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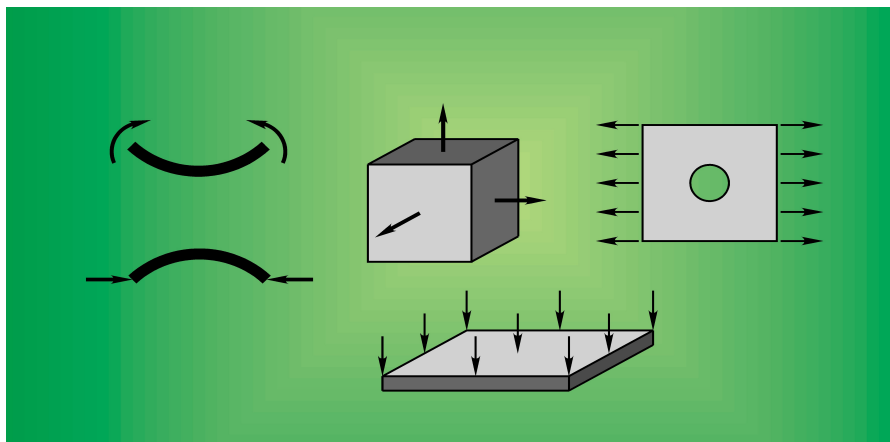
В. М. СОКОВ

**Lectures for the Subject
"Structural Mechanics for Ship 1"**

Навчальний посібник

Рекомендовано Методичною радою НУК

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Представлено лекційний та практичний матеріал з курсів "Теорія пружності" та "Будівельна механіка" англійською мовою, на який відведено 64 години. В рамках курсу "Теорія пружності" розглянуто класичну лінійну теорію пружності. В рамках курсу "Будівельна механіка" розглянуті наступні питання: згин і стійкість прямих стержнів та їх систем; згин і стійкість жорстких прямокутних пластин. Наведені початкові відомості про числові методи в теорії пружності та будівельній механіці. Лекційна частина містить 32 години. Практична частина містить 11 задач і також запланована в обсязі 32 годин.

Призначено для початкового ознайомлення студентів з технічними дисциплінами англійською мовою.

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СОКОВ Валерій Миколайович

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"Structural Mechanics for Ship 1"**

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SYLLABUS FOR THE DISCIPLINE "STRUCTURAL MECHANICS OF SHIP 1"

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3	<i>Lecture 3. Major Equations and Relations of the Theory of Elasticity. Direct and Contrary Tasks</i> § 3.1. Equilibrium Equations of Elementary Tetrahedron § 3.2. Equilibrium Equations of Elementary Parallelepiped	2

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3	§ 3.3. Hook's Law for Isotropic Bodies § 3.4. Cauchy Geometrical Relations Between Strains and Displacements. Compatibility Conditions § 3.5. Definition of Displacements for Known Strains § 3.6. Direct and Contrary Problems of the Theory of Elasticity Saint-Venant Principle	2
4	Lecture 4. Exploration of the Strain-Stress State in Point Of Elastic Body. Strength Theories § 4.1. Principal Planes and Principal Stresses. Stress Ellipsoid § 4.2. Maximum Shear Stresses and Its Planes § 4.3. Principal Linear and Maximum Shear (Angular) Strains § 4.4. Mechanical Properties of Strength and Plasticity and Ways of Definition of It § 4.5. Basic (Classic) Theories of Strength and Plasticity. Allowable Stresses and Conditions of Strength	2
5	Lecture 5. Basic Equations and Relations for Flat Strain-Stress State. Stress Concentration in Plates Near Holes (Necklines) § 5.1. Basic Concepts of Flat Strain-Stress State. Major Equations and Relations for Flat Strain-Stress State § 5.2 Schemes of Solution of Direct Problem of Flat Strain-Stress State. Stress Function § 5.3. Some Partial Analytic Solutions for the Problem of Flat Strain-Stress State and Its Practical Usage § 5.4. The Problem of Flat Strain-Stress State in Polar Coordinates. Stress Concentrations in Plates With Holes	2
6	PART II – BENDING OF RODS AND ROD SYSTEMS Lecture 6. Basic Determinations, Concepts and Differential Relations of Technical Theory of Flat Orthogonal Bending of Direct Beams	2

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7	Lecture 7. Flat Orthogonal Bending of Beams with One Span § 7.1. Start Parameter Method § 7.2. Superposition Principle. Usage of Reference Books for Calculations of Elements of Bending. Strength Condition. The Coefficient of "Support Pair"	2
8	Lecture 8. Methods of Resolving of Static Uncertainties. Bending of Beams with Many Spans. Considering Of Shear Strains for Definition of Kinematic Parameters of Bending § 8.1. Methods of Resolving of Static Uncertainties § 8.2. Bending of Beams with Many Spans § 8.3. Consideration of Shear Strains for Deflections and Rotation Angles in Beams	2
9	Lecture 9. Bending of Frames of Ship Hull § 9.1. Classification of Frames in Composition of Ship Hull § 9.2. The Method of Angular Displacements	2
10	Lecture 10. Bending of Beams on Elastic Foundation § 10.1. Practical Tasks Which Provide Using the Model of Beam on Elastic Foundation	2
11	Lecture 11. Bending of Flat Grillages § 11.1. Classification of Grillages as Rod Systems § 11.2. The Navier Method for Calculation of Grillages	2

Syllabus for the discipline "Structural Mechanics of Ship 1"

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13	Lecture 13. Stability of Rods and Its Systems § 13.1. Basic Concepts and Statements of Stability of Elastic Systems § 13.2. Stability of Rods with One Span. The Common Algorithm of Definition of Eulerian Collapse Forces for Beams with One Span § 13.3. Consideration of Deviation on Linear Hook's Law § 13.4. Stability of Grillages	2
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# of lecture	Themes of the lectures and its content	Number of hours
16	Lecture 16. Modern Numeric Methods of Solutions for the Problems of the Theory of Elasticity and Ship Structural Mechanics § 16.1. The General Description of Structures Strength Analysis Methods § 16.2. The Finite-Difference Method § 16.3. The Finite Element Method	2
	Total number of hours for lectures	32

PRACTICE

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4	Task#4. Calculation of beams with many spans under bending for strength	4
5	Task#5. Calculation of frames under bending for strength	4
6	Task#6. Calculation of beams on elastic foundation under bending for strength	2
7	Task#7. Calculation of grillages under bending for strength	4
8	Task#8. Calculation of beams under bending with axial forces for strength	2
9	Task#9. Calculation of beam for stability	2
10	Task#10. Calculation of rectangular plate for strength and stability	2
11	Task#11. Calculation of beam on elastic foundation under bending for strength using the finite difference method	2
	Total number of hours for practices	≈32÷36

Lecture 1. Introduction. About Structural Mechanics

§ 1.1. The Problems of the Ship Structural Mechanics

Structural mechanics of ships is the basic shipbuilding scientific discipline that deals with the problems of the **strength** and **rigidity** of ship hulls and warships and hull structures and their **stability** problem additionally. Also structural mechanics deals with problems of **fatigue**, **vibration** and **shock** loads of ship hull in common and ship hull constructions in particulars. The last three courses are studied in special parts of structural mechanics. Influence of temperature, radiation, electromagnetic fields, chemistry factors (for example corrosion) etc. can be considered.

The static **strength** of the ship hull and its structures we should understand as the ability of solids to resist fracture (separation into parts) under the action of operational and other loads that arise during the life of the solid.

The required **rigidity** of the ship hull and its structures assumes low level of deformations in these structures. Such deformations don't have to block carriage of assigned ship functions in the operation process.

Stability of a structure is its ability to support a given load (which can the structure experience during its life) without appearance of a sudden change in its configuration or shape (i.e. without collapse).

Fatigue is connected with ability of constructions to keep its wholeness under cyclic external loads for given number of cycles. If the fatigue strength failed there are cracks appear which may grow and to cause failure of the construction. Failure of fatigue strength is the reason of many accidents with ships.

Calculation of **vibration** is mainly connected with fatigue strength but not only. For vibration calculations we calculate generally free vibration frequencies of constructions and other factors optionally to avoid resonance. Resonance cause increasing of displacements and

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as consequence increasing of stresses and strains. Resonance may cause: fatigue strength failure both for the whole ship hull and separate ship hull constructions; threat for crew healthy.

Shock loads calculations are also important. Shock loads may cause serious damages of ship hull shells and cargo (even loss of ship). Shock loads are caused by slamming in natural conditions of exploitation, but also shock loads are connected with anthropogenic factors: such as detonation, ship collisions, etc. Shock loads caused by slamming cause great percentage of ship accidents.

The ship designing is performed according to "Rules" of classification societies, which based on generalized great ships operating experience and research results. Nevertheless, significant damage and sinking of ships still occur. The examples of such accidents are presented on the fig. 1.1.1. The reasons of crashes presented on the fig. 1.1.1 are different: failure of static strength, loss of stability, failure of fatigue strength etc.

Local damage (cracks, buckling) in ship structures and their elements also may lead to fatal consequences (loss of the ship). The examples of such damage cases are shown on the fig. 1.1.2.



Fig. 1.1.1. The ship hulls failures:
a – Liberty Tanker (16 January 1943); *b* – tanker "Prestige" (19 November 2002); *c* – the ship hull broken in two halves; *d* – MOL Comfort container ship (17 June 2013); *e* – bulk carrier MV Smart (August 2013)

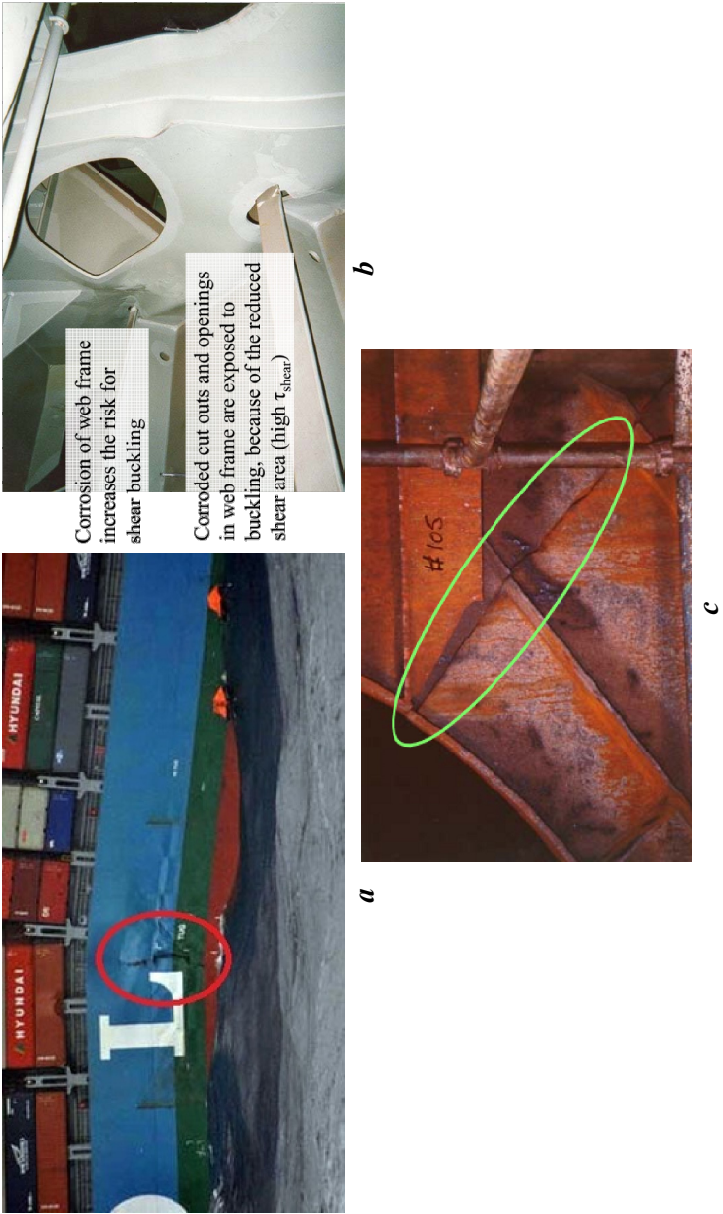


Fig. 1.1.2. Local damage in ship structures:
a – the hull cracking near the amidships of container ship MOL Comfort; *b* – shear buckling of the web frame areas where shear stress is high in the oil tanker (Det Norske Veritas); *c* – cracks in web frame (Det Norske Veritas)

§ 1.2. The Brief Description and the General Scheme of the Strength

Analyses of a Ship Hull and its Structures

There are several simplifying model bodies for the solid deformable bodies: rods, plates, shells (fig. 1.2.1). These simplifying models have its kinds, divisions etc. It's important to know where to apply one model approximation or another. For example the same solid body may be considered as rod and plate. There are definitions for rods, plates and shells, but sometimes given solid deformable body may be considered to be plate and rod simultaneously, etc.

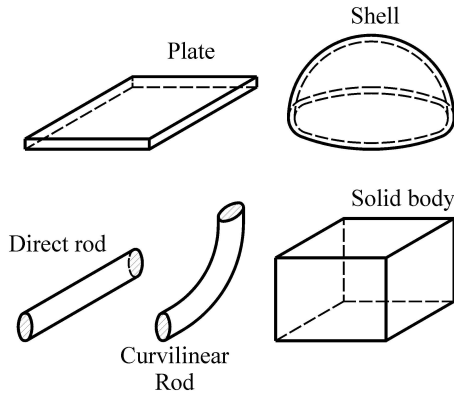


Fig. 1.2.1. Simplifying models of solid bodies

A ship hull is a complex solid three-dimensional structure. Shell plating provides impermeability to a ship. It is a thin covering that have top deck, sides and bottom. To provide rigidity (and in some cases static strength) of ship hull for given external loads (water pressure for example) it is reinforced by framing. Framing connected with shell form shared system which is called grillage.

All the continuous shell plating is usually separated into discrete flat plates by framing which can be reviewed independently from each other (for the simplest approaches). The form of those discrete plates

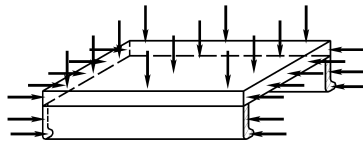
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is usually rectangular, but also other forms are possible. So called "support contour" for these rectangular plates (areas) is mutually perpendicular beams (framing) affixed to ship hull shell.

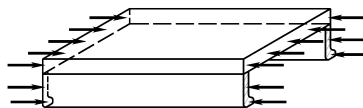
Plate plane may experience two kinds of deformation: in its own plane (tension-compression) and in orthogonal directions to own plane (deflections caused by bending or/and buckling). Deformations in own plane are caused by interaction with adjacent areas. Deformations in orthogonal directions are caused mainly by orthogonal pressure or compressive external loads acting along own plane, which cause loss of stability (fig. 1.2.2).

Stiffeners and other beams of the framing also have two kinds of deformations: axial ones caused by axial forces; deflections caused by orthogonal loads acting to plates or by compressive axial forces, which cause buckling (loss of stability) (see fig. 1.2.2).

Loading which cause bending (deflections) and axial strain-stress state of plate and beams



Loading which cause loss of stability or axial-strain stress state of plate and beams



Loading which cause only bending (deflections) of plate and beams

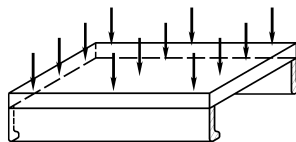


Fig. 1.2.2. The plate area and stiffeners and acting loads

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Beams in composition of ship hull are usually reviewed together with adjacent plating in place of fixing. This adjacent plating is caused the joined girde and considered in geometrical properties of beams. Other particulars which take place for mutual work of beams with adjacent plating aren't discussed here.

In fact for most of ship types ship hull length is more than in 5 times than its side height. When a ship hull is loaded both on still water and by waves, it experience bending in centerline plane (C.P.). Thus we can consider the ship hull as the beam with variable cross-section along the ship length and without any supports as a first approximation when the common strength of hull girder is investigated (fig. 1.2.3).

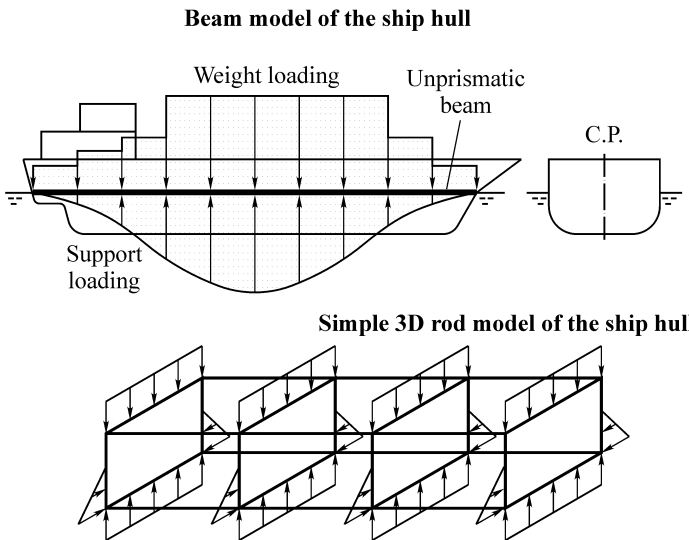


Fig. 1.2.3. Several models of the same ship hull

The same ship hull may be considered as reinforced shell, 3D rod grillage or several 2D flat grillages, 3D solid body, etc (see fig. 1.2.3). These methods may have different complexity. Flat areas of shell can be considered as plates.

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Considering the ship hull to be 3D solid body is the most precise approximation in strength estimation, but very labor-consuming and on reality almost is not used. Considering the ship hull to be reinforced shell gives the same results as presenting the ship hull as 3D solid body. That's why it's often used in practical calculations. The adequacy of the results applying rod models for ship hull or its parts (3D or 2D rod models, different grillages) is less, but appropriate for imprecise strength estimations when we don't need particulars. Rod systems provide the least labor-consuming and the fastest results in comparison with methods described above. Combining different physical approximations we shall have complex reliable figure in strength estimation.

The strength assessment of a real structure is usually performed in such sequence:

1. Development of the idealized physical presentation, possibly simplification model (rod, shell, plate) of real solid body and loading diagram basing on the analysis of the real structure and given loads (the physical model development).

2. Creation of the mathematical description of the accepted physical model behavior (the mathematical model development). The efficient method for the investigation of the mathematical model should be chosen.

3. Analysis of the obtained solution results. Strength, rigidity and stability of the investigated structure are verified according to valid "Rules" and regulations.

Strength problems can be solved using different mathematical models and approaches. The simplest discipline which allows us to calculate strength is "Strength of materials". But this discipline is based on simple models and assumptions. Methods of "Strength of materials" are insufficient to solve complex problems which take place in structural mechanics of ships.

On the contrary the discipline "The theory of elasticity" allows us to calculate static strength very particular. But for engineering

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calculations the discipline "The theory of elasticity" is very complex and connected with using of complex mathematical computations. But engineers have to know "The theory of elasticity" and to be able to solve simple tasks for this discipline to understand what is going on in structural mechanics.

In structural mechanics we mostly operate with rods, plates and shells, but sometimes solid body models are applied for very complex geometries and loads.

In order strength disciplines have to be studied in next sequence: "Strength of materials" → "The theory of elasticity" → "Structural mechanics" (rod systems, plates, shells).

It's assumed that reader already know the discipline "Strength of materials". Within this work the brief overview of disciplines "The theory of elasticity" and "Structural mechanics" is presented.

PART I. FUNDAMENTALS OF THE THEORY OF ELASTICITY

Lecture 2. Basic Concepts, Definitions and Denotations in the Theory of Elasticity

Mechanic of solid deformable body (MSDB) is science which explores strain-stress state of real bodies under external forces and temperature field. General assumptions of MSDB are: *continuity* and *elasticity*.

Continuity is ability of solid bodies to fill in by material all available volume of the body without emptiness. The material of the body is continuum which have no discrete (molecular or atomic) structure.

