ship model (7) consisting of the unknown dynamics ζ, the observer (11), the proposed controller (36) could guarantee all signals in the closed-loop system are bounded and converge within $t = t_f$.

Proof: Consider the total Lyapunov function as

$$V_3 = V_2 + V_o, \quad (37)$$

along with (15) and (30), the derivative of V3 is

$$\dot{V}_3 \le 2z_1^T(z_2 - \xi_1 + \bar{\alpha}) + 2z_2^T(M^{-1}\tau - \dot{\bar{\alpha}} - \zeta - \dot{v}_d) - \frac{2\bar{k}_0\sqrt{V_0}\left(e^{\sqrt{V_0}} - \hat{k}_t\right)}{e^{\sqrt{V_0}}(t_f - t)}. \quad (38)$$

By applying the controller to (38),

$$\dot{V}_3 \le -2z_1^T\xi_1 + 2z_1^T\bar{\alpha} - 2z_2^T\xi_2 + 2z_2^T\left(\tilde{\xi} - \frac{1}{2}z_2\right) - \frac{2\bar{k}_0\sqrt{V_0}\left(e^{\sqrt{V_0}} - \bar{k}_t\right)}{e^{\sqrt{V_0}}(t_f - t)}. \quad (39)$$

Using Young's inequality and (15),

$$2z_2^T\left(\tilde{\xi} - \frac{1}{2}z_2\right) \le \tilde{\xi}^T\tilde{\xi} \le \dot{\tilde{v}}^T\dot{\tilde{v}} \le \left[\frac{2\bar{k}_0\left(e^{\sqrt{V_0}} - \bar{k}_t\right)}{e^{\sqrt{V_0}}(t_f - t)}\right]^2 = \varepsilon. \quad (40)$$

Using (35) and (40),

$$\dot{V}_3 \le -\frac{2\bar{k}\sqrt{V_2}\left(e^{\sqrt{V_2}} - \bar{k}_{12}\right)}{e^{\sqrt{V_2}}(t_f - t)} - \frac{2\bar{k}_0\sqrt{V_0}\left(e^{\sqrt{V_0}} - \bar{k}_t\right)}{e^{\sqrt{V_0}}(t_f - t)} + \varepsilon. \quad (41)$$

Own to the definition of V3 and V0, it can be seen that $\sqrt{V_3} \ge \sqrt{V_2}$, $\sqrt{V_3} \ge \sqrt{V_0}$. (41) can be written as

$$\dot{V}_3 \le -\frac{2\bar{k}_z\sqrt{V_3}\left(e^{\sqrt{V_3}} - \bar{k}_l\right)}{e^{\sqrt{V_3}}(t_f - t)} + \varepsilon, \quad (42)$$

where $\bar{k}_z = 2\bar{k} + 2\bar{k}_0$, $\bar{k}_l = \max\{\bar{k}_{12}, \bar{k}_t\}$.

From (42) we have $\frac{\dot{V}_3}{2\sqrt{V_3}} \le -\frac{\bar{k}_z\left(e\sqrt{V_3} - \bar{k}_l\right)}{e\sqrt{V_3}(t_f - t)} + \frac{\varepsilon}{2\sqrt{V_3}}$.

Define $\varepsilon_t = \sqrt{V_3}$,

then $\dot{\varepsilon}_t = \frac{\dot{V}_3}{2\sqrt{V_3}} \le \frac{\bar{k}_z\left(e\sqrt{V_3} - \bar{k}_l\right)}{e\sqrt{V_3}(t_f - t)} + \frac{\varepsilon}{2\sqrt{V_3}}$,

$$\dot{\varepsilon}_t \le -\frac{\bar{k}_z\left(e^{\varepsilon_t} - \bar{k}_l\right)}{e^{\varepsilon_t}(t_f - t)} + \frac{\varepsilon}{2\sqrt{V_3}}$$

can be given. Note that ε is bounded, because t, tf, $\bar{k}_0$ and $\bar{k}_t$ are bounded, $\varepsilon \to 2\bar{k}_0$ if $V_0 \to \pm\infty$, $\varepsilon = 0$, if $t = t_f$. Obviously there is an equilibrium point at $t = t_f$. Suppose that there is another positive equilibrium point at $t \ne t_f$, it means that $\varepsilon \ne 0$, then V_3 can not maintain a steady state. Apparently contradicting the hypothesis that there is another positive equilibrium point at $t \ne t_f$. Therefore, at $t = t_f$, $\dot{V}_3 = 0$ can be sure by choosing appropriate parameters $\bar{k}_z$ and $\bar{k}_l$.