Elasticity is ability of solid bodies to recreate their initial sizes and forms after external loadings are removed. Thus initial state of body is suggested to be unstressed. This state is usually named *natural*.

There are three general branches of MSDB: theory of elasticity, theory of plasticity and theory of creep.

Theory of elasticity is science which explores behavior of solid deformable bodies in which there are only elastic strains.

Theory of plasticity is science which explores behavior of solid deformable bodies in which alongside with elastic strains there are also irreversible plastic strains.

Theory of creep is science which explores behavior of constructions under static loadings which cause creep strains.

There are several kinds of the theory of elasticity: so called linear theory of elasticity (or classic theory of elasticity), nonlinear theories of elasticity, different moment's theories of elasticity. But the most simple and historically first theory of elasticity is classic or linear theory of elasticity which is presented in these lectures.

§ 2.1. Initial Remarks

Further we shall use Cartesian coordinated system $Oxyz$ for getting of common relations in mechanics of solid deformable body. Coordinates of internal body points we shall denote as x, y, z . But coordinates of outer body surface points we shall denote as $\bar{x}, \bar{y}, \bar{z}$.

Cross section of the body is any mental surface which divides body into two pieces. For every section there are two appropriate coinciding inside surfaces. These surfaces belong to two pieces of body and situated on both different sides of the body section.

Outer normal vector $\mathbf{v}$ is direction which is orthogonal to outer or to some internal surface of body in given point and coming out. Direction of outer normal vector usually set by direction cosines $l = \cos(\vec{\mathbf{v}}, \wedge \vec{\mathbf{x}})$, $m = \cos(\vec{\mathbf{v}}, \wedge \vec{\mathbf{y}})$, $n = \cos(\vec{\mathbf{v}}, \wedge \vec{\mathbf{z}})$. *Moreover all vectors which are presented here are set by direction cosines.* Sometimes vectors are denoted by bold font.

Outer plane is piece of outer surface of body. Inside plane is piece of some internal surface (section) of body.

Elementary plane is infinitely small plane. Elementary plane can always be considered to be flat. Position of elementary plane set by coordinates of one of points which belong to it, and orientation in space by outer normal vector $\mathbf{v}(l; m; n)$. All quantities relating to elementary plane with set normal vector will be denoted with index $\mathbf{v}$.

Fiber is continuous aggregate (totality) of material points situated along some line.

Linear element of body is infinitely small segment of fiber. It can be considered direct.

Elementary volume is infinitely small piece of body, which have infinitely small sizes and volume. There are two kinds of elementary volumes: elementary parallelepiped and elementary tetrahedron (fig. 2.1.1). The body consists on these two elementary volumes. Outer surface is usually approximated by elementary tetrahedrons but inside volume consists on numerous of elementary parallelepipeds. All faces of elementary parallelepiped and internal faces of elementary tetrahedron are always parallel to coordinate planes and mutually orthogonal to each other.

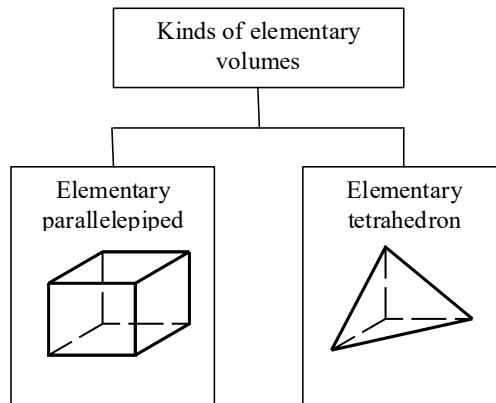


Fig. 2.1.1. Kinds of elementary volumes

The body is called closed one time if it occupies fully the volume inside of one closed outer surface. If the volume occupied by body is enclosed between two closed surfaces one of which is fully accommodated inside of another one and form inner cavity in body, such body is called closed two times. At existence of two, three and more similar inner cavities the body is called appropriately closed three times, closed four times etc.

To set body means that there are equations which define surfaces of that body: outer and internal ones. Also elastic properties of the body are known.

§ 2.2. Properties of Elastic Bodies

There are four common properties of ideal elastic bodies in frames of classic theory of elasticity:

1. **Homogeneity** means that elastic properties in all points of body along the same free direction are the same. At the test for tension-compression of specimens cut out in the same direction in different points of such body, these specimens will have the same strains under the same loadings.

2. **Isotropy** means that elastic properties of specimens cut in locality of some point are the same in all directions. Combining properties of homogeneity and isotropy we can consider that in ideally elastic body elastic properties of specimen don't depend on that in what point of body and in what direction it was cut.

3. **Physical linearity** means that there is direct dependency between strains and stresses which appear in body. For example when we change stresses in n times, the stresses also change in n times. If stresses disappear strains also disappear.

4. **Geometrical linearity** or relative rigidity means that relative linear strains and rotation angles of elements are small in comparison with one; also linear displacements of body points are small in comparison with sizes of this body.

§ 2.3. Factors Which Cause Strain-Stress State

There are several factors which causes strain-stress state: external forces and temperature field (fig. 2.3.1). External forces divide into two kinds: external surface forces and external volume forces. In the theory of elasticity they usually use projections of forces into coordinate axes. Projections of full intensity of surface external forces $\mathbf{p}_v$ into coordinate axes x, y, z we shall denote as p_{vx}, p_{vy}, p_{vz} (N/m²). Surface forces are functions of coordinates of body surface points: $p_{vx} = p_{vx}(\bar{x}, \bar{y}, \bar{z}), p_{vy} = p_{vy}(\bar{x}, \bar{y}, \bar{z}), p_{vz} = p_{vz}(\bar{x}, \bar{y}, \bar{z})$. Projections

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of full intensity of external volume forces $\mathbf{R}$ into coordinate axes x, y, z we shall denote as X, Y, Z (N/m^3). Volume external forces are also functions of body points: $X = X(x, y, z)$, $Y = Y(x, y, z)$, $Z = Z(x, y, z)$. Surface external forces act only to surface points of body, so these forces act only for elementary tetrahedrons. But volume external forces act for every point of body. Volume forces act both for elementary tetrahedrons and elementary parallelepipeds.

Temperature field usually set as function which depends on coordinates of points of body $T = T(x, y, z)$, K. Temperature field acts for every point of body, but it cause changes of linear strains only. Linear strains depend on temperature linearly.

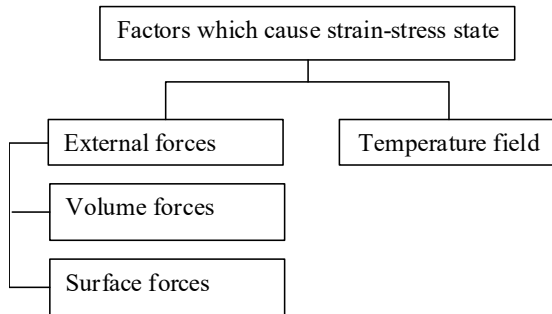


Fig. 2.3.1. Factors which cause strain-stress state

§ 2.4. Elastic Displacements and Strains

For example we have body loaded by external forces and temperature field (fig. 2.4.1). These factors cause deformation of body. Deformation means movements of points. Distance between start position of some point of body before deformation and end position of this point after deformation is called full displacement. We shall denote full displacement vector as $\mathbf{p}$. There are three projections of full displacements $\mathbf{p}$ into three coordinate axes x, y, z : $u = u(x, y, z)$, $v = v(x, y, z)$, $w = w(x, y, z)$, m .

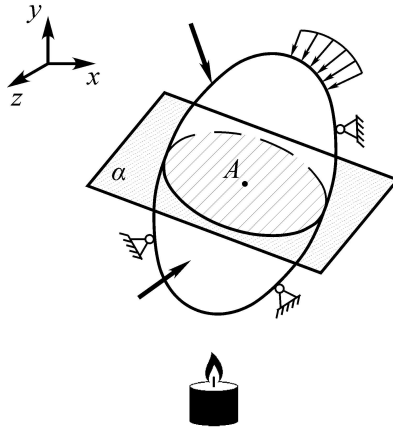


Fig. 2.4.1. Body loaded by external loadings and temperature field

On the fig. 2.4.2 there is full displacement ρ and its projections into coordinate axes are shown. Point A_1 is end position of point A after deformation. Distance between points A and A_1 is full displacement ρ of point A . Full displacement as vector have its module and direction cosines, which can be found using expressions

$$l_\rho = \frac{u}{\rho}; \quad m_\rho = \frac{v}{\rho}; \quad n_\rho = \frac{w}{\rho}. \quad (2.4.1)$$

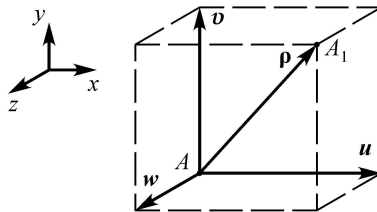


Fig. 2.4.2. Projections of displacement in some point A of body

Also there are other cinematic parameters of body. One of them is relative linear strain. Let's review what that is. For example we have some linear element in body AB (fig. 2.4.3) before deformation. After deformation points A and B move and will take new positions A_1 and B_1 . Difference between end length of some linear element after deformation and start length of the same element before deformation is called absolute linear strain. Absolute linear strain can be calculated using expression

$$\Delta l_{AB} = A_1B_1 - AB, \text{ m}, \quad (2.4.2)$$

where A_1B_1 – length of linear element AB after deformation (length of vector $\overrightarrow{A_1B_1}$); AB – length of linear element AB before deformation (length of vector $\overrightarrow{AB}$). Points A_1 and B_1 are new positions of points A and B appropriately after deformation.

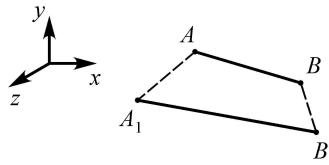


Fig. 2.4.3. For explaining of linear strains

Relative linear strain is relation of absolute linear strain to initial length of linear element before deformation. It can be calculated using expression (see fig. 2.4.3)

$$\varepsilon_{AB} = \frac{\Delta l_{AB}}{AB} = \frac{A_1B_1 - AB}{AB}. \quad (2.4.3)$$

Let's review what angular (shear) strains are. For example we have two mutually orthogonal linear elements before deformation $AB \perp BC$ (fig. 2.4.4). After deformation these linear elements will change its lengths and directions. The angle between these linear elements will be another than orthogonal. So change of initial orthogonal angle between two mutually orthogonal linear elements after deformation called angular strain.

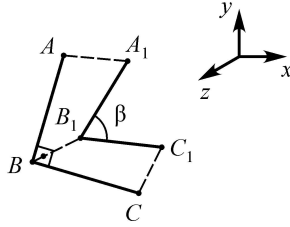


Fig. 2.4.4. For explaining of angular strains

Angular strains can be computed using formula

$$\gamma_{AB,A_1B_1} = \frac{\pi}{2} - \beta, \text{ rad.} \quad (2.4.4)$$

As conclusion at scheme presented on the fig. 2.4.5 the classification of strains is presented.

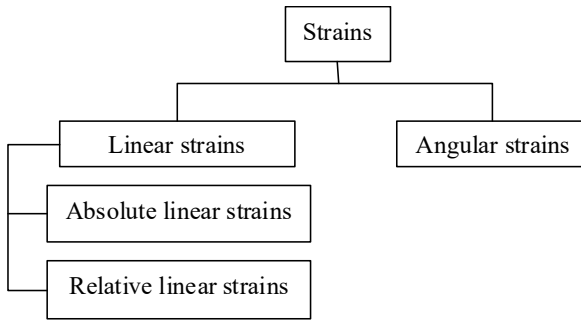


Fig. 2.4.5. Classification of strains

There are projections of linear and angular strains into coordinate axes x, y, z and coordinate planes. These projections are functions depending on coordinates of body points: $\epsilon_x = \epsilon_x(x, y, z)$, $\epsilon_y = \epsilon_y(x, y, z)$, $\epsilon_z = \epsilon_z(x, y, z)$, $\gamma_{xy} = \gamma_{xy}(x, y, z)$, $\gamma_{yz} = \gamma_{yz}(x, y, z)$, $\gamma_{zx} = \gamma_{zx}(x, y, z)$.

§ 2.5. Stresses. Stress and Strain Tensors

Stresses. For example we have body loaded by external forces and temperature field (see fig. 2.4.1). These factors cause deformation of body. Let's cut mentally our body by flat cross plane α and take some point A on plane which belong to one of pieces of body. Within elementary plane defined in locality of the point A there is internal force. Directions of given elementary plane and internal force are different. Relation of the value of this internal force to the area of elementary plane is called full stresses $\mathbf{p}_v$, Pa (fig. 2.5.1). So stresses are measure of intensity of internal forces.

There are three projections of full stresses $\mathbf{p}_v$ into coordinate axes x, y, z : $\mathbf{p}_{vx}, \mathbf{p}_{vy}, \mathbf{p}_{vz}$. Projection of full stresses into normal direction vector v of elementary plane is called full normal stresses and denoted as σ_v . Projection of full stresses into cross section plane α is called full shear stresses τ_v . Full stresses $\mathbf{p}_v$, full normal stresses σ_v and full shear stresses τ_v are connected by the next relations:

$$p_v^2 = p_{vx}^2 + p_{vy}^2 + p_{vz}^2 = \sigma_v^2 + \tau_v^2, \quad (2.5.1)$$

$$\sigma_v = p_{vx}l + p_{vy}m + p_{vz}n, \quad (2.5.2)$$

$$\tau_v^2 = p_v^2 - \sigma_v^2. \quad (2.5.3)$$

Direction cosines of full stresses can be defined by relations

$$l_p = \frac{p_{vx}}{p_v}; \quad m_p = \frac{p_{vy}}{p_v}; \quad n_p = \frac{p_{vz}}{p_v}. \quad (2.5.4)$$

Let's review coordinate planes of elementary parallelepiped. At every such plane there are full stresses, full normal stresses, and full shear stresses. If to project full shear stresses into coordinate axes we shall have that for every coordinate plane there are three kinds of stresses: two kinds of shear stresses and one kind of normal stresses.

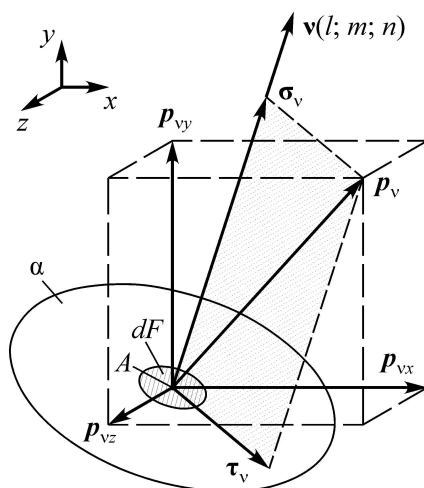


Fig. 2.5.1. For explaining of stresses

If to review all coordinate planes of elementary parallelepiped defined in locality of some point we shall have ordered system of stresses on planes of this parallelepiped (fig. 2.5.2). Index of normal stresses shows the direction of that normal stresses and the direction of normal vector of plane in which normal stresses situated. First index of shear stresses shows the direction of normal vector of plane in which shear stresses act. But second index of shear stresses shows its direction. For normal stresses there is next sign rule: normal stresses have to be considered positive if they cause tension of elementary parallelepiped along given axis. The rule for shear stresses we shall explain at example of τ_{xy} : if *directions of normal vector* of plane in which shear stresses act *and* appropriate coordinate axis x of this normal vector are the same (opposite) for the shear stresses to be positive its direction have to fit (to be opposite) to direction of its appropriate axis y .

Stresses are functions which depends on coordinates of body points: $\sigma_x = \sigma_x(x, y, z)$, $\sigma_y = \sigma_y(x, y, z)$, $\sigma_z = \sigma_z(x, y, z)$,
 $\tau_{xy} = \tau_{xy}(x, y, z)$, $\tau_{yz} = \tau_{yz}(x, y, z)$, $\tau_{zx} = \tau_{zx}(x, y, z)$.

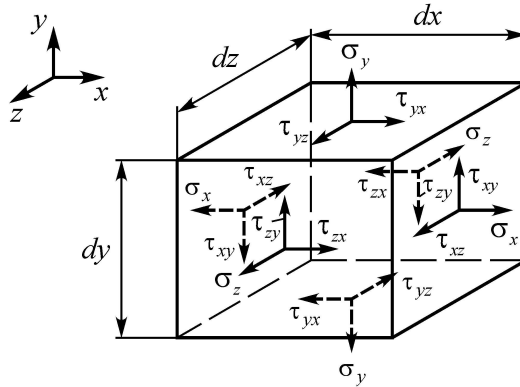


Fig. 2.5.2. For explaining of sign rule for stresses

Stress and strain tensors. So in every point of body we can define elementary parallelepiped or elementary tetrahedron. At faces of such elementary volumes there are stresses. Stresses raise appearance of strains. To describe strain-stress state in point tensors usually are used. There are two kinds of tensors: stress tensors and strain tensors.

Stress tensor can be presented as

$$T_{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \quad (2.5.5)$$

Strain tensor can be presented as

$$T_{\varepsilon} = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{bmatrix} \quad (2.5.6)$$

There is one general property of stress and strain tensors: these tensors are symmetrical relatively main diagonal owing to the *law of twinning shear stresses* (see lecture 3). So in every point we have six independent stresses and strains.

§ 2.6. Elastic Properties of Materials

Elastic properties of materials are: modulus of elasticity in tension, shear modulus, Poisson's ratio.

Modulus of elasticity in tension E can be defined experimentally at test of tension of specimen described in [1]. It can be calculated using expression (fig. 2.6.1)

$$E = \operatorname{tg} \alpha, \text{ Pa.} \quad (2.6.1)$$

Shear modulus can be defined both experimentally and analytically. Using experimental diagram (fig. 2.6.2) shear modulus G can be computed using expression

$$G = \operatorname{tg} \alpha, \text{ Pa.} \quad (2.6.2)$$

Analytically shear modulus can be found using formula

$$G = \frac{E}{2(1 + \mu)}, \text{ Pa,} \quad (2.6.3)$$

where μ is Poisson's ration.

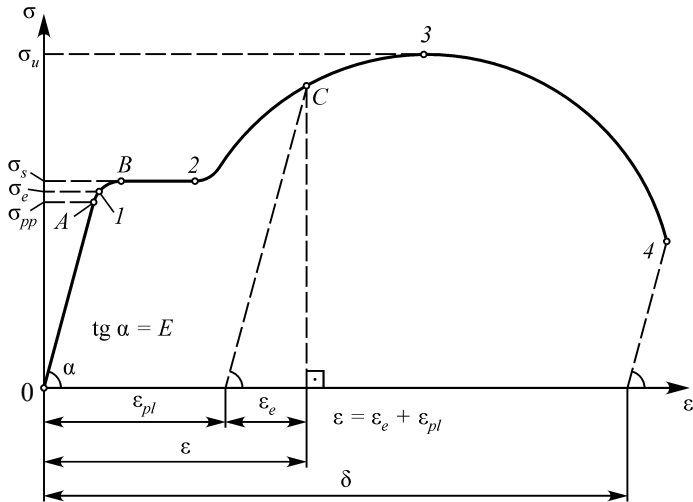


Fig. 2.6.1. The conditional diagram for specimen under tension:
 ε_e – is elastic linear strain; ε_{pl} – is plastic linear strain

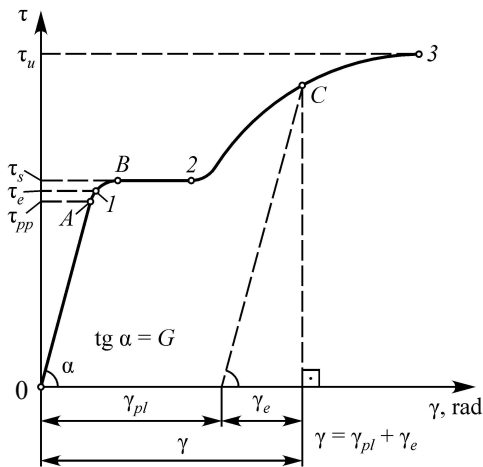


Fig. 2.6.2. Conditional diagram for specimen under pure shear:
 γ_e – is elastic linear strain; γ_{pl} – is plastic linear strain

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Poisson's ratio can be defined experimentally in conditions of flat strain stress state for plate under axial tension-compression. Poisson's ratio can be calculated using relation

$$\mu = \left| \frac{\varepsilon'}{\varepsilon} \right|, \quad (2.6.4)$$

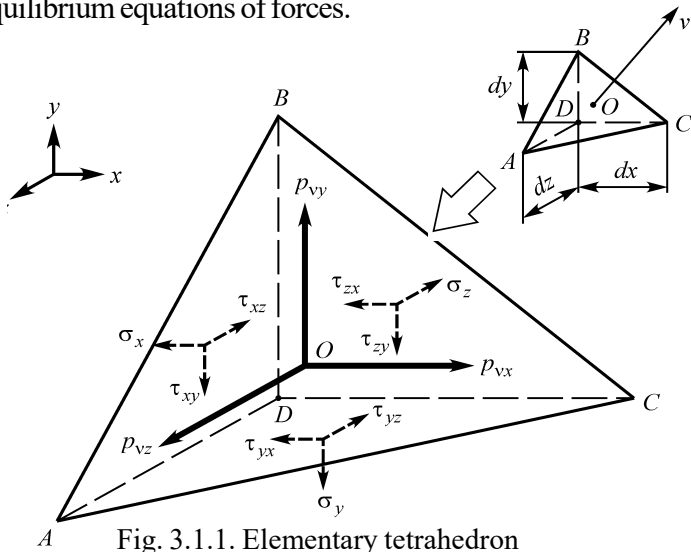
where ε' – is orthogonal linear strain; ε – longitudinal linear strain.

Lecture 3. Major Equations and Relations of the Theory of Elasticity. Direct and Contrary Problems

As it was told above the body consists of numerous of elementary parallelepipeds and elementary tetrahedrons. If body is affected by factors which cause strain-stress state (external loadings and temperature field) there are internal forces of elasticity appear in that body by which body resists to external factors mentioned above. So the body is always equilibrated by systems of external and internal forces. If the whole body is equilibrated it means that every elementary volume is equilibrated.

§ 3.1. Equilibrium Equations of Elementary Tetrahedron

Let's define in locality of some point D elementary tetrahedron (fig. 3.1.1). There are stresses and projections of external surface forces shown. Neglecting with volume forces it is necessary to compose three equilibrium equations of forces.



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For example we shall review one of these equations

$$\begin{aligned} \Sigma \text{ Forces}_x = 0: & -\sigma_x \cdot F_{ADB} - \tau_{yx} \cdot F_{ADC} - \\ & -\tau_{zx} \cdot F_{CDB} + p_{vx} \cdot F_{ABC} = 0, \end{aligned} \quad (3.1.1)$$

where F_{ADB} – is the area of plane ADB , etc.

Denotation $\Sigma \text{ Forces}_x = 0$ means sum of forces in projection of axis x equals zero.

If to divide (3.1.1) by F_{ABC} we shall have next:

$$\begin{aligned} -\sigma_x \cdot l - \tau_{yx} \cdot m - \tau_{zx} \cdot n + p_{vx} &= 0 \Rightarrow p_{vx} = \\ &= \sigma_x \cdot l + \tau_{yx} \cdot m + \tau_{zx} \cdot n. \end{aligned} \quad (3.1.2)$$

Because ADB, ADC, CDB are projections of surface (plane) ABC into coordinate axes. If to accept $F_{ADB} = F_x$, $F_{ADC} = F_y$, $F_{CDB} = F_z$ and $F_{ABC} = F_v$ we shall have next:

$$l = \frac{F_x}{F_v}; \quad m = \frac{F_y}{F_v}; \quad n = \frac{F_z}{F_v}.$$

Composing equations like (3.1.1) for axes y and z we shall have two lack equations. Finally we have three equilibrium equations of elementary tetrahedron or so called the first group of equilibrium equations of linear mechanics of deformable body

$$\left. \begin{aligned} \sigma_x l + \tau_{yx} m + \tau_{zx} n &= p_{vx}, \\ \tau_{xy} l + \sigma_y m + \tau_{zy} n &= p_{vy}, \\ \tau_{xz} l + \tau_{yz} m + \sigma_z n &= p_{vz}. \end{aligned} \right\}. \quad (3.1.3)$$

§ 3.2. Equilibrium Equations of Elementary Parallelepiped

In locality of some point A we define elementary parallelepiped (fig. 3.2.1). As it was told above stresses are functions of coordinates of points. These functions of stresses can be expanded in Taylor series in locality of some point. We have to expand function like $f(x + dx, y + dy, z + dz)$. Within linear mechanics we neglect with nonlinear members of this row and take into account only linear ones. At opposite faces of parallelepiped stresses distinguish for very small quantity. It is necessary to compose three equations of forces and three equations of moments.

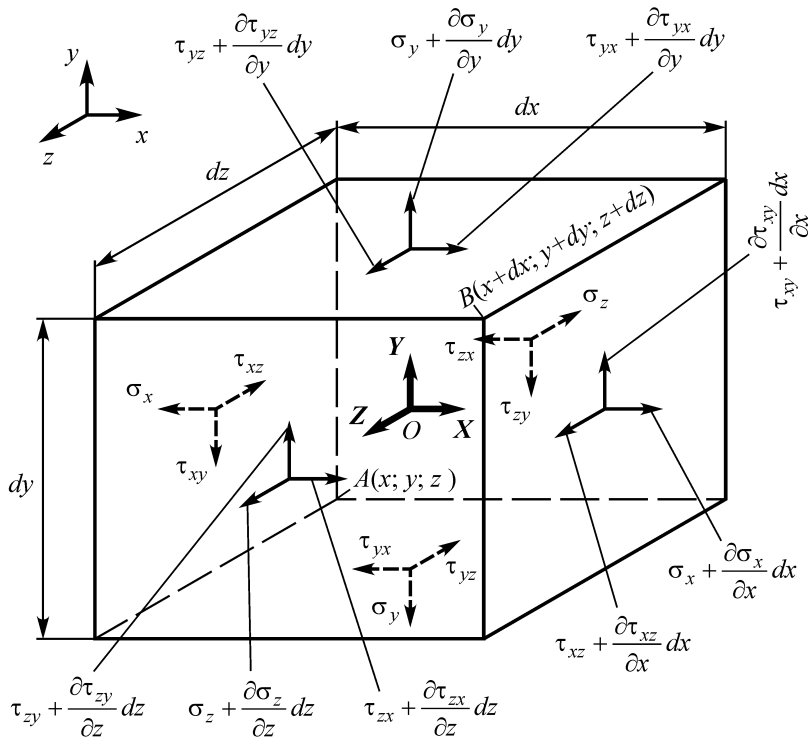


Fig. 3.2.1. Elementary parallelepiped

Lecture 3

One of equilibrium equations of forces can be written:

$$\left. \begin{aligned} \Sigma \text{ Forces}_x = 0: & -\sigma_x \cdot dy \, dz - \tau_{yx} \cdot dx \, dz - \tau_{zx} \cdot dx \, dy + \\ & + \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} dz \right) dx \, dy + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy \right) dx \, dz + \\ & + \left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx \right) dy \, dz + X \cdot dx \, dy \, dz - \rho \frac{\partial^2 u}{\partial t^2} \cdot dx \, dy \, dz = 0. \end{aligned} \right\} (3.2.1)$$

Opening brackets, gathering similar variables and dividing by multiplier $dx \, dy \, dz$ in (3.2.1), we shall have first equilibrium equation

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} - \rho \frac{\partial^2 u}{\partial t^2} = -X. \quad (3.2.2)$$

Composing equations like (3.2.1) for axes y and z we shall have two lack equations. Finally we have three equilibrium equations of elementary parallelepiped or so called the second group of equilibrium equations of linear mechanics of deformable body

$$\left. \begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} - \rho \frac{\partial^2 u}{\partial t^2} &= -X, \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} - \rho \frac{\partial^2 v}{\partial t^2} &= -Y, \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} - \rho \frac{\partial^2 w}{\partial t^2} &= -Z. \end{aligned} \right\} (3.2.3)$$

Equilibrium equations of moments we shall compose relatively mass centre of parallelepiped situated in point O . One of the equilibrium equations of moments can be written:

$$\left. \begin{aligned} \Sigma \text{ Moments}_{xO} = 0: & \quad -\tau_{zy} \cdot dx dy \frac{1}{2} dz + \tau_{yz} dx dz \frac{1}{2} dy - \\ & - \left(\tau_{zy} + \frac{\partial \tau_{zy}}{\partial z} dz \right) dx dy \frac{1}{2} dz + \left(\tau_{yz} + \frac{\partial \tau_{yz}}{\partial y} dy \right) dx dz \frac{1}{2} dy = 0. \end{aligned} \right\} (3.2.4)$$

Denotation $\Sigma \text{ Moments}_{xO} = 0$ means sum of moments in projection of axis x relatively point O equals zero.

Opening brackets, gathering similar values, dividing by multiplier $dx \cdot dy \cdot dz$ and eliminating smidgens of higher order in (3.2.4), we shall have

$$\tau_{zy} = \tau_{yz}. \quad (3.2.5)$$

Composing equilibrium equations of moments in projection on axes y and z relatively point O , similar to (3.2.4), we shall have two equations

$$\tau_{zx} = \tau_{xz}, \quad \tau_{xy} = \tau_{yx}. \quad (3.2.6)$$

As a result after composing of equilibrium equations of moments in projections on axes x, y, z relatively point O we shall have the next system of equations

$$\tau_{zy} = \tau_{yz}; \quad \tau_{zx} = \tau_{xz}; \quad \tau_{xy} = \tau_{yx}. \quad (3.2.7)$$

The system of equations (3.2.7) is mathematical formulation of the *law of twinning shear stresses*: for two decussate planes shear stresses acting on those planes which are orthogonal to line of intersection of those planes are directed to each other or from each other.

§ 3.3. Hook's Law for Isotropic Bodies

Natural tests show that there is linear dependency between components of stresses and strains when we have little deformation of elastic bodies

$$\left. \begin{aligned} \varepsilon_x &= c_{11}\sigma_x + c_{12}\sigma_y + c_{13}\sigma_z + c_{14}\tau_{xy} + c_{15}\tau_{yz} + c_{16}\tau_{zx} \\ \varepsilon_y &= c_{21}\sigma_x + \dots & \dots + c_{26}\tau_{zx} \\ \varepsilon_z &= c_{31}\sigma_x + \dots & \dots + c_{36}\tau_{zx} \\ \gamma_{xy} &= c_{41}\sigma_x + \dots & \dots + c_{46}\tau_{zx} \\ \gamma_{yz} &= c_{51}\sigma_x + \dots & \dots + c_{56}\tau_{zx} \\ \gamma_{zx} &= c_{61}\sigma_x + \dots & \dots + c_{66}\tau_{zx} \end{aligned} \right\} (3.3.1)$$

The system (3.3.1) is called the common Hook's law, but factors c_{ij} are elastic constants of material. The consequence of linearity of equations (3.3.1) is so called the principle of independency of action of forces: components of strains caused by some totality of stresses equal to sum of components of strains caused by action of every component of stresses separately. Dependencies (3.3.1) contain 36 elastic constants, but only 24 of them are independent owing to the energy conservation law, because $c_{ij} = c_{ji}$.

For ideal elastic isotropic homogenous bodies Hook's law, where strains expressed through stresses can be written

$$\left. \begin{aligned} \varepsilon_x &= \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)] + \alpha T; & \gamma_{xy} &= \frac{\tau_{xy}}{G}; \\ \varepsilon_y &= \frac{1}{E}[\sigma_y - \mu(\sigma_z + \sigma_x)] + \alpha T; & \gamma_{yz} &= \frac{\tau_{yz}}{G}; \\ \varepsilon_z &= \frac{1}{E}[\sigma_z - \mu(\sigma_x + \sigma_y)] + \alpha T; & \gamma_{zx} &= \frac{\tau_{zx}}{G}. \end{aligned} \right\} (3.3.2)$$

There is another form of Hook's law, where stresses expressed through strains

$$\left. \begin{aligned} \sigma_x &= 2G \left(\varepsilon_x + \frac{\mu\theta}{1-2\mu} - \frac{1+\mu}{1-2\mu} \alpha T \right), & \tau_{xy} &= G\gamma_{xy}, \\ \sigma_y &= 2G \left(\varepsilon_y + \frac{\mu\theta}{1-2\mu} - \frac{1+\mu}{1-2\mu} \alpha T \right), & \tau_{yz} &= G\gamma_{yz}, \\ \sigma_z &= 2G \left(\varepsilon_z + \frac{\mu\theta}{1-2\mu} - \frac{1+\mu}{1-2\mu} \alpha T \right), & \tau_{zx} &= G\gamma_{zx}. \end{aligned} \right\} \quad (3.3.3)$$

where E – modulus of elasticity in tension, Pa; $G = \frac{E}{2(1+\mu)}$ – shear modulus, Pa; μ – Poisson's ratio; $\theta = \varepsilon_x + \varepsilon_y + \varepsilon_z$; α – factor of linear thermal extension, K^{-1} .

Hook's law is the third group of basic equations of the theory of elasticity. These equations are called physical equations.

§ 3.4. Cauchy Geometrical Relations Between Strains and Displacements. Compatibility Conditions

Between displacements and strains there are relations. To get these relations we shall take in locality of some point A (fig. 3.4.1) three linear elements which are parallel to coordinate axes and mutually orthogonal to each other. There are coordinates of points A, B, C, D : $A(x; y; z)$, $B(x + dx; y; z)$, $C(x; y + dy; z)$, $D(x; y; z + dz)$. After deformation points A, B, C, D will take new positions appropriately A_1, B_1, C_1, D_1 . There are new coordinates of points: $A_1(x + u_A; y + v_A; z + w_A)$, $B_1(x + dx + u_B; y + v_B; z + w_B)$, $C_1(x + u_C; y + dy + v_C; z + w_C)$, $D_1(x + u_D; y + v_D; z + dz + w_D)$. Displacements in body are set by functions: $u = u(x, y, z)$, $v = v(x, y, z)$, $w = w(x, y, z)$. Extending

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in Taylor series functions for displacements u, v, w for points A, B, C, D and taking into consideration only linear members we can get values of components of displacements in those points. After extending in Taylor series displacements for points A, B, C, D we shall have:

$$\left. \begin{aligned} u_A = u, \quad u_B = u + \frac{\partial u}{\partial x} dx, \quad u_C = u + \frac{\partial u}{\partial y} dy, \quad u_D = u + \frac{\partial u}{\partial z} dz, \\ v_A = v, \quad v_B = v + \frac{\partial v}{\partial x} dx, \quad v_C = v + \frac{\partial v}{\partial y} dy, \quad v_D = v + \frac{\partial v}{\partial z} dz, \\ w_A = w, \quad w_B = w + \frac{\partial w}{\partial x} dx, \quad w_C = w + \frac{\partial w}{\partial y} dy, \quad w_D = w + \frac{\partial w}{\partial z} dz. \end{aligned} \right\} (3.4.1)$$

First we shall review linear element dx between points A and B . Coordinates of points A and B are presented above. Coordinates of points A_1 and B_1 considering (3.4.1) are:

$$\left. \begin{aligned} A_1(x + u; y + v; z + w) \\ B_1\left(x + dx + u + \frac{\partial u}{\partial x} dx; y + v + \frac{\partial v}{\partial x} dx; z + w + \frac{\partial w}{\partial x} dx\right). \end{aligned} \right\}$$

Distance between points A_1 and B_1 is

$$A_1B_1 = dx \sqrt{\left(1 + \frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2}. \quad (3.4.2)$$

But distance between points A and B : $AB = dx$.

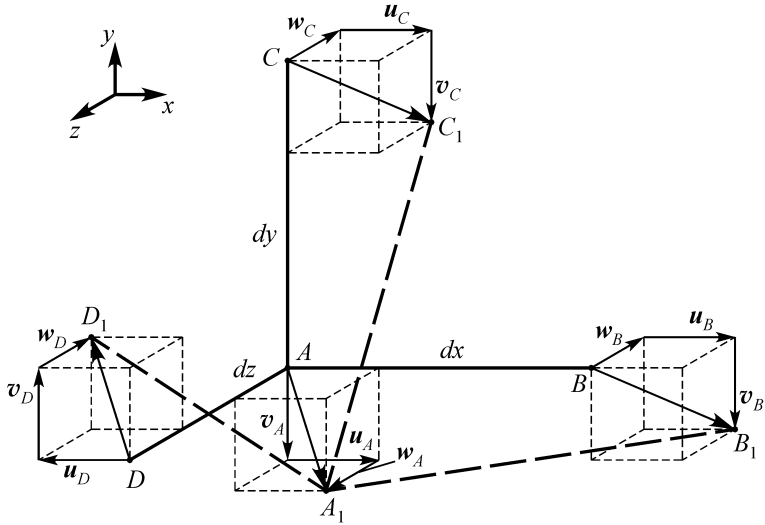


Fig. 3.4.1. For getting of Cauchy relations

Linear relative strain in direction AB (in direction of axis x) can be found using expression

$$\varepsilon_{AB} = \varepsilon_x = \frac{A_1B_1 - AB}{AB}. \quad (3.4.3)$$

Substituting (3.4.2) into (3.4.3) we shall have expression for ε_x

$$\varepsilon_x = \sqrt{\left(1 + \frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2} - 1. \quad (3.4.4)$$

Analogically reviewing linear elements AC, AD we can get expressions for $\varepsilon_x, \varepsilon_z$. Expressions for $\varepsilon_x, \varepsilon_z$ can be got also using circular reordering of characters. Circular reordering of characters means to put instead given character next neighboring. Neighboring characters for example are: $x \rightarrow y \rightarrow z \rightarrow x; u \rightarrow v \rightarrow w \rightarrow u$, etc.

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Further we shall define angular strain between AB and AC (between linear elements dx and dy , i.e. in directions which are parallel appropriately to axes x and y). Required angular strain between directions AB and AC can be found using expression

$$\sin \gamma_{xy} = \cos\left(\frac{\pi}{2} - \gamma_{xy}\right) = \cos\left(\overrightarrow{A_1B_1}, \wedge \overrightarrow{A_1C_1}\right). \quad (3.4.5)$$

Cosine of angle between vectors A_1B_1 and A_1C_1 can be found using known formula

$$\cos\left(\overrightarrow{A_1B_1}, \wedge \overrightarrow{A_1C_1}\right) = \cos\left(\overrightarrow{A_1B_1}, \wedge \vec{x}\right) \cdot \cos\left(\overrightarrow{A_1C_1}, \wedge \vec{x}\right) + \left. \begin{aligned} &+ \cos\left(\overrightarrow{A_1B_1}, \wedge \vec{y}\right) \cdot \cos\left(\overrightarrow{A_1C_1}, \wedge \vec{y}\right) + \\ &+ \cos\left(\overrightarrow{A_1B_1}, \wedge \vec{z}\right) \cdot \cos\left(\overrightarrow{A_1C_1}, \wedge \vec{z}\right). \end{aligned} \right\} \quad (3.4.6)$$

In compact notice (3.4.6) can be presented

$$\begin{aligned} \cos\left(\overrightarrow{A_1B_1}, \wedge \overrightarrow{A_1C_1}\right) = \\ = l_{A_1B_1} \cdot l_{A_1C_1} + m_{A_1B_1} \cdot m_{A_1C_1} + n_{A_1B_1} \cdot n_{A_1C_1}, \end{aligned} \quad (3.4.6')$$

where l, m, n are direction cosines of vectors A_1B_1 and A_1C_1 . Direction cosines of vectors A_1B_1 and A_1C_1 can be found in the way presented below.