Then the stability of the system can be established. Along with the conclusion of theorem 1, it can be observed that the system (7) can be stabilized within a time *tf*.

Simulation results

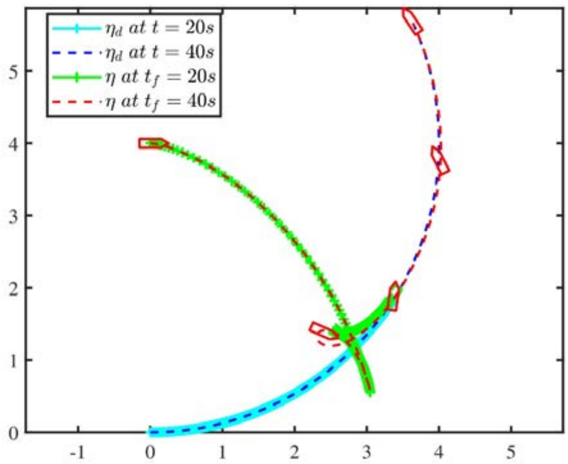

Fig. 2. Trajectory tracking of the desired path using the proposed controller

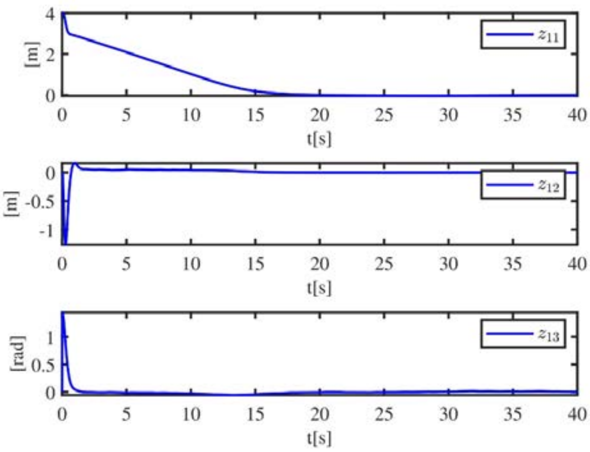

Fig. 3. Time evolution of the trajectory tracking errors (*tf* = 40s)

To verify the validity of controllers, the parameters of vessels are used and taken from the works of [36]. The simulation time is set to 20s and 40s, respectively. The parameters of virtual ship are chosen as u_d = 0.2m/s, v_d = 0m/s, rd = 0.05rad/s. Note that if controller gain *k* is constant, when $t \to t_f$, it causes a huge gain $\frac{k}{t_f - t}$, and may lead to deterioration of system performance. To avoid this phenomena, the parameters of controllers are chosen as $k_{11} = 8\bar{t} + 0.01$, $k_{12} = 8\bar{t} + 0.01$, $k_{13} = 8\bar{t} + 0.01$, $k_{21} = 8\bar{t} + 0.01$, $k_{22} = 8\bar{t} + 0.01$ and $k_{23} = 8\bar{t} + 0.01$. The parameters of observers are chosen as $k_{01} = 8\bar{t} + 0.01$, $k_{02} = 8\bar{t} + 0.01$, $k_{03} = 8\bar{t} + 0.01$. The convergence time is set as t_f = 20 and t_f = 40. The look-ahead distance is set as Δ = 1.2.

As shown in Figure 2, it can see that the vessel is enabled to track successfully the virtual ship when t_f are

20s and 40s. Figure 3 presents that the trajectory tracking errors are convergent within a time tf 40s. It is worth noting that $z_{13} = \psi - \psi_a \neq \psi - \psi_d$. Figure 4 shows the time evolution of velocities. Figure 5 and Figure 6 shows that the proposed observer gives an accurate estimation of the unknown dynamics. Figure 7 shows that control inputs of the vessel under the proposed controller combined with the proposed observer.