Using coordinates of points A_1 and B_1 and (3.4.1) we can find that vector A_1B_1 have next projections for coordinate axes:

$$\overrightarrow{A_1B_1} \left(\left(1 + \frac{\partial u}{\partial x}\right) dx; \left(\frac{\partial v}{\partial x}\right) dx; \left(\frac{\partial w}{\partial x}\right) dx \right),$$

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but the length of A_1B_1 using (3.4.3) is $A_1B_1 = (1 + \varepsilon_x)dx$. Hence direction cosines of vector A_1B_1 are

$$\left. \begin{aligned} \cos(\overrightarrow{A_1B_1}, \vec{x}) &= l_{A_1B_1} = \frac{1 + \frac{\partial u}{\partial x}}{1 + \varepsilon_x}; \\ \cos(\overrightarrow{A_1B_1}, \vec{y}) &= m_{A_1B_1} = \frac{\frac{\partial v}{\partial x}}{1 + \varepsilon_x}; \\ \cos(\overrightarrow{A_1B_1}, \vec{z}) &= n_{A_1B_1} = \frac{\frac{\partial v}{\partial x}}{1 + \varepsilon_x}. \end{aligned} \right\} \quad (3.4.7)$$

Analogically direction cosines of vector A_1C_1 can be found

$$\left. \begin{aligned} \cos(\overrightarrow{A_1C_1}, \vec{x}) &= l_{A_1C_1} = \frac{\frac{\partial u}{\partial y}}{1 + \varepsilon_y}; \\ \cos(\overrightarrow{A_1C_1}, \vec{y}) &= m_{A_1C_1} = \frac{1 + \frac{\partial v}{\partial y}}{1 + \varepsilon_y}; \\ \cos(\overrightarrow{A_1C_1}, \vec{z}) &= n_{A_1C_1} = \frac{\frac{\partial w}{\partial y}}{1 + \varepsilon_y}. \end{aligned} \right\} \quad (3.4.8)$$

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Substituting (3.4.7) and (3.4.8) into right part of (3.4.6) we shall have

$$\begin{aligned} \sin \gamma_{xy} &= \cos(\overrightarrow{A_1 B_1}, \wedge \overrightarrow{A_1 C_1}) = \\ &= \frac{\left(1 + \frac{\partial u}{\partial x}\right) \frac{\partial u}{\partial y} + \left(1 + \frac{\partial v}{\partial y}\right) \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y}}{(1 + \varepsilon_x)(1 + \varepsilon_y)}. \end{aligned} \quad (3.4.9)$$

Analogically γ_{yz}, γ_{zx} can be found. Expressions of γ_{yz}, γ_{zx} can also be written using circular reordering of characters, basing on the expression (3.4.9).

Expressions like (3.4.4) and (3.4.9) which connect strains and displacements are accurate expressions of nonlinear theory of elasticity. Expressions like (3.4.4) and (3.4.9) are also called geometrical relations. Simplifying these precise expressions in several approaches we shall have simple dependencies for classic theory of elasticity

$$\left. \begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x}, & \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \\ \varepsilon_y &= \frac{\partial v}{\partial y}, & \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \\ \varepsilon_z &= \frac{\partial w}{\partial z}, & \gamma_{zx} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}. \end{aligned} \right\} \quad (3.4.10)$$

Relations (3.4.10) are called Cauchy geometrical relations for linear theory of elasticity or simply Cauchy relations.

Six equations of Cauchy (3.4.10) set dependencies between nine functions: three displacements and six strains. Hence there are only three quantities which can be set independently. It is impossible to set six strains arbitrarily, because after substituting those into (3.4.10) three

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required displacements are connected by six expressions which in common occasion may be incompatible.

Incompatibility indicates the impossibility of appearance of given strains in body. Therefore strains can't be set arbitrarily. They have to satisfy to some relations, which can be got by the way of excluding of displacements from (3.4.10). Excluding displacements from (3.4.10) we shall have next relations

$$\left. \begin{aligned} \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} &= \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}, \\ \frac{\partial}{\partial x} \left(\frac{\partial \gamma_{xy}}{\partial z} - \frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} \right) &= 2 \frac{\partial^2 \varepsilon_x}{\partial y \partial z}, \\ \frac{\partial^2 \varepsilon_y}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial y^2} &= \frac{\partial^2 \gamma_{yz}}{\partial y \partial z}, \\ \frac{\partial}{\partial y} \left(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} \right) &= 2 \frac{\partial^2 \varepsilon_y}{\partial z \partial x}, \\ \frac{\partial^2 \varepsilon_z}{\partial x^2} + \frac{\partial^2 \varepsilon_x}{\partial z^2} &= \frac{\partial^2 \gamma_{zx}}{\partial z \partial x}, \\ \frac{\partial}{\partial z} \left(\frac{\partial \gamma_{zx}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yz}}{\partial x} \right) &= 2 \frac{\partial^2 \varepsilon_z}{\partial x \partial y}. \end{aligned} \right\} \quad (3.4.11)$$

Equations (3.4.11) are called Saint-Venant equations or equations of compatibility of strains or equations of continuity. Physical meaning of equations of compatibility (3.4.11) can be explained in the next way. For example we have body cut for numerous of elementary parallelepipeds before deformation. Then every parallelepiped was

deformed in accordance with strains laws, which were set for the whole body. Assembling all parallelepipeds together after deformation we shall have the whole body after deformation. If given strains of parallelepipeds satisfy to compatibility conditions (3.4.11) we shall have that all parallelepipeds are closely adjacent to each other without voids between them after assembling. In opposite case there will be a lot of cracks and exfoliations in body. Therefore in body which has property of continuity it is impossible appearance of strains which don't satisfy (3.4.11). Strains defined for given displacements using Cauchy relations satisfy to equations of compatibility of strains automatically.

§ 3.5. Definition of Displacements for Known Strains

Using Cauchy relations it is easy to define strain for known displacements. Let's define contrary task: to define displacement for known strains. Solution of this task is connected with integration of geometrical relations, which connect with each other components of strains and displacements. To get formulas for displacements for known strains it is necessary to use known formula for definition of function knowing its full differential. After geometrical transformations we shall have next formulas for displacements

$$\left. \begin{aligned}
 u &= u_0 + (y - y_0) \left(\frac{\partial u}{\partial y} \right)_0 + (z - z_0) \left(\frac{\partial u}{\partial z} \right)_0 + \int_{x_0}^x \varepsilon_x dx + \\
 &\int_{y_0}^y \int_{x_0}^x \left(\frac{\partial \gamma_{xy}}{\partial y} - \frac{\partial \varepsilon_y}{\partial x} \right) dy^2 + \int_{z_0}^z \int_{x_0}^x \left(\frac{\partial \gamma_{zx}}{\partial z} - \frac{\partial \varepsilon_z}{\partial x} \right) dz^2 + \\
 &+ \frac{1}{2} (y - y_0) \int_{z_0}^z \left(\frac{\partial \gamma_{xy}}{\partial z} - \frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} \right) dz.
 \end{aligned} \right\} (3.5.1)$$

$$\left. \begin{aligned}
 v = v_0 + (z - z_0) \left(\frac{\partial v}{\partial z} \right)_0 + (x - x_0) \left(\frac{\partial v}{\partial x} \right)_0 + \int_{y_0}^y \varepsilon_y dy + \\
 + \int_{z_0}^z \int_{y_0}^y \left(\frac{\partial \gamma_{yz}}{\partial z} - \frac{\partial \varepsilon_z}{\partial y} \right) dz^2 + \int_{x_0}^x \int_{y_0}^y \left(\frac{\partial \gamma_{xy}}{\partial x} - \frac{\partial \varepsilon_x}{\partial y} \right) dx^2 + \\
 + \frac{1}{2} (z - z_0) \int_{x_0}^x \left(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} \right) dx.
 \end{aligned} \right\} \quad (3.5.2)$$

$$\left. \begin{aligned}
 w = w_0 + (x - x_0) \left(\frac{\partial w}{\partial x} \right)_0 + (y - y_0) \left(\frac{\partial w}{\partial y} \right)_0 + \int_{z_0}^z \varepsilon_z dz + \\
 + \int_{x_0}^x \int_{z_0}^z \left(\frac{\partial \gamma_{zx}}{\partial x} - \frac{\partial \varepsilon_x}{\partial z} \right) dx^2 + \int_{y_0}^y \int_{z_0}^z \left(\frac{\partial \gamma_{yz}}{\partial y} - \frac{\partial \varepsilon_y}{\partial z} \right) dy^2 + \\
 + \frac{1}{2} (x - x_0) \int_{y_0}^y \left(\frac{\partial \gamma_{zx}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yz}}{\partial x} \right) dy,
 \end{aligned} \right\} \quad (3.5.3)$$

where x_0, y_0, z_0 – are initial point for integration; u_0, v_0, w_0 – are initial displacements considering the given body to be completely rigid; $(\partial u / \partial y)_0$ – is the rotation angle of the element dy in direction of axis x (in plane xOy) considering given body to be completely rigid, etc.

Initial rotation angles are connected with angular displacements by the next relations

$$\left. \begin{aligned} \left(\frac{\partial u}{\partial y} \right)_0 + \left(\frac{\partial v}{\partial x} \right)_0 &= (\gamma_{xy})_0, & \left(\frac{\partial v}{\partial z} \right)_0 + \left(\frac{\partial w}{\partial y} \right)_0 &= (\gamma_{yz})_0, \\ \left(\frac{\partial w}{\partial x} \right)_0 + \left(\frac{\partial u}{\partial z} \right)_0 &= (\gamma_{zx})_0. \end{aligned} \right\} \quad (3.5.4)$$

§ 3.6. Direct and Contrary Problems of the Theory of Elasticity. Saint-Venant Principle

There are two common problems in the theory of elasticity: direct and contrary problems. On the fig. 3.6.1 there is scheme where problems of structural mechanics are shown.

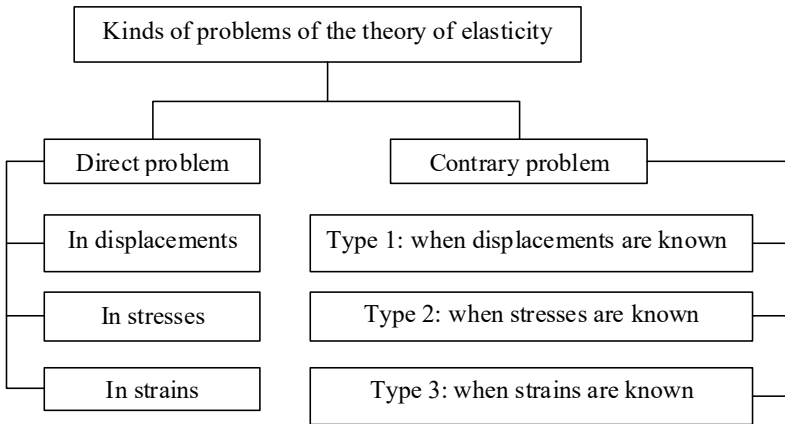


Fig. 3.6.1. Kinds of problems of the theory of elasticity

For direct problem it is necessary for given body, set constraints, external loadings and temperature field to define displacements, strains and stresses. For contrary problem it is necessary for given body with

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its known constraints and displacements or strains or stresses to define the rest two lacking groups of components of strain-stress state and external loadings with temperature field.

Direct problem in displacements means that displacements are unknown values for the first step of solution of this problem. To solve direct task in displacements it is necessary to solve so called Lamé equations which can be written as

$$\left. \begin{aligned} \nabla^2 u + \frac{1}{1-2\mu} \frac{\partial \theta}{\partial x} &= -\frac{X}{G} + \frac{2(1+\mu)}{1-2\mu} \frac{\partial T}{\partial x} \alpha, \\ \nabla^2 v + \frac{1}{1-2\mu} \frac{\partial \theta}{\partial y} &= -\frac{Y}{G} + \frac{2(1+\mu)}{1-2\mu} \frac{\partial T}{\partial y} \alpha, \\ \nabla^2 w + \frac{1}{1-2\mu} \frac{\partial \theta}{\partial z} &= -\frac{Z}{G} + \frac{2(1+\mu)}{1-2\mu} \frac{\partial T}{\partial z} \alpha. \end{aligned} \right\} \quad (3.6.1)$$

Boundary conditions for Lamé equations are

$$\left. \begin{aligned} p_{vx} &= G \left(\frac{\partial u}{\partial v} + \frac{\partial p}{\partial x} + \frac{2l}{1-2\mu} [\mu\theta - (1+\mu)\alpha T] \right); \\ p_{vy} &= G \left(\frac{\partial v}{\partial v} + \frac{\partial p}{\partial y} + \frac{2m}{1-2\mu} [\mu\theta - (1+\mu)\alpha T] \right); \\ p_{vz} &= G \left(\frac{\partial w}{\partial v} + \frac{\partial p}{\partial z} + \frac{2n}{1-2\mu} [\mu\theta - (1+\mu)\alpha T] \right). \end{aligned} \right\} \quad (3.6.2)$$

where $\theta = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$; $\frac{\partial()}{\partial v} = l \frac{\partial()}{\partial x} + m \frac{\partial()}{\partial y} +$

$+ n \frac{\partial()}{\partial z}$ – derivative along normal vector to surface;

$\rho = lu + mv + nw$ – projection of displacement into normal vector to

surface; $\nabla^2() = \frac{\partial^2()}{\partial x^2} + \frac{\partial^2()}{\partial y^2} + \frac{\partial^2()}{\partial z^2}$ – Laplace operator; α –

factor of linear thermal extension.

There are also kinematic boundary conditions for the parts of surface, i.e. some laws of displacements are set at the parts S_i of body surface

$$\left. \begin{aligned} u(\bar{x}, \bar{y}, \bar{z}) &= u_i(\bar{x}, \bar{y}, \bar{z}); \\ v(\bar{x}, \bar{y}, \bar{z}) &= v_i(\bar{x}, \bar{y}, \bar{z}); \\ w(\bar{x}, \bar{y}, \bar{z}) &= w_i(\bar{x}, \bar{y}, \bar{z}), \end{aligned} \right\} \quad (3.6.3)$$

where $i = 1, 2, \dots, n$ – index of given part of body surface; n – total number of parts.

Equations (3.6.1) and (3.6.2) are equilibrium equations of elementary parallelepiped and elementary tetrahedron appropriately but expressed through displacements using sequentially Hook's law for getting strains and Cauchy geometrical relations. Expressions (3.6.2) are force boundary conditions (BCs) and (3.6.3) are kinematical BCs.

On the fig. 3.6.2 there are major steps in solution of direct problem of the theory of elasticity in displacements are presented.

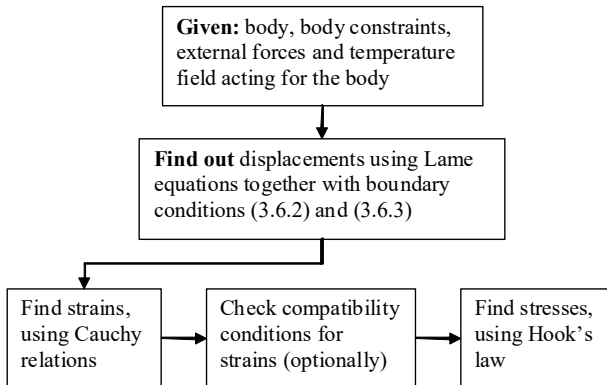


Fig. 3.6.2. Sequence in solution of the direct problem of the theory of elasticity in displacements

Direct problem in stresses means that stresses are unknown values for the first step of solution of this problem. There are six components of stresses. So we have to have at least six equations to find these stresses: three equations of elementary parallelepiped and three equations of elementary tetrahedron as boundary conditions. If there are displacements set on parts of body surface like (3.6.3) it is necessary to express these displacements through stresses using sequentially Cauchy relations to get strains and Hook's law to get stresses. But these six stresses aren't independent (analogously as strains are). There are compatibility conditions for stresses, which are called Beltrami-Mitchell compatibility conditions and can be written as (without considering of temperature)

$$\left. \begin{aligned} \nabla^2 \sigma_x + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial x^2} &= -2 \frac{\partial X}{\partial x} - \frac{\mu}{1-\mu} \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} \right); \\ \nabla^2 \sigma_y + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial y^2} &= -2 \frac{\partial Y}{\partial y} - \frac{\mu}{1-\mu} \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} \right); \\ \nabla^2 \sigma_z + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial z^2} &= -2 \frac{\partial Z}{\partial z} - \frac{\mu}{1-\mu} \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} \right); \\ \nabla^2 \tau_{xy} + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial x \partial y} &= - \left(\frac{\partial X}{\partial y} + \frac{\partial Y}{\partial x} \right); \\ \nabla^2 \tau_{yz} + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial y \partial z} &= - \left(\frac{\partial Y}{\partial z} + \frac{\partial Z}{\partial y} \right); \\ \nabla^2 \tau_{zx} + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial z \partial x} &= - \left(\frac{\partial Z}{\partial x} + \frac{\partial X}{\partial z} \right), \end{aligned} \right\} \quad (3.6.4)$$

where $\Theta = \sigma_x + \sigma_y + \sigma_z$; $\nabla^2() = \frac{\partial^2()}{\partial x^2} + \frac{\partial^2()}{\partial y^2} + \frac{\partial^2()}{\partial z^2}$ –

the Laplace operator.

Expressions (3.6.4) are compatibility conditions for stresses. They can be got using Saint-Venant compatibility conditions applying Hook's law and equilibrium equations of elementary parallelepiped.

On the fig. 3.6.3 there are major steps in solution of the direct problem of the theory of elasticity in stresses are presented.

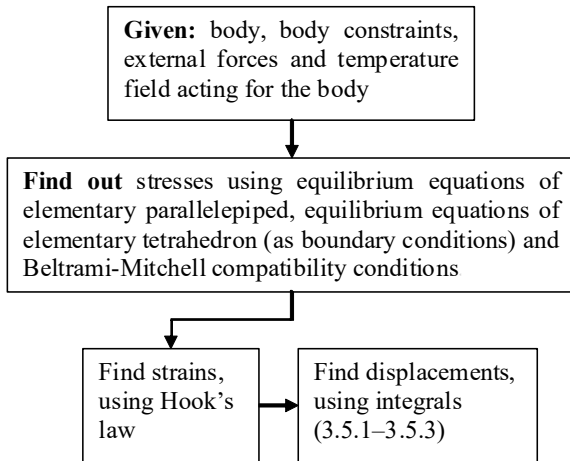


Fig. 3.6.3. Sequence in solution of direct problem of the theory of elasticity in stresses

Direct problem in strains (see fig. 3.6.1) can be solved in the same way as direct problem in stresses. But first it is necessary to express strains through stresses using Hook's law.

As it is shown on the fig. 3.6.1 there are three kinds of the contrary problem. For the first kind of the contrary problem there are laws of displacements are known. It is necessary to find stresses and strains.

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Using Cauchy relations we can find the laws for strains and after applying Hook's law for known strains we can find stresses. Using equilibrium equations for known stresses or displacements (Lame equations) we can find external volume and surface forces. Also temperature field can be found using compatibility equations for stresses where temperature field is taken into consideration (these equations are not presented here).

For the second kind of the contrary problem there are laws of stresses are known. It is necessary to find strains and displacements. Using Hook's law we can find strains. For known strains we can find displacements using relations (3.5.1–3.5.3). Using equilibrium equations for known stresses or displacements (Lame equations) we can find external volume and surface forces.

For the third kind of the contrary problem there are laws of strains are known. It is necessary to find the laws for displacements and stresses. Using integrals (3.5.1–3.5.3) we can find displacements. Also using Hook's law we can find stresses. Using equilibrium equations for known stresses or displacements (Lame equations) we can find external volume and surface forces.

Saint-Venant principle can be formulated in the next way. If there is body loaded by external statically equivalent systems of forces, thus sizes of field of applying of loadings are relatively small in comparison with sizes of body, in cross sections which are situated far enough from places of applying of external forces, stresses depends little on way of loading. Statically equivalent systems are systems which have the same principal vectors and principal moments.

This principle is widely used in the theory of elasticity in strength of materials in structural mechanics.

The principle of independency of force action (superposition method) is true only for linear mechanics. It can be formulated in next way. If there is system of external loadings, which consists of several forces acting for the body simultaneously, the displacement (stresses, strain) in some point, caused by all external forces equals to the sum

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of displacements (stresses, strains) caused by every external force acting separately.

Mathematically this principle can be formulated

$$u_A(P_1, P_2, \dots, P_n) = u_A(P_1) + u_A(P_2) + \dots + u_A(P_n), \quad (3.6.5)$$

where $P_1, P_2, \dots, P_n$ are external forces both concentrated and distributed ones; u_A – is some displacement (stresses, strain) in some point A .

Equilibrium equations for solid body (for right coordinate system) can be written

$$\left. \begin{aligned} \int_F p_{vx} dF + \int_V X dV &= 0, \\ \int_F p_{vy} dF + \int_V Y dV &= 0, \\ \int_F p_{vz} dF + \int_V Z dV &= 0, \\ \int_F (p_{vz}y - p_{vy}z) dF + \int_V (Zy - Yz) dV &= 0, \\ \int_F (p_{vx}z - p_{vz}x) dF + \int_V (Xz - Zx) dV &= 0, \\ \int_F (p_{vy}x - p_{vx}y) dF + \int_V (Yx - Xy) dV &= 0, \end{aligned} \right\} \quad (3.6.6)$$

where F is the surface field for integral and V is the volume field for integral.

**Lecture 4. Exploration of the Strain-Stress State in Point
Of Elastic Body. Strength Theories**

§ 4.1. Principal Planes and Principal Stresses. Stress Ellipsoid

Principal planes and principal stresses. Let's review elementary parallelepiped defined in locality of some point. As it was told in lecture #2 faces of elementary parallelepiped are always parallel to coordinate planes. Rotating coordinate planes we shall have different stresses and appropriate strains at its faces. At last we can rotate our coordinate axes in such way that shear stresses at all six faces or three coordinate planes disappear, but there will be only normal stresses. These planes are called principal. So principal planes are planes on which there are no shear stresses. Principal normal stresses are stresses acting in principal planes.

Values of principal stresses in point can be found solving next so called cubical equation

$$\sigma_v^3 - I_1 \sigma_v^2 + I_2 \sigma_v - I_3 = 0, \quad (4.1.1)$$

where

$$\left. \begin{aligned} I_1(T_\sigma) &= \sigma_x + \sigma_y + \sigma_z = \sigma_1 + \sigma_2 + \sigma_3, \\ I_2(T_\sigma) &= \sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \\ &\quad - \tau_{zx}^2 = \sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1, \\ I_3(T_\sigma) &= \sigma_x \sigma_y \sigma_z + 2 \tau_{xy} \tau_{yz} \tau_{zx} - \sigma_x \tau_{yz}^2 - \\ &\quad - \sigma_y \tau_{zx}^2 - \sigma_z \tau_{xy}^2 = \sigma_1 \sigma_2 \sigma_3. \end{aligned} \right\} \quad (4.1.2)$$

Quantities I_1, I_2, I_3 are called stress invariants. Stress invariants don't depend on orientation of coordinate system in space relatively body. It means that reviewing strain-stress states in some point relatively

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different coordinate axes we shall have that stress and strain tensors will be different, but stress invariants will always be the same.

Also values $\sigma_1, \sigma_2, \sigma_3$ are principal stresses, which are roots of cubical equation (4.1.1). Principal stresses $\sigma_1, \sigma_2, \sigma_3$ are always real quantities. Presenting principal normal stresses we order them applying next rule: $\sigma_1 \geq \sigma_2 \geq \sigma_3$.

Applying equilibrium equations of elementary tetrahedron we can get equations for defining of direction cosines l, m, n of normal vectors of principal planes relatively given comparison axes. Direction cosines can be found using next equations

$$\left. \begin{aligned} (\sigma_x - \sigma_v)l + \tau_{yx}m + \tau_{zx}n &= 0, \\ \tau_{xy}l + (\sigma_y - \sigma_v)m + \tau_{zy}n &= 0, \\ \tau_{xz}l + \tau_{yz}m + (\sigma_z - \sigma_v)n &= 0 \end{aligned} \right\}, \quad (4.1.3)$$

where σ_v – is one of the principle normal stresses.

There are numerous solutions of (4.1.3) including zero solution. To get necessary solution we need additional condition

$$l^2 + m^2 + n^2 = 1. \quad (4.1.4)$$

Solving together (4.1.3) and (4.1.4) we can find direction cosines for given principle stresses. Substituting sequentially $\sigma_v = \sigma_1, \sigma_v = \sigma_2, \sigma_v = \sigma_3$ into (4.1.3) and (4.1.4) we can get direction cosines for all three normal vectors of principle planes.

Let's review how to get direction cosines for σ_1 . As determinant of (4.1.3) equals zero there are only two independent equations in (4.1.3) from which two direction cosines can be expressed through third one. For example

$$m_1 = a_1 l_1, \quad n_1 = b_1 l_1, \quad (4.1.5)$$

where a_1, b_1 are some factors which have to be found.

Substituting (4.1.5) into (4.1.4) we shall have next relations:

$$\begin{aligned} l_1 &= \pm \frac{1}{\sqrt{1+a_1^2+b_1^2}}, \quad m_1 = \pm \frac{a_1}{\sqrt{1+a_1^2+b_1^2}}, \\ n_1 &= \pm \frac{b_1}{\sqrt{1+a_1^2+b_1^2}}. \end{aligned} \quad (4.1.6)$$

Also substituting (4.1.5) into (4.1.3) we shall have next relations

$$\left. \begin{aligned} \tau_{yx}a_1 + \tau_{zx}b_1 &= \sigma_1 - \sigma_x, \\ (\sigma_y - \sigma_1)a_1 + \tau_{zy}b_1 &= -\tau_{xy}, \\ \tau_{yz}a_1 + (\sigma_z - \sigma_1)b_1 &= -\tau_{xz}. \end{aligned} \right\} \quad (4.1.7)$$

Solving mutually two any equations of three ones given in (4.1.7) we have to find a_1 and b_1 . After using (4.1.6) for known a_1 and b_1 we can find direction cosines l_1, m_1, n_1 of normal vector of principal plane where σ_1 takes place. Analogously direction cosines for σ_2 and σ_3 can be found.

Found direction cosines for three principle planes have to satisfy for the next relations:

$$\left. \begin{aligned} \sum_{i=1}^{i=3} l_i^2 &= 1, \quad \sum_{i=1}^{i=3} m_i^2 = 1, \quad \sum_{i=1}^{i=3} n_i^2 = 1, \\ l_i^2 + m_i^2 + n_i^2 &= 1, \quad \text{where } i=1, 2, 3; \\ \sum_{i=1}^{i=3} l_i m_i &= 0, \quad \sum_{i=1}^{i=3} m_i n_i = 0, \quad \sum_{i=1}^{i=3} n_i l_i = 0, \\ l_i l_j + m_i m_j + n_i n_j &= 0, \quad \text{where } ij=12, 23, 31. \end{aligned} \right\} \quad (4.1.8)$$

Taking into consideration absence of shear stresses on principle planes stress tensor in principle coordinate axes $1, 2, 3$ can be written as

$$T_{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}. \quad (4.1.9)$$

Stress ellipsoid. Let's review stress state in some point defined by stress tensor (4.1.9) in principle axes $1, 2, 3$ and some plane in the same point, direction cosines l, m, n of which are defined relatively principle axes $1, 2, 3$. Using equilibrium equations of elementary tetrahedron and changing designations x, y, z there into $1, 2, 3$ accordingly also applying tensor components (4.1.9) we shall have projections of full stresses p_v into coordinate axes in given plane

$$p_{v1} = \sigma_1 l, \quad p_{v2} = \sigma_2 m, \quad p_{v3} = \sigma_3 n. \quad (4.1.10)$$

From (4.1.10) we have that

$$l = \frac{p_{v1}}{\sigma_1}, \quad m = \frac{p_{v2}}{\sigma_2}, \quad n = \frac{p_{v3}}{\sigma_3}. \quad (4.1.11)$$

Taking into account that $l^2 + m^2 + n^2 = 1$ we shall have next relation

$$\left(\frac{p_{v1}}{\sigma_1} \right)^2 + \left(\frac{p_{v2}}{\sigma_2} \right)^2 + \left(\frac{p_{v3}}{\sigma_3} \right)^2 = 1. \quad (4.1.12)$$

As components p_{v1}, p_{v2}, p_{v3} can be considered as coordinates of end of vector of full stresses $\mathbf{p}_v$, there is conclusion based on the (4.1.12) that this end of vector of full stresses defines closed surface which is called ellipsoid. Axes of this ellipsoid match principle axes and equal to according principle strains σ_1, σ_2 , and σ_3 . This ellipsoid is called the Lamé ellipsoid (fig. 4.1.1).

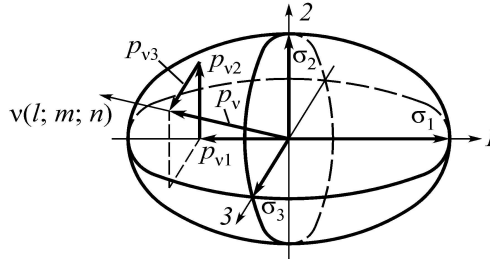


Fig. 4.1.1. Stress ellipsoid of Lamé

§ 4.2. Maximum Shear Stresses and Its Planes

Among all planes, which can be defined in some point there is the one in which shear stresses have the largest value on modulus. It is convenient to find solution in principle axes $1, 2, 3$, considering stress tensor (4.1.9) to be known. For some plane defined in some point there is normal vector $\mathbf{v}$ which have direction cosines l, m, n relatively principle axes. In that plane there are components of full stresses p_{v1}, p_{v2}, p_{v3} defined by (4.1.10).

Squares of full stresses, full normal stresses and full shear stresses can be found using obvious expressions

$$\left. \begin{aligned} p_v^2 &= p_{v1}^2 + p_{v2}^2 + p_{v3}^2 = \\ &= \sigma_1^2 l^2 + \sigma_2^2 m^2 + \sigma_3^2 n^2, \\ \sigma_v^2 &= (p_{v1}l + p_{v2}m + p_{v3}n)^2 = \\ &= (\sigma_1 l^2 + \sigma_2 m^2 + \sigma_3 n^2)^2, \\ \tau_v^2 &= p_v^2 - \sigma_v^2 = \\ &= \sigma_1^2 l^2 + \sigma_2^2 m^2 + \sigma_3^2 n^2 - (\sigma_1 l^2 + \sigma_2 m^2 + \sigma_3 n^2)^2. \end{aligned} \right\} \quad (4.2.1)$$

Lecture 4

There is task to find optimal values of direction cosines l, m, n relatively principle axes which provide maximum to τ_v^2 . Reviewing the most common occasion, when $\sigma_1, \sigma_2, \sigma_3$ are different we shall have next optimal values of direction cosines l, m, n which provide only maximums to τ_v^2 in (4.2.1):

$$\left. \begin{aligned} l = 0, \quad m = \pm \frac{1}{\sqrt{2}}, \quad n = \pm \frac{1}{\sqrt{2}}, \\ l = \pm \frac{1}{\sqrt{2}}, \quad m = 0, \quad n = \pm \frac{1}{\sqrt{2}}, \\ l = \pm \frac{1}{\sqrt{2}}, \quad m = \pm \frac{1}{\sqrt{2}}, \quad n = 0. \end{aligned} \right\} \quad (4.2.2)$$

Substituting direction cosines (4.2.2) into third equation in (4.2.1) we can get next values of maximum shear stresses acting in according planes defined by (4.2.2)

$$\tau_1 = \frac{\sigma_2 - \sigma_3}{2}, \quad \tau_2 = \tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}, \quad \tau_3 = \frac{\sigma_1 - \sigma_2}{2}, \quad (4.2.3)$$

where $\tau_{\max}$ – are the largest shear stresses of maximum shear stresses.

So τ_1, τ_2, τ_3 are maximum shear stresses, which act in planes parallel to according principal axes $1, 2, 3$. These planes divide orthogonal angle in two between the rest principle directions (fig. 4.2.1).

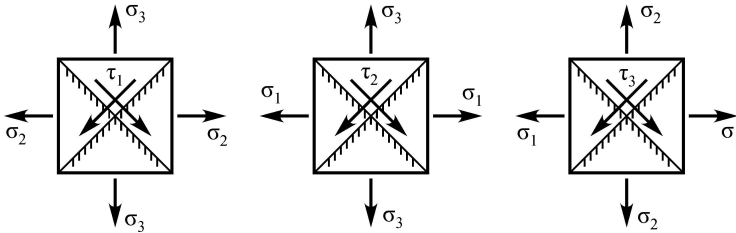


Fig. 4.2.1. Planes of the maximum shear stresses

§ 4.3. Principal Linear and Maximum Shear (Angular) Strains

There are several ways to find principal linear strains. One of the ways is to calculate principal linear strains $\varepsilon_1, \varepsilon_2, \varepsilon_3$ using known values of principal stresses using next expressions

$$\left. \begin{aligned} \varepsilon_1 &= \frac{1}{E} [\sigma_1 - \mu(\sigma_2 + \sigma_3)]; \\ \varepsilon_2 &= \frac{1}{E} [\sigma_2 - \mu(\sigma_3 + \sigma_1)]; \\ \varepsilon_3 &= \frac{1}{E} [\sigma_3 - \mu(\sigma_1 + \sigma_2)]; \\ \gamma_{12} &= \gamma_{23} = \gamma_{31} = 0, \end{aligned} \right\} \quad (4.3.1)$$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ – principle linear strains; $\gamma_{12}, \gamma_{23}, \gamma_{31}$ – are principle shear strains.

But if there are only strains are known it is possible to find principle strains solving next equation (analogical to (4.1.1))

$$\varepsilon_v^3 - I_1 \varepsilon_v^2 + I_2 \varepsilon_v - I_3 = 0, \quad (4.3.2)$$

where

$$\left. \begin{aligned} I_1(T_\varepsilon) &= \varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_1 + \varepsilon_2 + \varepsilon_3, \\ I_2(T_\varepsilon) &= \varepsilon_x \varepsilon_y + \varepsilon_y \varepsilon_z + \varepsilon_z \varepsilon_x - \varepsilon_{xy}^2 - \\ &\quad - \varepsilon_{yz}^2 - \varepsilon_{zx}^2 = \varepsilon_1 \varepsilon_2 + \varepsilon_2 \varepsilon_3 + \varepsilon_3 \varepsilon_1, \\ I_3(T_\varepsilon) &= \varepsilon_x \varepsilon_y \varepsilon_z + 2\varepsilon_{xy} \varepsilon_{yz} \varepsilon_{zx} - \varepsilon_x \varepsilon_{yz}^2 - \\ &\quad - \varepsilon_y \varepsilon_{zx}^2 - \varepsilon_z \varepsilon_{xy}^2 = \varepsilon_1 \varepsilon_2 \varepsilon_3. \end{aligned} \right\} \quad (4.3.3)$$

Quantities I_1, I_2, I_3 are called strain invariants. Strain invariants don't depend on orientation of coordinate system in space relatively

body. There are always only three real roots of (4.3.2) which are principal linear strains. Directions of principal linear strains are the same as directions of principal normal stresses.

Maximum shear strains $\gamma_1, \gamma_2, \gamma_3$ can be found using expressions

$$\left. \begin{aligned} \gamma_1 &= \frac{\tau_1}{G} = \frac{(\sigma_2 - \sigma_3)}{2G} = \varepsilon_2 - \varepsilon_3, \\ \gamma_2 &= \gamma_{\max} = \frac{\tau_2}{G} = \frac{(\sigma_1 - \sigma_3)}{2G} = \varepsilon_1 - \varepsilon_3, \\ \gamma_3 &= \frac{\tau_3}{G} = \frac{(\sigma_1 - \sigma_2)}{2G} = \varepsilon_1 - \varepsilon_2. \end{aligned} \right\} \quad (4.3.4)$$

Directions of maximum shear strains are the same as directions of maximum shear stresses.

§ 4.4. Mechanical Properties of Strength and Plasticity and Ways of Definition of It

With the exception of the elastic properties there are also strength properties of materials. Some of them can be defined using diagrams like presented on the figures 2.6.1 and 2.6.2. So called strength properties include such quantities shown on figures 2.6.1 and 2.6.2:

σ_e, τ_e – elastic limit (point), Pa, the largest value of stresses on modulus before ones there are no plastic strains;

σ_{pp}, τ_{pp} – the limit of proportionality, Pa, is the largest value of stresses on modulus, after ones there is no following of Hook's law: $\sigma = \varepsilon E$ or $\tau = \gamma G$;

σ_s, τ_s (R_{ch} or $R_{p0.2}$) – yield limit (point), Pa, is the smallest value of stresses on modulus, when there is growth of strains under constant axial force; but if there is no explicit section of plasticity we have so called conditional yield limit $\sigma_{0.2}, \tau_{0.3}$, which defines the level of stresses on modulus causing appearance of plastic strains the value of which

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equals to set quantity. The set quantity for $\sigma_{0,2}$ equals 0,2 % = 0,002, but for $\tau_{0,3}$ the set quantity equals 0,3 % = 0,003 rad;

$\sigma_u (R_m)$ – ultimate strength the largest possible value of stresses on modulus;

$\delta (A_5)$ – relative elongation after break is the value of plastic strains after specimen breaks. Relative elongation can be defined both using graphs like fig. 2.6.1 and also using specimen. If to use specimen the value of relative elongation δ can be defined using expression

$$\delta = \frac{l_0 - l_b}{l_0} \cdot 100 \%, \quad (4.4.1)$$

where l_0 – is initial length before test of tension of specimen; l_b – is the value of length after breaking of the specimen.

Relative narrowing of cross section after break is the value which can be defined using specimen. It can be defined using expression

$$\psi = \frac{F_0 - F_b}{F_0} \cdot 100 \%, \quad (4.4.2)$$

where F_0 – is the value of cross section before test of tension; F_b – is the value of the cross section of specimen after break of specimen.

§ 4.5. Basic (Classic) Theories of Strength and Plasticity. Allowable Stresses and Conditions of Strength

Solution of direct problem of the theory of elasticity, i.e. definition of stresses, strains and displacements is not end in itself. We solve direct or contrary problem in frames of projection of some construction. In strength of materials we compare real stresses in construction with allowable ones. Thus we have to find the most tensed point in body and to compare it with some admissible stresses. So there is problem to define the most tensed i.e. the most dangerous point in body.

Sometimes it is necessary to compare several points in loaded body for their stress state. It is necessary to know which point is more

tensed but which is less tensed. There is conclusion that it is necessary to have some unique parameter which would be able to estimate stress state in point uniquely. This parameter is usually called *equivalent stresses*.

For every point of body there is complex stress state in the most common occasion because there are six independent components of stresses there. The problem of finding of unique parameter or equivalent stresses mentioned above can be simplified if to set the criteria of comparability of different complex stress states with simple ones such as tension-compression or shear. Definition of mentioned criteria is the part of the problem of so called theories or hypothesis of strength and plasticity. There are a lot of such theories. But we shall review only several of them.

There are four classic theories of strength.

The 1-st strength theory is called the theory of maximum normal stresses. For the first strength theory equivalent stresses are the largest principle stresses on modulus but taken with its sign. Two stress states are identical if the largest principal stresses for these two states are equal. If we have ordered principle stresses $\sigma_1 \geq \sigma_2 \geq \sigma_3$ equivalent stresses can be found in few steps. For the first step we have to find maximum value on modulus between σ_1 , σ_2 and σ_3 . For the second step we have to find equivalent stresses as maximum value found in first step but taken with its own sign.