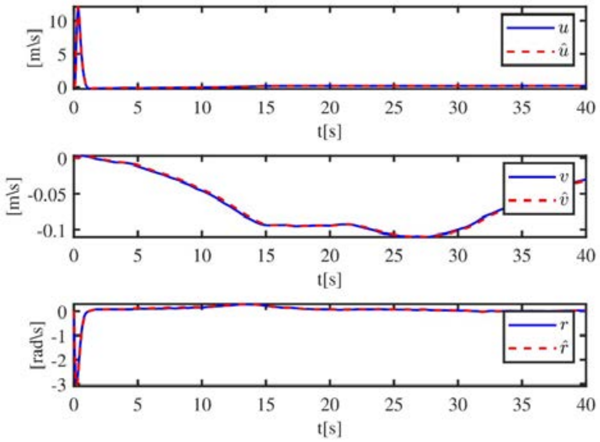

Fig. 4. Time evolution of velocities (t_f= 40s)

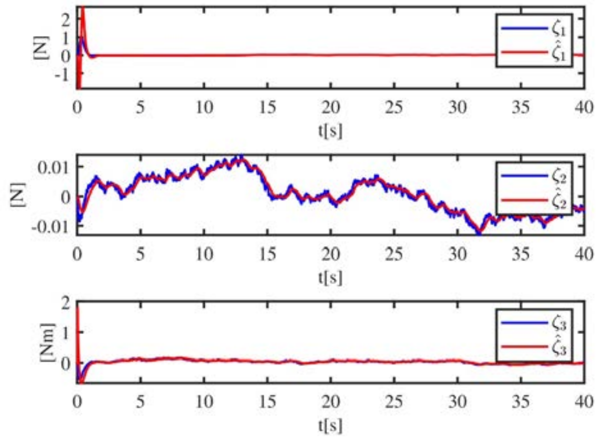

Fig. 5. Time evolution of the unknown dynamics (t_f= 40s)

Figure 2–7 shows that the vessel is enable to track successfully the virtual ship within a time tf. Figure 6 shows that the precise estimations of unknown dynamics can be quickly given by using the proposed observer. It can be sure that after some time moment a controller is using precise estimation.

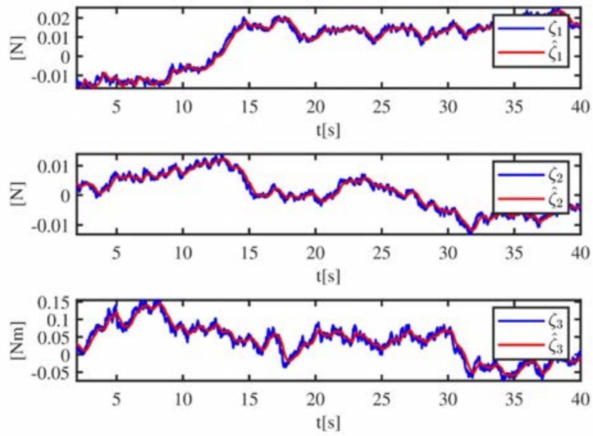

Fig. 6. Local enlarged drawing: time evolution of the unknown dynamics (t_f= 40s)

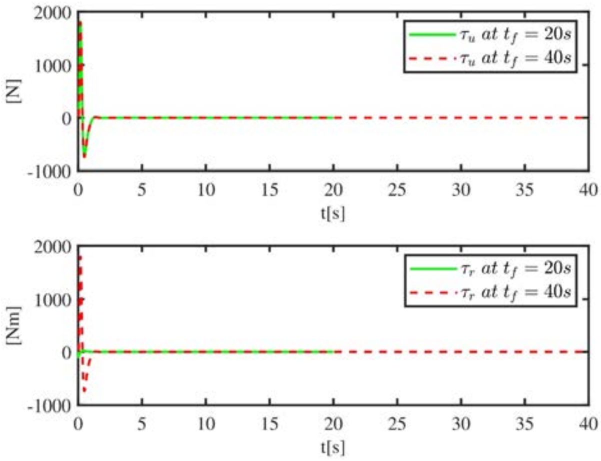

Fig. 7. Time evolution of control inputs (t_f=40s)

CONCLUSION

In this paper, arbitrary convergence time is introduced in trajectory tracking controller of DP vessel. A observer with arbitrary convergence time is proposed, and a controller is designed based on the observer. The proposed controller ensure not only fixed time convergence, but also ensure that convergence time is available to choose.

REFERENCES

[1] Lei T, Wen Y, Yu Y, et al. (2024) Predictive trajectory tracking control for the USV in networked environments with communication constraints. Ocean Engineering, 298117185.