The 2-nd strength theory is called the theory of maximum linear strains. There are two expressions for equivalent stresses

$$\left. \begin{aligned} \sigma_{eq} &= \sigma_1 - \mu(\sigma_2 + \sigma_3) > 0, \\ \sigma'_{eq} &= \sigma_3 - \mu(\sigma_1 + \sigma_2) < 0. \end{aligned} \right\} \quad (4.5.1)$$

First expression of (4.5.1) defines equivalent stresses for simple tension but the second one defines equivalent stresses for simple compression. Equivalent stresses defined by (4.5.1) have to be reviewed together in estimation of strength.

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The 3-rd strength theory is called the theory of maximum shear stresses. Equivalent stresses for this theory can be found using expression

$$\sigma_{eq} = \sigma_1 - \sigma_3. \quad (4.5.2)$$

The 4-th strength theory is called the theory of specific potential energy of deformation of the body form. Equivalent stresses for this theory can be found using expression

$$\left. \begin{aligned} \sigma_{eq} = \sigma_i &= \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \\ &= \frac{2}{\sqrt{2}} \sqrt{\tau_1^2 + \tau_2^2 + \tau_3^2} = \frac{1}{\sqrt{2}} \times \\ &\times \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}, \end{aligned} \right\} (4.5.3)$$

where σ_i – is intensity of stresses, which is used in the theory of plasticity.

Third and fourth theories are widely used because these theories give satisfactory results. Third theory fits well for brittle materials but fourth one for plastic materials.

Equivalent stresses have to be compared with allowable stresses. Allowable stresses are stresses which are considered to be dangerous. It is forbidden for the construction to have equivalent stresses more than allowable ones. In opposite case we consider that our construction don't carry its functional possibilities.

In the common occasion the expression for allowable stresses can be written as

$$[\sigma] = \frac{\sigma_0}{n}, \quad (4.5.4)$$

where σ_0 – are some dangerous stresses, defined in diagrams like on figures 2.6.1 or 2.6.2; n – is some safety factor.

Admissible stresses are usually connected with some points in diagram of axial tension-compression or shear. Usually these points are yield point or ultimate (tensile) point. Allowable stresses can be found using expressions

$$[\sigma]_s = \frac{\sigma_s}{n_s}, \quad [\sigma]_u = \frac{\sigma_u}{n_u}, \quad [\tau]_s = \frac{\tau_s}{n_s}, \quad [\tau]_u = \frac{\tau_u}{n_u}, \quad (4.5.5)$$

where $[\sigma]_s, [\tau]_s$ – are allowable (admissible) stresses; σ_s – is yield limit (point) in diagram of axial tension (see fig. 2.6.1); τ_s – is yield limit (point) in diagram of pure shear (see fig. 2.6.2); σ_u – is ultimate limit (point) in diagram of axial tension (see fig. 2.6.1); τ_u – is ultimate limit (point) in diagram of pure shear (see fig. 2.6.2); n – is the safety factor, which is always is more than one. Additionally there are allowable stresses for compression and for tension.

The value of safety factor depends on many factors and experience of exploitation of constructions. There are special tables and dependencies for safety factors.

The condition of satisfactory strength can be written in common view

$$\sigma_{eq} \leq [\sigma], \quad (4.5.6)$$

where σ_{eq} – are equivalent stresses, defined using one of the strength theories; $[\sigma]$ – are allowable stresses, defined using (4.5.4) or one of expressions in (4.5.5).

Usually for plastic materials they use 1-st and 3-rd expressions of (4.5.5) for allowable stresses, but for brittle materials they use usually 2-nd and 4-th expressions for definition of allowable stresses. But this is not the rule, because for brittle materials there are so called conditional yield points, which can be considered to be dangerous stress levels in (4.5.4).

Lecture 5. Basic Equations and Relations for Flat Strain-Stress State. Stress Concentration in Plates Near Holes (Necklines)

**§ 5.1. Basic Concepts of Flat Strain-Stress State.
Major Equations and Relations for Flat Strain-Stress State**

Flat strain-stress state is reviewed for thin plates the contour of which satisfies to next equation $f(\bar{x}, \bar{y}) = 0$ (fig. 5.1.1).

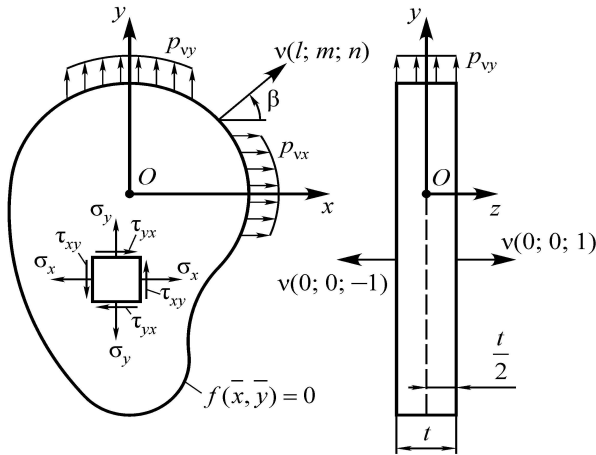


Fig. 5.1.1. For explaining of flat strain-stress state

Contour of that plane is considered to be boundary surface which have normal direction vector $\mathbf{v}(l = \cos \beta, m = \sin \beta, n = 0)$. Flat strain-stress state can be applied for sheeting of ships for different pipes and volumes with thin walls.

Flat strain-stress state takes place in thin plates when there are next conditions provided:

Lecture 5

1. There are next surface loadings on boundary surface:

$$p_{vx} = p_{vx}(\bar{x}, \bar{y}), \quad p_{vy} = p_{vy}(\bar{x}, \bar{y}), \quad p_{vz} = 0. \quad (5.1.1)$$

2. There are no external surface loadings at foundation planes of our plate, i.e. side planes are free

$$p_{vx} = 0, \quad p_{vy} = 0, \quad p_{vz} = 0. \quad (5.1.2)$$

3. Volume external forces and temperature field distributed uniformly for thickness, functions of which don't depends on coordinate z

$$X = X(x, y), \quad Y = Y(x, y), \quad Z = Z(x, y), \quad T = T(x, y). \quad (5.1.3)$$

4. There are only σ_x , σ_y , τ_{xy} are nonzero stresses. The rest components of stresses are zero $\sigma_z = 0$, $\tau_{yz} = 0$, $\tau_{zx} = 0$. So stress tensor can be written

$$T_\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}. \quad (5.1.4)$$

Equations and relations for massive body can be simplified for flat strain-stress state.

Equilibrium equations of elementary tetrahedron can be written for flat strain-stress state

$$\left. \begin{aligned} \sigma_x l + \tau_{yx} m &= p_x, \\ \tau_{xy} l + \sigma_y m &= p_y. \end{aligned} \right\} \quad (5.1.5)$$

Equilibrium equations of elementary parallelepiped can be written

$$\left. \begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} - \rho \frac{\partial^2 u}{\partial t^2} &= -X, \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} - \rho \frac{\partial^2 v}{\partial t^2} &= -Y. \end{aligned} \right\} \quad (5.1.6)$$

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Hook's law for flat strain-stress state can be written (without considering of temperature)

$$\left. \begin{aligned} \varepsilon_x &= \frac{1}{E}(\sigma_x - \mu\sigma_y); & \varepsilon_y &= \frac{1}{E}(\sigma_y - \mu\sigma_x); \\ \varepsilon_z &= -\frac{\mu}{E}(\sigma_x + \sigma_y); & \gamma_{xy} &= \frac{\tau_{xy}}{G}; & \gamma_{yz} &= 0; & \gamma_{zx} &= 0. \end{aligned} \right\} \quad (5.1.7)$$

From (5.1.7) there is conclusion can be made

$$\varepsilon_z = -\frac{\mu}{1-\mu}(\varepsilon_x + \varepsilon_y). \quad (5.1.8)$$

In another form Hook's law can be presented (without considering of temperature)

$$\left. \begin{aligned} \sigma_x &= \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y); & \sigma_y &= \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x); \\ \sigma_z &= 0; & \tau_{xy} &= G\gamma_{xy}; & \tau_{yz} &= 0; & \tau_{zx} &= 0. \end{aligned} \right\} \quad (5.1.9)$$

Compatibility conditions expressed through strains and stresses can be written

$$\left. \begin{aligned} \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} &= \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}, \\ \nabla^2(\sigma_x + \sigma_y) &= 0. \end{aligned} \right\} \quad (5.1.10)$$

Expressions (5.1.10) are simplifications of compatibility conditions of Saint-Venant's and Beltrami-Mitchell's.

Lame equations can also be simplified. Without considering of temperature and volume external forces Lame equations can be written

$$\left. \begin{aligned} \nabla^2 u + \frac{1+\mu}{1-\mu} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right) &= 0, \\ \nabla^2 v + \frac{1+\mu}{1-\mu} \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} \right) &= 0. \end{aligned} \right\} \quad (5.1.11)$$

Cubic nonlinear equation for definition of principle stresses can be written

$$[\sigma_v^2 - \sigma_v(\sigma_x + \sigma_y) + (\sigma_x \sigma_y - \tau_{xy}^2)] \sigma_v = 0. \quad (5.1.12)$$

Roots of the equation (5.1.12) are principal stresses defined by next expressions

$$\left. \begin{aligned} \sigma_1 &= \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}, \\ \sigma_2 &= \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}, \\ \sigma_3 &= 0. \end{aligned} \right\} \quad (5.1.13)$$

It's obvious that any orthogonal plane to axis z in which $\sigma_z = \tau_{yz} = \tau_{zx} = 0$ is principle. The rest two principle planes are parallel to axis z . For definition of angles of inclination to axis x the next expression can be used

$$\operatorname{tg} 2\beta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}. \quad (5.1.14)$$

The maximum shear stresses can be found using expressions

$$\left. \begin{aligned} \tau_1 &= \pm \frac{\sigma_2}{2}, \quad \tau_2 = \pm \frac{\sigma_1}{2}, \\ \tau_3 &= \pm \frac{\sigma_1 - \sigma_2}{2} = \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}. \end{aligned} \right\} \quad (5.1.15)$$

The largest on modulus value of maximum shear stresses can be defined by expression

$$\tau_{\max} = \max(|\tau_1|, |\tau_2|, |\tau_3|). \quad (5.1.16)$$

Principle strains and maximum shear strains can be found using expressions

$$\left. \begin{aligned} \varepsilon_1 &= \frac{1}{E}(\sigma_1 - \mu\sigma_2), \quad \varepsilon_2 = \frac{1}{E}(\sigma_2 - \mu\sigma_1), \\ \varepsilon_3 &= -\frac{\mu}{E}(\sigma_1 + \sigma_2), \quad \gamma_{\max} = \frac{\tau_{\max}}{G}. \end{aligned} \right\} \quad (5.1.17)$$

The expression for the 4-th theory of strength can be written

$$\sigma_{eq} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}. \quad (5.1.18)$$

Expressions for definition of displacements for known strains can be presented

$$\left. \begin{aligned} u &= u_0 + (y - y_0) \left(\frac{\partial u}{\partial y} \right)_0 + \int_{x_0}^x \varepsilon_x dx + \int_{y_0}^y \int_{x_0}^x \left(\frac{\partial \gamma_{xy}}{\partial y} - \frac{\partial \varepsilon_y}{\partial x} \right) dy^2, \\ v &= v_0 + (x - x_0) \left(\frac{\partial v}{\partial x} \right)_0 + \int_{y_0}^y \varepsilon_y dy + \int_{x_0}^x \int_{y_0}^y \left(\frac{\partial \gamma_{xy}}{\partial x} - \frac{\partial \varepsilon_x}{\partial y} \right) dx^2. \end{aligned} \right\} \quad (5.1.19)$$

§ 5.2. Schemes of Solution of Direct Problem of Flat Strain-Stress State. Stress Function

To solve the direct problem in stresses of the flat strain-stress state there is direct way to solve the system of equations (5.1.6) with second equation in (5.1.10)

$$\left. \begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} - \rho \frac{\partial^2 u}{\partial t^2} &= -X, \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} - \rho \frac{\partial^2 v}{\partial t^2} &= -Y, \\ \nabla^2 (\sigma_x + \sigma_y) &= 0. \end{aligned} \right\} \quad (5.2.1)$$

Boundary conditions in addition to the system (5.2.1) are equilibrium equations of elementary tetrahedron (5.1.5) and also kinematic boundary conditions at parts f_i of contour $f(\bar{x}, \bar{y}) = 0$ of the body set on contour

$$\left. \begin{aligned} u(\bar{x}, \bar{y}) &= u_i(\bar{x}, \bar{y}); \\ v(\bar{x}, \bar{y}) &= v_i(\bar{x}, \bar{y}). \end{aligned} \right\} \quad (5.2.2)$$

But the solution of the direct problem of the flat strain-stress state can be simplified if to use so called stress function $\psi(x, y)$. This function is connected with stresses by relations

$$\sigma_x = \frac{\partial^2 \psi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 \psi}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 \psi}{\partial x \partial y}. \quad (5.2.3)$$

Substituting (5.2.3) into the 3-rd of (5.2.1) we have the next biharmonic equation

$$\frac{\partial^4 \psi}{\partial x^4} + 2 \frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial y^4} = \nabla^2 \nabla^2 \psi = 0. \quad (5.2.4)$$

The function $\psi = \psi(x, y)$, which satisfy to the biharmonic equation (5.2.4) is called *stress function*.

Additionally for the stress function there are boundary conditions

$$\frac{\partial^2 \psi}{\partial y^2} l - \frac{\partial^2 \psi}{\partial x \partial y} m = p_{vx}, \quad -\frac{\partial^2 \psi}{\partial x \partial y} l + \frac{\partial^2 \psi}{\partial x^2} m = p_{vy}. \quad (5.2.5)$$

§ 5.3. Some Partial Analytic Solutions for the Problem of Flat Strain-Stress State and Its Practical Usage

There is no common solution for biharmonic equation (5.2.4) for free contour of plate and free loadings and temperature field. But for some occasions for orthogonal, circular and elliptical plates there are partial analytic solutions, which can be used in practice.

For rectangular plates there are next boundary conditions at edges (fig. 5.3.1)

$$\left. \begin{aligned} &\text{when } x = 0, v(l = -1; m = 0) : \sigma_x = -p_{vx}, \\ &\tau_{xy} = -p_{vy}; \\ &\text{when } x = a, v(1; 0) : \sigma_x = p_{vx}, \tau_{xy} = p_{vy}; \\ &\text{when } y = 0, v(0; -1) : \sigma_y = -p_{vy}, \tau_{xy} = -p_{vx}; \\ &\text{when } y = b, v(0; 1) : \sigma_y = p_{vy}, \tau_{xy} = p_{vx}, \end{aligned} \right\} \quad (5.3.1)$$

where p_{vx}, p_{vy} are projections of external surface loadings into accordant axes.

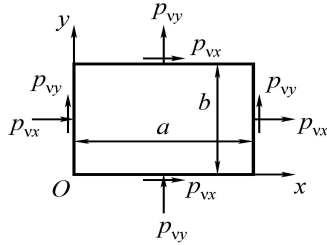


Fig. 5.3.1. For boundary conditions

There is so called solution *in polynomials* for rectangular plates. To get it is necessary to set the stress function in next view

$$\left. \begin{aligned} \psi(x, y) = & a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 + \\ & + a_6x^3 + a_7x^2y + a_8xy^2 + a_9y^3 + a_{10}xy^3 + a_{11}x^3y \end{aligned} \right\}, \quad (5.3.2)$$

where $a_i (i = 0, 1, 2, \dots, 11)$ are constant factors.

Substituting stress function (5.3.2) into biharmonic equation (5.2.4) we can insure that stress function satisfy to biharmonic equation identically.

Considering some factors a_i to be zero but some a_i to be nonzero we shall have different strain-stress states for plate: tension-compression, bending etc.

Using polynomials of higher order than (5.3.2) we shall have sequence of other solutions. For this case polynomial factors are chosen for it to satisfy biharmonic equation (5.2.4) and given boundary conditions.

Using solutions in polynomials it's impossibility to consider break loadings and concentrated forces.

Also there is so called solutions *in trigonometric series*. To get this solution it's necessary to set the stress function as trigonometric series

$$\begin{aligned}\psi(x, y) &= \sum_{n=1}^{n=\infty} f_n(y) \sin\left(\frac{n\pi x}{a}\right), \\ \psi(x, y) &= \sum_{n=1}^{n=\infty} f_n(y) \cos\left(\frac{n\pi x}{a}\right).\end{aligned}\quad (5.3.3)$$

Substituting (5.3.3) into stress function (5.2.4) we shall have the stresses presented as series with unknown factors. To get the solution in trigonometric series it's necessary to present external surface loadings in Fourier series and after comparing accordant factors for extended surface loadings and for stresses we can get unknown factors in series for stresses.

Applying first expression in (5.3.3) we shall have the solution which satisfies to the fig. 5.3.2,*a*. Applying second expression in (5.3.3) we shall have the solution which satisfies to the fig. 5.3.2,*b*.

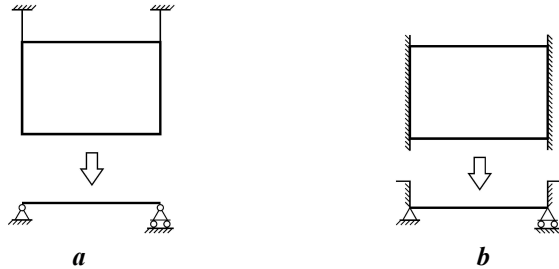


Fig. 5.3.2. For explanation of solutions in trigonometric series

Analytical solutions of the problem of flat strain-stress state can be used for theoretical explorations of bending of beams and also for the problem of so called joined girdle of beams or effective width of girdles of beams.

Let's explain the last problem: the problem of definition of effective width of girdles of beams. For example we have usual orthogonal bending of beams presented on the fig. 5.3.3.

On the fig. 5.3.3 we can see the real distribution of normal stresses on girdles. It's obvious the distribution is nonuniform. So the formulas

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of the strength of materials can't be applied for normal stresses. So the task for this occasion is to define the effective (equivalent) conditional width of the girdle for the normal stresses to apply the expressions of the strength of materials. Because within joined girdle the distribution of normal stresses is considered to be uniform. This effective width will be reduced in comparison with real width. Within this reduced girdle we consider that normal stresses are distributed uniformly.

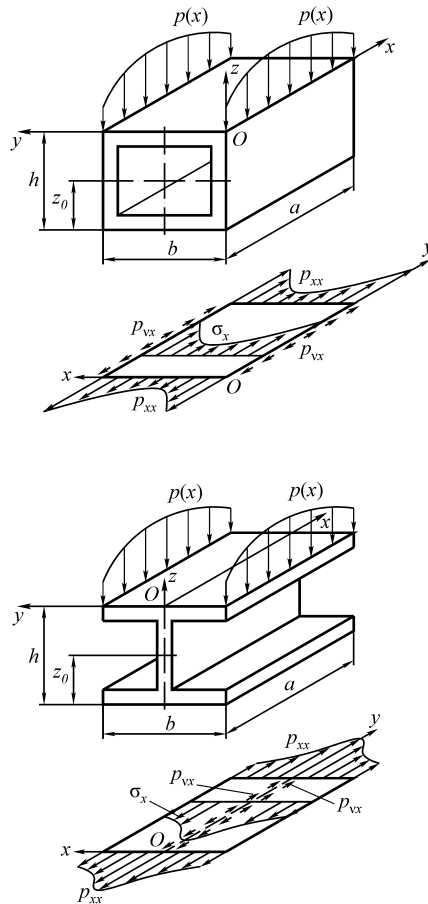


Fig. 5.3.3. For explaining of joined girdle

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For joined girdle there are next demands:

1) longitudinal linear strains and normal stresses have to be distributed uniformly within width of girdle (the hypothesis of flat cross sections is true);

2) for equal longitudinal forces in appropriate orthogonal cross sections of joined and real girdles linear strains of joined girdle have to equal to the linear strains of real girdle for lines of connection of girdle with wall.

Because for joined girdle it's necessary to apply the same force as for real one we shall have next

$$\sigma s t = t \int_0^b \sigma_x dy, \quad (5.3.4)$$

where σ – are normal stresses distributed uniformly within width of joined girdle; s – width of joined girdle; t – thickness of joined girdle; σ_x – is nonuniform distribution of normal stresses within width of real girdle.

Using (5.3.4) the width of joined girdle s can be written

$$s = \frac{1}{E \varepsilon_{xk}} \int_0^b \sigma_x dy, \quad (5.3.5)$$

where ε_{xk} – are linear strains in real girdle for line of connection of girdle and wall.

For convince there is reductive factor ψ which is relation of width of joined girdle and width of real girdle

$$\psi = \frac{s}{b}, \quad s = \psi b. \quad (5.3.6)$$

For σ_x they use analytic solutions in trigonometric series based on expressions (5.3.3). Stresses σ_x depends of constraints of beams and kinds of external loadings. For different conditions of constraints of beams and for different external loadings there are different values for reductive factor ψ . These values of ψ can be found in [2].

§ 5.4. The Problem of Flat Strain-Stress State in Polar Coordinates. Stress Concentrations in Plates With Holes

The formulas of connection between orthogonal Cartesian and polar coordinate systems are these

$$\left. \begin{aligned} x &= r \cos \varphi, \quad y = r \sin \varphi, \quad \frac{\partial r}{\partial x} = \cos \varphi, \\ \frac{\partial r}{\partial y} &= \sin \varphi, \quad r = \sqrt{x^2 + y^2}, \quad \varphi = \arctg \frac{y}{x}, \\ \frac{\partial \varphi}{\partial x} &= -\frac{\sin \varphi}{r}, \quad \frac{\partial \varphi}{\partial y} = \frac{\cos \varphi}{r}. \end{aligned} \right\} \quad (5.4.1)$$

Substituting expressions (5.4.1) into expressions of elements of flat strain-stress state written in § 5.1 and § 5.2 we shall have expressions for flat strain-stress state in polar coordinates.

The stresses, strains and displacements for flat strain-stress state in polar coordinate system are shown on the fig. 5.4.1.

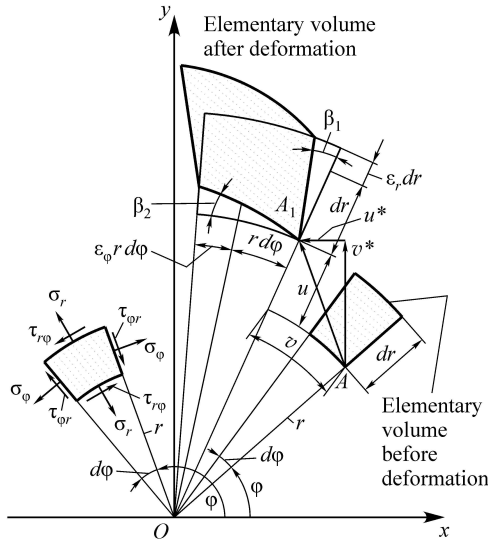


Figure 5.4.1. For explanation of elements for flat strain-stress state

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There is connection between displacements u and v in polar coordinate system and between displacements u^* and v^* in Cartesian orthogonal coordinate system

$$\left. \begin{aligned} u^* &= u \cos \varphi - v \sin \varphi, \\ v^* &= v \cos \varphi + u \sin \varphi. \end{aligned} \right\} \quad (5.4.2)$$

The linear strain of radial elements dr is notified as ε_r (radial linear strain) but the linear strain of tangential elements $r d\varphi$ is ε_φ (tangential linear strain). The shear angle between radial element dr and tangential one $r d\varphi$ is $\gamma_{r\varphi} = \gamma_{\varphi r} = \beta_1 + \beta_2$. Normal stresses σ_r act in planes, which are perpendicular to radius r , but σ_φ act on radial planes.

Considering transversity of coordinate directions r and φ , Hook's law for flat strain-stress state in polar coordinate system can be written

$$\left. \begin{aligned} \varepsilon_r &= \frac{1}{E}(\sigma_r - \mu\sigma_\varphi), \quad \varepsilon_\varphi = \frac{1}{E}(\sigma_\varphi - \mu\sigma_r), \\ \gamma_{r\varphi} &= \frac{\tau_{r\varphi}}{G}, \quad \sigma_r = \frac{E}{1-\mu^2}(\varepsilon_r + \mu\varepsilon_\varphi), \\ \sigma_\varphi &= \frac{E}{1-\mu^2}(\varepsilon_\varphi + \mu\varepsilon_r), \quad \tau_{r\varphi} = G\gamma_{r\varphi}. \end{aligned} \right\} \quad (5.4.3)$$

There are Cauchy relations, which connect displacements and strains in polar coordinates for flat strain-stress state

$$\left. \begin{aligned} \varepsilon_r &= \frac{\partial u}{\partial r}, \quad \varepsilon_\varphi = \frac{1}{r} \left(u + \frac{\partial v}{\partial \varphi} \right), \\ \gamma_{r\varphi} &= \frac{1}{r} \frac{\partial u}{\partial \varphi} + \frac{\partial v}{\partial r} - \frac{v}{r}. \end{aligned} \right\} \quad (5.4.4)$$

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Biharmonic equation (5.2.4) in polar coordinate system can be written

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} \right) \left(\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \varphi^2} \right) = 0. \quad (5.4.5)$$

Stresses in polar coordinates for flat strain-stress state can be written

$$\left. \begin{aligned} \sigma_r &= \frac{1}{r} \left(\frac{\partial \psi}{\partial r} + \frac{1}{r} \frac{\partial^2 \psi}{\partial \varphi^2} \right), & \sigma_\varphi &= \frac{\partial^2 \psi}{\partial r^2}, \\ \tau_{r\varphi} &= \frac{1}{r} \left(\frac{1}{r} \frac{\partial \psi}{\partial \varphi} - \frac{\partial^2 \psi}{\partial r \partial \varphi} \right) = - \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial \varphi} \right). \end{aligned} \right\} \quad (5.4.6)$$

Equilibrium equations of elementary tetrahedron can be written

$$\left. \begin{aligned} \sigma_r l + \tau_{\varphi r} m &= p_{vr}, \\ \tau_{r\varphi} l + \sigma_\varphi m &= p_{v\varphi} \end{aligned} \right\}, \quad (5.4.7)$$

where l, m – are direction cosines of outer normal vector v relatively radial direction r and tangential direction φ ; $p_{vr}, p_{v\varphi}$ – are projections of external forces into directions r and φ .

Using stress function (5.4.5) in polar coordinates and appropriate boundary conditions some useful analytic solutions can be got. These solutions have practical usage mostly for exploring of stress concentrations for plates with holes.

Stress concentrators are places of steep transitions of geometry in solid body or in constructions.

Stress concentration is local enlarging of stresses in stress concentrators in comparison with nominal stresses acting for enough from concentrators.

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The level of stress concentration can be estimated by stress concentration factor

$$k = \frac{\sigma_c}{\sigma_{nom}}, \tag{5.4.8}$$

where σ_c – are stresses in concentrator; σ_{nom} – are nominal stresses defined far enough from concentration zone. Stresses in (5.4.8) may be normal, shear, some equivalent, etc., but the nature in numerator and denominator in (5.4.8) has to be the same.

There are often constructions met which have plates with holes of different forms, and these plates are in flat strain-stress state. It's necessary to estimate the level of stress concentrations. For some occasions analytic solutions of flat strain-stress state in polar coordinates can be used.

In the table 5.4.1 there are stress concentration factors are presented for different occasions of flat strain-stress state for plates with holes.

Table 5.4.1. Stress concentration factors for plates with holes

#	The calculation scheme	The expression for stress concentration factor in stress concentrators in points <i>A</i> and <i>B</i>
1	Infinite plate with round hole under normal nominal stresses	$k = 3$

Table 5.4.1

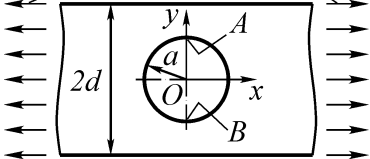
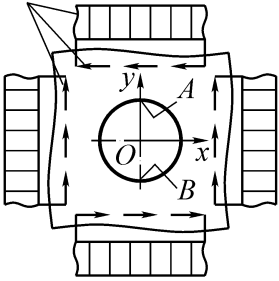
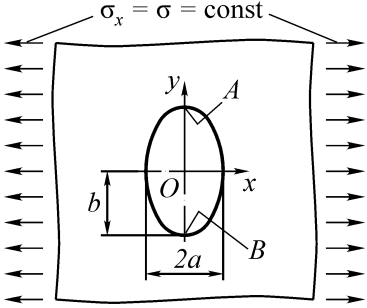
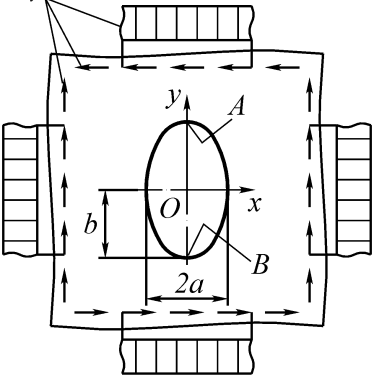
#	The calculation scheme	The expression for stress concentration factor in stress concentrators in points <i>A</i> and <i>B</i>
2	Finite plate with round hole under normal nominal stresses (but the length along <i>x</i> is infinite) $\sigma_x = \sigma = \text{const}$ 	$k = \frac{6}{\left(1 - \frac{a^2}{d^2}\right) \left(2 + \frac{a^2}{d^2}\right)}$
3	Infinite plate with round hole under shear nominal stresses $\tau_{xy} = \tau = \text{const}$ 	$k = \frac{4}{\sqrt{3}} = 2,31$

Table 5.4.1

#	The calculation scheme	The expression for stress concentration factor in stress concentrators in points A and B
4	Infinite plate with elliptical hole under normal nominal stresses $\sigma_x = \sigma = \text{const}$ 	$k = 1 + 2 \frac{b}{a}$
5	Infinite plate with elliptical hole under shear nominal stresses $\tau_{xy} = \tau = \text{const}$ 	$k = \frac{4}{\sqrt{3}(1-m^2)},$ $m = \frac{a-b}{a+b}$

PART II. BENDING OF RODS AND ROD SYSTEMS

Lecture 6. Basic Determinations, Concepts and Differential Relations of Technical Theory of Flat Orthogonal Bending of Direct Beams

§ 6.1. Basic Concepts of Flat Orthogonal Bending of Beams with One and with Many Spans. Kinds of External Loadings. Constraints and Its Properties

Bending is one of kinds of deformation. Flat bending of rods means that there are loadings and appropriate kinematical parameters are defined only in one of two principle planes (fig. 6.1.1). Also we consider there are no axial forces, reactions and displacements for flat orthogonal bending. Orthogonal axes of rod match for principal central axes and longitudinal axis comes through centers of masses of cross sections. We use right Cartesian coordinate system. Further we shall deal only with direct rods.

On the fig. 6.1.1 there are next designations: y and z are principal axes; x is longitudinal axis; α and β are principal planes. All external loading are situated in principle plane α .

There are four types of external loadings which take part for flat orthogonal bending. These external loadings are: concentrated forces P_i , [N], concentrated moments M_i , [N·m] distributed loadings $q(x)$, [N/m], and distributed bending moment loadings $m(x)$, [N]. In practice

there are (almost) no distributed bending moment loadings. It exists only mathematically in theoretical explorations. It's necessary to create special laboratory conditions to realize this loading. So further we shall not use distributed bending moment loadings. Also we shall not take into consideration nonuniform temperature fields. External loadings are applied to the mass centers of elementary volumes.

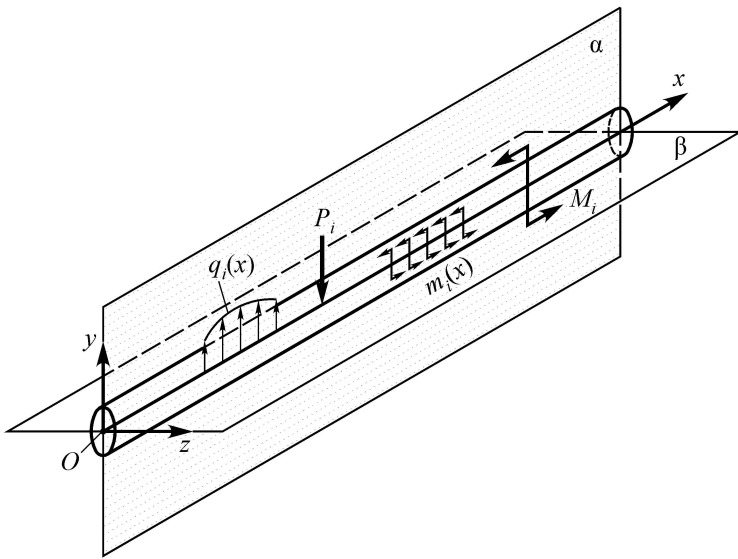


Fig. 6.1.1. External loadings for orthogonal flat bending

On the fig. 6.1.1 there are positive values of external loadings are shown. So for forces positive direction is up, and for bending moments positive direction is anticlockwise.

For beams there are different constraints which influence for parameters of bending and have to be considered for boundary conditions (BCs). Constraints can be situated both at ends and inside of beams. There are two principle constraints which influence for

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parameters of bending: elastic restraints which influence for rotation angles and elastic supports which influence for deflections (fig. 6.1.2).

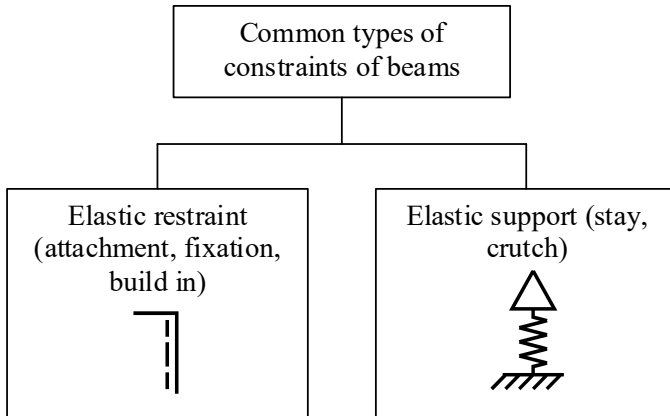


Fig. 6.1.2. Kinds of constraints of beams

Constraints of beams are movable (fig. 6.1.3). On the fig. 6.1.3 it is shown: P – is active force, which cause deflection v . M – is active bending moment which cause appearance of rotation angle α .

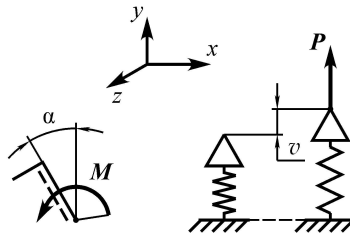


Fig. 6.1.3. Kinematical and force parameters for constraints of beams

On the fig. 6.1.3 there are positive directions for deflections and rotation angles are shown. So positive direction for deflections is up and positive direction for rotation angles is anticlockwise.

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Between force parameters P , M and kinematical parameters v , α there are linear dependencies

$$\alpha = M \cdot A_r, \quad v = P \cdot A_s, \quad M = \alpha \cdot K_r, \quad P = v \cdot K_s, \quad (6.1.1)$$

where α – is a rotation angle, rad; v – is a deflection, m; A_r is the suppleness coefficient of elastic restraint which shows the value of rotation angle under bending moment which equals one, [rad/(N·m)]; A_s is the suppleness coefficient of elastic support which shows the value of deflection under force which equals one, [m/N]; K_r is stiffness coefficient of elastic restraint which shows the value of bending moment causing appearance of rotation angle which equals one, [N·m/rad]; K_s is stiffness coefficient of elastic support which shows the value of force causing appearance of deflection which equals one, [N/m].

There is connection between suppleness coefficients A , and stiffness coefficients K

$$K = \frac{1}{A}, \quad A = \frac{1}{K}. \quad (6.1.2)$$

Elastic restraints and elastic supports can transform into another types of constraints within some conditions (fig. 6.1.4).

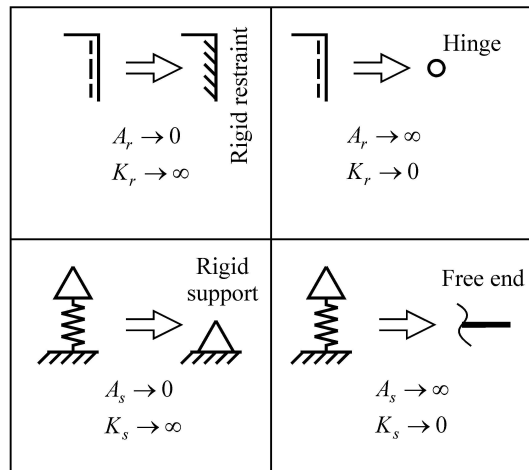


Fig. 6.1.4. Transformations of beam constraints

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Different combinations of restraints and supports generate numerous constraints. Some of them are presented in the table 6.1.1. Information in the last column of the table 6.1.1 will be explained later.