[2] Li B, Gao X, Huang H, et al. (2024) Improved adaptive twisting sliding mode control for trajectory tracking of an AUV subject to uncertainties. Ocean Engineering, 297116204.

[3] Weijun Z, Zishi X, Yongxin W, et al. (2023) Energy-based trajectory tracking control of under-actuated unmanned surface vessels. Ocean Engineering, no. 288(P2):

[4] Li J, Xiang X, Zhang Q, et al. (2024) Robust practical prescribed time trajectory tracking of USV with guaranteed performance. Ocean Engineering, 302117622.

[5] Peng H, Huang B, Jin M, et al. (2024) Distributed finite-time bearing-based formation control for underactuated surface vessels with Levant differentiator. ISA transactions, 147239-251.

[6] Mwaffo V. (2024) Formation control and collision avoidance of unmanned water surface vehicles in maritime environments. Journal of the Franklin Institute, no. 361(7): 106791.

[7] Liu Y, Lin X, Liu Y, et al. (2024) Affine formation maneuver control of underactuated surface vessels: Guaranteed safety under moving obstacles in narrow channels. Ocean Engineering, 303117721.

[8] Xia Guoqing, Xia Xiaoming, Zheng Zhi. (2022) Formation tracking control for underactuated surface vehicles with actuator magnitude and rate saturations. Ocean Engineering, 260: 111965.

[9] Xia Guoqing, Xia Xiaoming, Zhao Bo, Sun Chuang, Sun Xianxin. (2020) Distributed Tracking Control for Connectivity-Preserving and Collision-Avoiding Formation Tracking of Underactuated Surface Vessels with Input Saturation. Applied sciences, no. 10(10): pp. 3372.

[10] Xia Guoqing, Sun Xianxin, Xia Xiaoming. (2022) Swarm Control for Connectivity-Preserving and Collision-Avoiding Unmanned Surface Vehicles Subject to Multiple Constraints. Journal of Marine Science and Engineering, no. 10(6): pp. 827.

[11] Xia Guoqing, Xia Xiaoming, Sun Xianxin. (2021) Formation control with collision avoidance for underactuated surface vehicles. Asian Journal of Control, no. 24(5): pp. 2244-2257.

[12] Benwei Zhang, Guoqing Xia. (2023) Output Feedback Tracking Control for Vessel with Collision-Avoidance and Performance Constraints. Appl. Sci, no. 13, pp. 11285.

[13] Zhang, B.; Xia, G. (2023) Output Feedback Tracking Control with Collision Avoidance for Dynamic Positioning Vessel under Input Constraint. Mar. Sci. Eng, no. 11, pp. 811.

[14] Zhang X, Yang X, Huang C, et al. (2024) Adaptive fuzzy finite-time PID backstepping control for chaotic systems with full states constraints and unmodeled dynamics. Information Sciences, 661120148.

[15] K Li, K Feng, Y Li. (2024) "Fuzzy Adaptive Fault-Tolerant Formation Control for USVs With Intermittent Actuator Faults," IEEE Transactions on Intelligent Vehicles, vol. 9, no.3, pp. 4445-4455.

[16] Xing G W, Yi D, Tezdogan T, et al. (2023) Adaptive neural network and extended state observer-based non-singular terminal sliding mode tracking control for an underactuated USV with unknown uncertainties. Applied Ocean Research, no. 135.

[17] Xia, G.; Xia, X.; Zhao, Bo.; Sun, C.; Sun X. (2020) Distributed Tracking Control for Connectivity-Preserving and Collision-Avoiding Formation Tracking of Underactuated Surface Vessels with Input Saturation. Applied sciences, no. 10, pp. 3372.

[18] Xia, G.; Sun, C.; Zhao, B.; Xia, X.; Sun, X. (2019) Neuroadaptive Distributed Output Feedback Tracking Control for Multiple Marine Surface Vessels With Input and Output Constraints. IEEE Access, no. 7, pp. 123076-123085.

[19] Xia, G.; Xia, X.; Zhao, Bo.; Sun C.; Sun X. (2020) Event-Triggered Controller Design for Autopilot with Input Saturation. Mathematical Problems in Engineering, 2020, pp. 1-7.

[20] Xia Guoqing, Xia Xiaoming, Zhao Bo, Sun Chuang, Sun Xianxin. (2020) Event-Triggered Controller Design for Autopilot with Input Saturation. Mathematical Problems in Engineering, 2020, pp. 1-7.

[21] Xia X, Xia C, Sun C. (2024) Distributed formation tracking control of underactuated surface vehicles based on event-trigged control. Transactions of the Institute of Measurement and Control, no. 46(4): pp. 729-740.

[22] Xia X, Yang Z, Yang T. (2023) Leader-Follower Formation Tracking Control of Underactuated Surface Vehicles Based on Event-Trigged Control. Applied Sciences, no. 13(12):

[23] Wang, N.; Qian, C.; Sun, J.; Liu, Y. (2016) Adaptive Robust Finite-Time Trajectory Tracking Control of Fully Actuated Marine Surface Vehicles. IEEE Transactions on Control Systems Technology, no. 24, pp. 1454-1452.

[24] Xia, G.; Sun, C.; Zhao, B.; Xia, X.; Sun, X. (2019) Cooperative control of multiple dynamic positioning vessels with input saturation based on finite-time disturbance observe. International Journal of Control, Automation and Systems, no. 17, pp. 370-379.

[25] Do, K.D. (2011) Practical formation control of multiple underactuated ships with limited sensing ranges. Robotics and Autonomous Systems, no. 59, pp. 457-471.

[26] Park, B.S.; Kwon, J.W.; Kim, H. (2017) Neural network-based output feedback control for reference tracking of underactuated surface vessels. Automatica, no. 77, pp. 353-359.

[27] Belleter, D.J.W.; Paliotta, C.; Maggiore, M.; Pettersen, K.Y. (2016) Path Following for Underactuated Marine Vessels. IFAC-PapersOnLine, no. 49, pp. 588-593.

[28] Ghommam, J.; Iftekhar, L.; Saad, M. P. (2019) Adaptive Finite Time Path-Following Control of Underactuated Surface Vehicle With Collision Avoidance. Journal of Dynamic Systems, Measurement, and Control, no. 141.

[29] Chen, L.; Cui, R.; Yang, C.; Yan, W. (2019) Adaptive Neural Network Control of Underactuated Surface Vessels With Guaranteed Transient Performance: Theory and Ex– perimental Results. IEEE Transactions on Industrial Electronics, no. 67, pp. 4024-4035.

[30] Zheng, Z.; Ruan, L.; Zhu, M.; Guo, X. (2020) Reinforcement learning control for underactuated surface vessel with output error constraints and uncertainties. IEEE Trans. Autom. Control, no. 399, pp. 479-490.

[31] Li,T.; Zhao, R.; Chen, C.L.P.; Fang, L.; Liu, C. (2019) Finite-Time Formation Control of Under-Actuated Ships Using Nonlinear Sliding Mode Control. IEEE Transactions on Cybernetics, no. 67, pp. 4889-4899.

[32] Zheng, J.; Yang, G. (2018) Fault-Tolerant Fixed-Time Trajectory Tracking Control of Autonomous Surface Vessels With Specified Accuracy. IEEE Transactions on Industrial Electronics, no. 48, pp. 3243-3253.

[33] Pal, A.K.; Kamal, S.; Nagar, S.K.; Bandyopadhyay, B.; Fridman, L. (2020) Design of controllers with arbitrary convergence time. Automatica, no. 112.

[34] Fossen, T.I.; Strand, J.P. (1999) Passive nonlinear observer design for ships using Lyapunov methods: Full-Scale experiments with a supply vessel. Automatica, no. 35, pp. 3-16.

[35] Belleter, D.; Maghenem, M.A.; Paliotta, C.; Pettersen, K.Y. (2019) Observer based path following for underactuated marine vessels in the presence of ocean currents: A global approach. Automatica, no. 100, pp.123-134.

[36] Gao, Z.; Guo, G. (2018) Command filtered path tracking control of saturated ASVs based on time-varying disturbance observer. Asian journal of control, no. 22, pp. 1197-1210.

[37] Skjetne, R.; Fossen, T.I.; Kokotovi, P. V. (2005) Adaptive maneuvering, with experiments, for a model ship in a marine control laboratory. Automatica, no. 41, pp. 289-298.

СПИСОК ВИКОРИСТАНОЇ ЛІТЕРАТУРИ

[1] Lei T, Wen Y, Yu Y, et al. (2024) Predictive trajectory tracking control for the USV in networked environments with communication constraints. Ocean Engineering, 298117185.

[2] Li B, Gao X, Huang H, et al. (2024) Improved adaptive twisting sliding mode control for trajectory tracking of an AUV subject to uncertainties. Ocean Engineering, 297116204.

[3] Weijun Z, Zishi X, Yongxin W, et al. (2023) Energy-based trajectory tracking control of under-actuated unmanned surface vessels. Ocean Engineering, no. 288(P2):

[4] Li J, Xiang X, Zhang Q, et al. (2024) Robust practical prescribed time trajectory tracking of USV with guaranteed performance. Ocean Engineering, 302117622.

[5] Peng H, Huang B, Jin M, et al. (2024) Distributed finite-time bearing-based formation control for underactuated surface vessels with Levant differentiator. ISA transactions, 147239-251.

[6] Mwaffo V. (2024) Formation control and collision avoidance of unmanned water surface vehicles in maritime environments. Journal of the Franklin Institute, no. 361(7): 106791.

[7] Liu Y, Lin X, Liu Y, et al. (2024) Affine formation maneuver control of underactuated surface vessels: Guaranteed safety under moving obstacles in narrow channels. Ocean Engineering, 303117721.

[8] Xia Guoqing, Xia Xiaoming, Zheng Zhi. (2022) Formation tracking control for underactuated surface vehicles with actuator magnitude and rate saturations. Ocean Engineering, 260: 111965.

[9] Xia Guoqing, Xia Xiaoming, Zhao Bo, Sun Chuang, Sun Xianxin. (2020) Distributed Tracking Control for Connectivity-Preserving and Collision-Avoiding Formation Tracking of Underactuated Surface Vessels with Input Saturation. Applied sciences, no. 10(10): pp. 3372.

[10] Xia Guoqing, Sun Xianxin, Xia Xiaoming. (2022) Swarm Control for Connectivity-Preserving and Collision-Avoiding Unmanned Surface Vehicles Subject to Multiple Constraints. Journal of Marine Science and Engineering, no. 10(6): pp. 827.

[11] Xia Guoqing, Xia Xiaoming, Sun Xianxin. (2021) Formation control with collision avoidance for underactuated surface vehicles. Asian Journal of Control, no. 24(5): pp. 2244-2257.

[12] Benwei Zhang, Guoqing Xia. (2023) Output Feedback Tracking Control for Vessel with Collision-Avoidance and Performance Constraints. Appl. Sci, no. 13, pp. 11285.

[13] Zhang, B.; Xia, G. (2023) Output Feedback Tracking Control with Collision Avoidance for Dynamic Positioning Vessel under Input Constraint. Mar. Sci. Eng, no. 11, pp. 811.

[14] Zhang X, Yang X, Huang C, et al. (2024) Adaptive fuzzy finite-time PID backstepping control for chaotic systems with full states constraints and unmodeled dynamics. Information Sciences, 661120148.

[15] K Li, K Feng, Y Li. (2024) "Fuzzy Adaptive Fault-Tolerant Formation Control for USVs With Intermittent Actuator Faults," IEEE Transactions on Intelligent Vehicles, vol. 9, no.3, pp. 4445-4455.

[16] Xing G W, Yi D, Tezdogan T, et al. (2023) Adaptive neural network and extended state observer-based non-singular terminal sliding mode tracking control for an underactuated USV with unknown uncertainties. Applied Ocean Research, no. 135.

[17] Xia, G.; Xia, X.; Zhao, Bo.; Sun, C.; Sun X. (2020) Distributed Tracking Control for Connectivity-Preserving and Collision-Avoiding Formation Tracking of Underactuated Surface Vessels with Input Saturation. Applied sciences, no. 10, pp. 3372.

[18] Xia, G.; Sun, C.; Zhao, B.; Xia, X.; Sun, X. (2019) Neuroadaptive Distributed Output Feedback Tracking Control for Multiple Marine Surface Vessels With Input and Output Constraints. IEEE Access, no. 7, pp. 123076-123085.

[19] Xia, G.; Xia, X.; Zhao, Bo.; Sun C.; Sun X. (2020) Event-Triggered Controller Design for Autopilot with Input Saturation. Mathematical Problems in Engineering, 2020, pp. 1-7.

[20] Xia Guoqing, Xia Xiaoming, Zhao Bo, Sun Chuang, Sun Xianxin. (2020) Event-Triggered Controller Design for Autopilot with Input Saturation. Mathematical Problems in Engineering, 2020, pp. 1-7.

[21] Xia X, Xia C, Sun C. (2024) Distributed formation tracking control of underactuated surface vehicles based on event-trigged control. Transactions of the Institute of Measurement and Control, no. 46(4): pp. 729-740.

[22] Xia X, Yang Z, Yang T. (2023) Leader-Follower Formation Tracking Control of Underactuated Surface Vehicles Based on Event-Trigged Control. Applied Sciences, no. 13(12):

[23] Wang, N.; Qian, C.; Sun, J.; Liu, Y. (2016) Adaptive Robust Finite-Time Trajectory Tracking Control of Fully Actuated Marine Surface Vehicles. IEEE Transactions on Control Systems Technology, no. 24, pp. 1454-1452.

[24] Xia, G.; Sun, C.; Zhao, B.; Xia, X.; Sun, X. (2019) Cooperative control of multiple dynamic positioning vessels with input saturation based on finite-time disturbance observe. International Journal of Control, Automation and Systems, no. 17, pp. 370-379.

[25] Do, K.D. (2011) Practical formation control of multiple underactuated ships with limited sensing ranges. Robotics and Autonomous Systems, no. 59, pp. 457-471.

[26] Park, B.S.; Kwon, J.W.; Kim, H. (2017) Neural network-based output feedback control for reference tracking of underactuated surface vessels. Automatica, no. 77, pp. 353-359.

[27] Belleter, D.J.W.; Paliotta, C.; Maggiore, M.; Pettersen, K.Y. (2016) Path Following for Underactuated Marine Vessels. IFAC-PapersOnLine, no. 49, pp. 588-593.

[28] Ghommam, J.; Iftekhar, L.; Saad, M. P. (2019) Adaptive Finite Time Path-Following Control of Underactuated Surface Vehicle With Collision Avoidance. Journal of Dynamic Systems, Measurement, and Control, no. 141.

[29] Chen, L.; Cui, R.; Yang, C.; Yan, W. (2019) Adaptive Neural Network Control of Underactuated Surface Vessels With Guaranteed Transient Performance: Theory and Ex– perimental Results. IEEE Transactions on Industrial Electronics, no. 67, pp. 4024-4035.

[30] Zheng, Z.; Ruan, L.; Zhu, M.; Guo, X. (2020) Reinforcement learning control for underactuated surface vessel with output error constraints and uncertainties. IEEE Trans. Autom. Control, no. 399, pp. 479-490.

[31] Li,T.; Zhao, R.; Chen, C.L.P.; Fang, L.; Liu, C. (2019) Finite-Time Formation Control of Under-Actuated Ships Using Nonlinear Sliding Mode Control. IEEE Transactions on Cybernetics, no. 67, pp. 4889-4899.

[32] Zheng, J.; Yang, G. (2018) Fault-Tolerant Fixed-Time Trajectory Tracking Control of Autonomous Surface Vessels With Specified Accuracy. IEEE Transactions on Industrial Electronics, no. 48, pp. 3243-3253.

[33] Pal, A.K.; Kamal, S.; Nagar, S.K.; Bandyopadhyay, B.; Fridman, L. (2020) Design of controllers with arbitrary convergence time. Automatica, no. 112.

[34] Fossen, T.I.; Strand, J.P. (1999) Passive nonlinear observer design for ships using Lyapunov methods: Full-Scale experiments with a supply vessel. Automatica, no. 35, pp. 3-16.

[35] Belleter, D.; Maghenem, M.A.; Paliotta, C.; Pettersen, K.Y. (2019) Observer based path following for underactuated marine vessels in the presence of ocean currents: A global approach. Automatica, no. 100, pp.123-134.

[36] Gao, Z.; Guo, G. (2018) Command filtered path tracking control of saturated ASVs based on time-varying disturbance observer. Asian journal of control, no. 22, pp. 1197-1210.

[37] Skjetne, R.; Fossen, T.I.; Kokotovi, P. V. (2005) Adaptive maneuvering, with experiments, for a model ship in a marine control laboratory. Automatica, no. 41, pp. 289-298.

Дата надходження статті до редакції: 12.10.2024
Дата затвердження статті до друку: 02.11.2024