Table 6.1.1. Constraints and its parameters

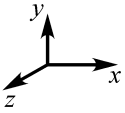
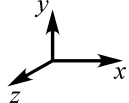
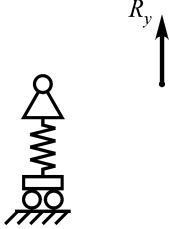
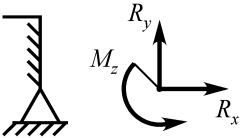
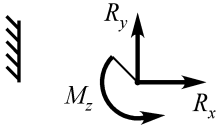
#	View of constraint and appropriate nonzero reactions for the coordinate system  and appropriate kinematical parameters	Boundary conditions for the flat orthogonal bending at ends of beam
1	Immovable hinge  $u = 0; v = 0; \alpha_z \neq 0$	$\left. \begin{aligned} EI(0)v''(0) &= 0, \\ v(0) &= 0, \\ EI(l)v''(l) &= 0, \\ v(l) &= 0. \end{aligned} \right\}$
2	Movable hinge  $u \neq 0; v = 0; \alpha_z \neq 0$	$\left. \begin{aligned} EI(0)v''(0) &= 0, \\ v(0) &= 0, \\ EI(l)v''(l) &= 0, \\ v(l) &= 0. \end{aligned} \right\}$
3	Immovable elastic hinge  $u = 0; v = R_y \cdot A_s; \alpha_z \neq 0$	$\left. \begin{aligned} EI(0)v''(0) &= 0, \\ -K_s^{\text{left}} v(0) - (EI(0)v''(0))' &= 0, \\ EI(l)v''(l) &= 0, \\ -K_s^{\text{right}} v(l) + (EI(l)v''(l))' &= 0. \end{aligned} \right\}$

Table 6.1.1

#	View of constraint and appropriate nonzero reactions for the coordinate system  and appropriate kinematical parameters	Boundary conditions for the flat orthogonal bending at ends of beam
4	Movable elastic hinge  $u \neq 0; v = R_y \cdot A_s; \alpha_z \neq 0$	$\left. \begin{aligned} EI(0)v''(0) &= 0, \\ -K_s^{\text{left}} v(0) - (EI(0)v''(0))' &= 0, \\ EI(l)v''(l) &= 0, \\ -K_s^{\text{right}} v(l) + (EI(l)v''(l))' &= 0. \end{aligned} \right\}$
5	Immovable restraint  $u = 0; v = 0; \alpha_z = 0$	$\left. \begin{aligned} v'(0) &= 0, \\ v(0) &= 0, \\ v'(l) &= 0, \\ v(l) &= 0. \end{aligned} \right\}$
6	Immovable restraint (simple denotation)  $u = 0; v = 0; \alpha_z = 0$	$\left. \begin{aligned} v'(0) &= 0, \\ v(0) &= 0, \\ v'(l) &= 0, \\ v(l) &= 0. \end{aligned} \right\}$

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Table 6.1.1

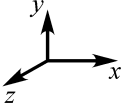
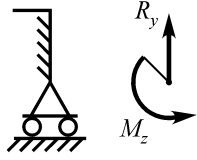
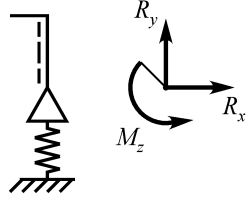
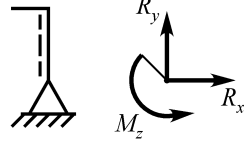
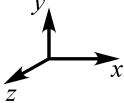
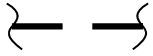
#	View of constraint and appropriate nonzero reactions for the coordinate system  and appropriate kinematical parameters	Boundary conditions for the flat orthogonal bending at ends of beam
7	Movable build in  $u \neq 0; v = 0; \alpha_z = 0$	$\left. \begin{aligned} v'(0) &= 0, \\ v(0) &= 0, \\ v'(l) &= 0, \\ v(l) &= 0. \end{aligned} \right\}$
8	Elastic restraint on elastic support  $u = 0; v = R_y \cdot A_s; \alpha_z = M_z \cdot A_r$	$\left. \begin{aligned} -K_r^{\text{left}} v'(0) + EI(0)v''(0) &= 0, \\ -K_s^{\text{left}} v(0) - (EI(0)v''(0))' &= 0, \\ -K_r^{\text{right}} v'(l) - EI(l)v''(l) &= 0, \\ -K_s^{\text{right}} v(l) + (EI(l)v''(l))' &= 0. \end{aligned} \right\}$
9	Elastic restraint on rigid support  $u = 0; v = 0; \alpha_z = M_z \cdot A_r$	$\left. \begin{aligned} -K_r^{\text{left}} v'(0) + EI(0)v''(0) &= 0, \\ v(0) &= 0, \\ -K_r^{\text{right}} v'(l) - EI(l)v''(l) &= 0, \\ v(l) &= 0. \end{aligned} \right\}$

Table 6.1.1

#	View of constraint and appropriate nonzero reactions for the coordinate system  and appropriate kinematical parameters	Boundary conditions for the flat orthogonal bending at ends of beam
10	Free end  $u \neq 0; v \neq 0; \alpha_z \neq 0$	$\left. \begin{aligned} EI(0)v''(0) &= 0, \\ (EI(0)v''(0))' &= 0, \\ EI(l)v''(l) &= 0, \\ (EI(l)v''(l))' &= 0. \end{aligned} \right\}$

Very often 3-rd and 4-th constraints are considered to be the same scheme. For this case we consider that there are attributes of 3-rd constraint.

§ 6.2. Internal Force Factors and Kinematical Parameters of Bending

Every loaded elastic body resists to external influences. If there are external forces applied to the body there are internal forces appears in that body as answer. So body resists to external influences by internal forces. External forces are applied to the mass centers of elementary volumes but internal forces are applied to mass centers of orthogonal cross sections of elementary volumes.

There are next internal forces which take part in orthogonal flat bending: shear forces Q , [N], and bending moments M , [N·m]. On the fig. 6.2.1 there are positive directions of internal forces Q and M for orthogonal flat bending are presented.

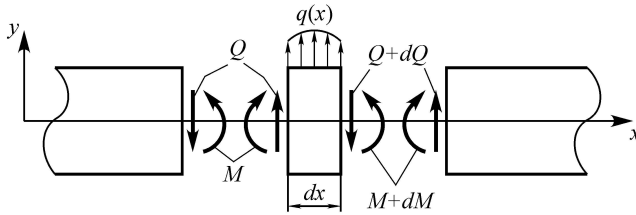


Fig. 6.2.1. Positive directions for internal shear forces and internal bending moments

If in some part of beam there are no shear forces i.e. $Q=0$, we tell that we have flat pure bending (or simply pure bending) on that part of beam having there only bending moments.

Also under external loadings every beam deforms. To know uniquely the deformed state of beam under external loadings we need to know deflection and rotation angle in every point of beam (fig. 6.2.2). Also on the fig. 6.2.2 point C and C_1 are mass centers of elementary volumes.

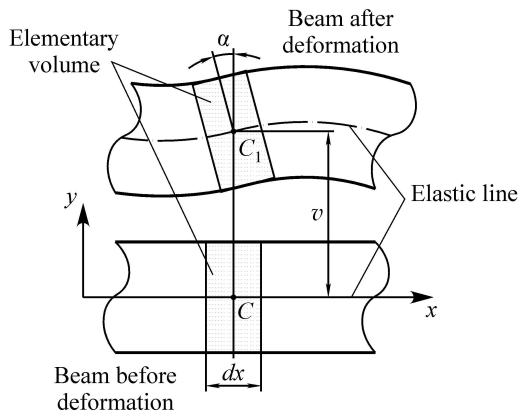


Fig. 6.2.2. Kinematic parameters of bending

Deflection v for given point (for our occasion this is point C) is distance between mass centers C and C_1 of the same elementary volume before deformation and after deformation in projection for given principle plane. The positive direction for deflections is up.

Rotation angle α for given point is defined between cross section plane before deformation and cross section plane after deformation in projection for given principle plane. The positive direction for rotation angles is anticlockwise.

§ 6.3. Hypothesis of Technical Theory of Flat Orthogonal Bending of Beams. Basic Relations Between Kinematical and Force Parameters of Bending

Usually physical processes described by some equations and relations. If to tell more accurately equations describe the behavior of model of some physical process or appearance. The same appearance or process can have several models. Some of those models are simple but some complicated. Every model always based on some assumptions. The same is for bending. Bending of rods is really complex process of massive body. But describing flat orthogonal bending of rods we accept several simplifying assumptions.

There are five general assumptions of the pure bending of beams:

1. Hypothesis of flat cross sections: cross sections flat and orthogonal before deformation still to be the same after deformation (fig. 6.3.1).

2. Elastic line in given point is the circular line with some defined radius. The curvature of elastic line depends on value of bending moments directly.

3. Hypothesis of physical linearity: the dependency between normal stresses σ and linear strains ε is linear i.e. Hook's $\sigma = E\varepsilon$ law is true.

4. Hypothesis of geometrical linearity of relative rigidity means that values of linear strains and rotation angles are much less than one, but deflections are much less in comparison with length of beam.

5. The hypothesis of linear strain-stress state means that longitudinal fibers are in axial linear strain-stress state. Deformation of fibers follows Hook's law. Longitudinal fibers can be only tensed or compressed. Neighboring fibers are independent from each other so they don't influence for each other. As result horizontal layers don't press for each other under orthogonal external loadings.

These five hypothesis are usually called the hypothesis of technical theory of bending of beams (TTBB).

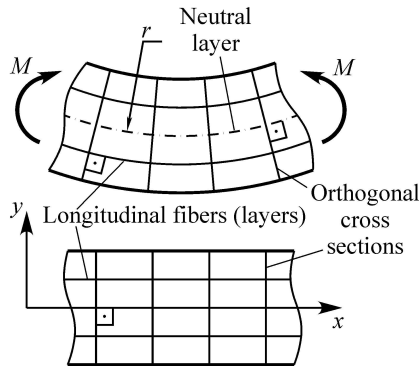


Fig. 6.3.1. For explanations of hypothesis of technical theory of flat orthogonal bending of beams

On the fig. 6.3.1 there are conditions of pure bending are shown, also M are external bending moments. Hypothesis of TTBB are true for pure bending, but for orthogonal bending, when $Q \neq 0$ they also can be applied.

There are next relations between kinematical and force parameters presented below

$$\left. \begin{aligned} N &= \frac{dM}{dx}, & q &= \frac{dQ}{dx}, & q &= \frac{d^2M}{dx^2}, \\ \alpha &= \frac{dv}{dx}, & \frac{d^2v}{dx^2} &= \frac{M}{EI}, & \frac{d\alpha}{dx} &= \frac{M}{EI}, \end{aligned} \right\} \quad (6.3.1)$$

where E – modulus of elasticity in tension, Pa; I – central principal moment of inertia relatively principal axis which is orthogonal to given principle plane in which flat orthogonal bending takes place (relatively axis y , (fig. 6.1.1)), m^4 .

Using (6.3.1) we can create differential equation for flat orthogonal bending of direct beams

$$[EI(x)v''(x)]' = q(x). \quad (6.3.2)$$

§ 6.4. Normal and Shear Stresses in Cross Sections of Beams

In cross sections of beams there are normal stresses σ caused by internal bending moments M , and shear stresses τ caused by internal shear forces Q . On the fig. 6.4.1 there is physical distribution shown of normal and shear stresses within orthogonal cross sections. We consider that loadings are applied in principal plane coming through axes yOx .

Neutral layer is the layer which doesn't change its length after deformation and normal stresses there equal zero but shear stresses have the largest value on modulus there.

Neutral axis is the line which is the result of intersection of the neutral layer plane and plane of orthogonal cross section in the given point of beam.

Normal stresses along neutral axis equals zero but shear stresses have the maximum value on modulus there. Moreover stresses (both normal and shear ones) have the same value along some line parallel to neutral axis in the cross section.

For practical use it is not convenient to show stresses in isometric view so they use schematic view presented on the fig. 6.4.2.

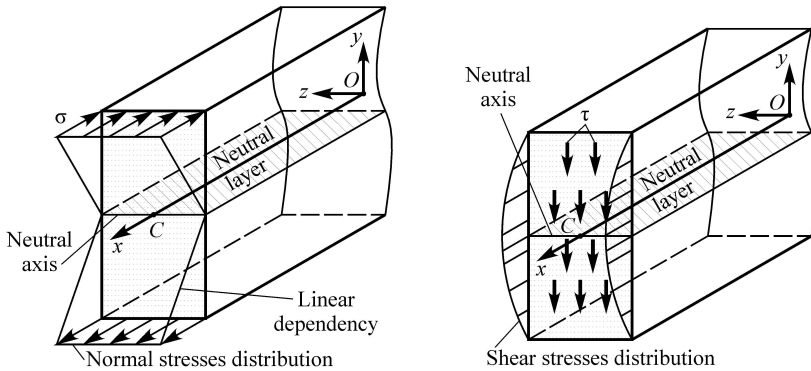


Fig. 6.4.1. Physical distribution of normal and shear stresses within cross sections

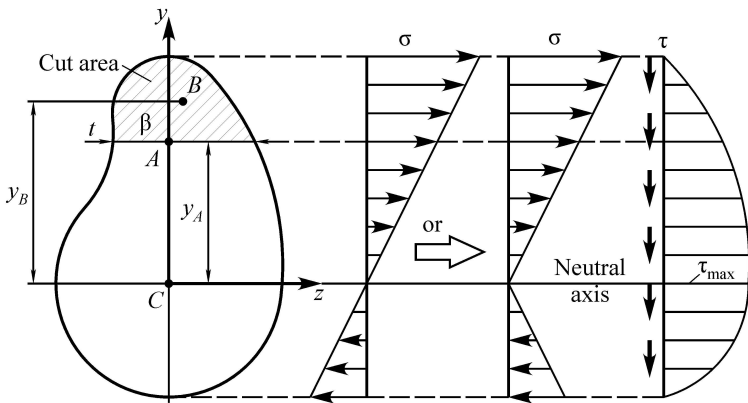


Fig. 6.4.2. Scheme distribution of stresses for orthogonal flat bending

Normal stresses can be defined using expression

$$\sigma = \sigma_x = -\frac{M \cdot y}{I}, \text{ Pa}, \quad (6.4.1)$$

where M – is the internal bending moment acting in the given cross section, $[\text{N} \cdot \text{m}]$; y – is the coordinate of given point relatively central principle axis (this is distance from neutral axis to given point in the

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direction parallel to axis y considering sign), [m]; I – is the central principle moment of inertia relatively principle axis y , [m⁴].

Shear stresses can be defined using expression

$$\tau = -\tau_{xy} = \frac{Q \cdot S^*}{I \cdot t}, \text{ Pa}, \quad (6.4.2)$$

where Q – is the shear force acting in given cross section, [N]; S^* – is the static moment of cut area of cross section relatively neutral axis, [m³]; I – is the central principle moment of inertia relatively principle axis y , [m⁴]; t – is the thickness of cross section at level of given point taken in cross section, [m].

Cut area is defined between two lines of the cross section: one of it comes through given point and another one is coming through the farthest point situated on the same side that given point relatively neutral axis. Sizing thickness t it is necessary to consider that level of given point is parallel to neutral axis.

Below on example it's shown how to define normal and shear stresses for some point A of the cross section (fig. 6.4.2).

It is easy to define normal stresses

$$\sigma_A = -\frac{M \cdot y_A}{I}.$$

Shear stresses can be defined using expression (6.4.2)

$$\tau_A = \frac{Q \cdot S_\beta^*}{I \cdot t},$$

where $S_\beta^* = F_\beta \cdot |y_B|$ is static moment of cut area β of the cross section relatively neutral axis (cut area β is hatched on the fig. 6.4.2); F_β is area of cut area β , [m²]; $|y_B|$ is the coordinate of mass centre B of cut area β relatively principle central axes (this is distance from neutral axis to mass centre B of cut area β in the direction parallel to axis y), [m]; point B is mass centre of cut area β .

Lecture 7. Flat Orthogonal Bending of Beams with One Span

§ 7.1. Start Parameter Method

In § 6.3 the differential equation for flat orthogonal bending was got

$$[EI(x)v''(x)]'' = q(x). \quad (7.1.1)$$

Values for formula (7.1.1) were explained above. There is accurate analytical solution for (7.1.1).

Integrating (7.1.1) we shall have the expressions for shear forces $Q(x)$, for bending moments $M(x)$, for rotation angles $\alpha(x)$, and for deflections $v(x)$ presented below

$$\left. \begin{aligned} Q(x) &= [EI(x)v''(x)]' = \int_0^x q(x) dx + A, \\ M(x) &= EI(x)v''(x) = \int_0^x \int_0^x q(x) dx^2 + Ax + B, \\ \alpha(x) &= v'(x) = \int_0^x \frac{\int_0^x \int_0^x q(x) dx^2}{EI(x)} dx + \int_0^x \frac{Ax}{EI(x)} dx + \\ &+ \int_0^x \frac{B}{EI(x)} dx + C, \\ v(x) &= \int_0^x \int_0^x \frac{\int_0^x \int_0^x q(x) dx^2}{EI(x)} dx^2 + \int_0^x \int_0^x \frac{Ax}{EI(x)} dx^2 + \\ &+ \int_0^x \int_0^x \frac{B}{EI(x)} dx^2 + Cx + D. \end{aligned} \right\} \quad (7.1.2)$$

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In (7.1.2) factors A, B, C, D are unknown which have to be found satisfying (7.1.2) to boundary conditions. There are four boundary conditions for beam with one span (i.e. two conditions for every end).

The physical nature of factors A, B, C, D are:

A – the initial internal shear force at left end, [N];

B – the initial internal bending moment at left end, [N·m];

C – the initial rotation angle at left end, [rad];

D – the initial deflection at left end, [m].

For beams there are more complicated occasions of loadings are possible. There may be concentrated forces and bending moments and also distributed loadings within some section (fig. 7.1.1). If to solve this task directly it is necessary to divide this beam into separate beams using positions of concentrated loadings and beginnings and ends of distributed external loadings as boundaries, after to write down the compatibility conditions at ends (for deflections and for rotation angles) for separate beams, or to write down equilibrium conditions for every node (node is place of connection of two ends of separate beams in one point), and to find out parameters of bending using additional manipulations not reviewed here (see below, for example beam on elastic foundation). If to apply direct method described above for solution of this task the number of unknown values is $4n$ for every separate beam.

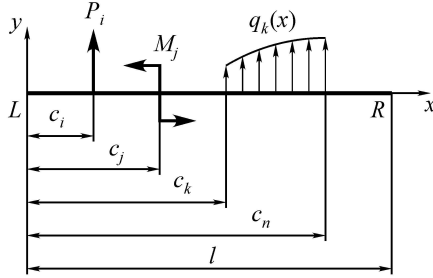
But there is more convenient method to solve this problem when we have more complicated loading described above. This method is called *start parameters method*. For start parameter method there are always only four unknown values A, B, C, D (see formula (7.1.2)).

On the fig. 7.1.1 the most common occasion for loading of beams is presented.

At ends L and R there may be different constraints (see table 6.1.1). There may be several loadings P_i, M_i and $q_k(x)$. Total number of concentrated external loadings P_i is shown as N_p , total number of

concentrated bending moments M_i is denoted as N_M , total number of external distributed loadings $q_k(x)$ is defined as N_q .

Fig. 7.1.1. For the explanation of the start parameters method



For external distributed loadings $q_k(x)$ there are some particulars. If the real loading $q_k(x)$ doesn't finish at right end, it's necessary to continue it (i.e. $q_k(x)$) up till right end, but on the section $(l - c_n)$ we continued our $q_k(x)$ it is necessary to apply the imagining loading $q_n(x) = -q_k(x)$, as it shown on the fig. 7.1.2, not to violate real load state of beam.

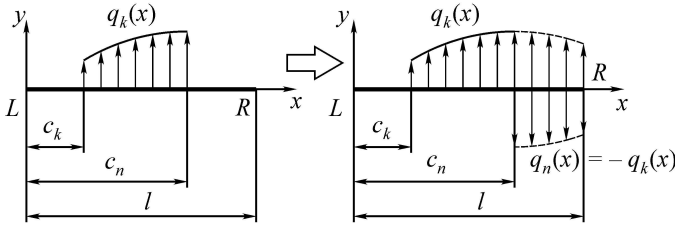


Fig. 7.1.2. For explanation of particulars of external distributed loadings

The differential equation for this occasion can be written

$$[EI(x)v''(x)]'' = \sum_{k=1}^{N_q} \left\| \begin{array}{c} q_k(x) \\ c_k \end{array} \right\| - \sum_{n=1}^{N_q} \left\| \begin{array}{c} q_n(x) \\ c_n \end{array} \right\|, \quad (7.1.3)$$

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where $\parallel$ is the conditional sign offered by Bubnov I.G. Its meaning is shown below on example

$$f(x) = \parallel_a 30(x-a) \text{ means } f(x) = \begin{cases} 0, & \text{if } x \leq a, \\ 30(x-a), & \text{if } x > a. \end{cases} \quad (7.1.4)$$

Also in (7.1.3) q_n is imagining loading necessary for working of start parameters method. Explanation for q_n see above. It can be defined in next way

$$\left. \begin{aligned} q_n(x) &= 0 & \text{if } c_n = l, \\ q_n(x) &= -q_k(x) & \text{if } c_n < l. \end{aligned} \right\} \quad (7.1.5)$$

Integrating (7.1.3) we shall have general relations for start parameter method (7.1.6).

All explanations for (7.1.6) are presented above within lecture #7. As it was told above at ends L and R of beam there may be different constraints. For every constraint in table 6.1.1 there are boundary conditions which have to be used for start parameter method together with expressions (7.1.6) to find unknown factors A, B, C, D . Of course more complicated occasions of boundary conditions at ends L and R are possible. For this case it's necessary to compose equilibrium equations for end elementary volumes taking into account kinematical relations there. These equilibrium equations are the expressions for boundary conditions.

The start parameters method can also be applied for calculation of beams with many spans, when the beam is situated for example on several hinges with elastic restraints. For this case it's necessary to throw away those constraints, after to show accordant reactions (see table 6.1.1) in points instead thrown constrains, and to write down appropriate kinematic and force conditions (for the same points) in addition to existing boundary conditions. After solution of the system of equations by start parameters method we shall have unknown factors A, B, C, D and reactions in points where constraints are.

$$\left. \begin{aligned}
 Q(x) &= [EI(x)v''(x)]' = \sum_{k=1}^{k=N_q} \left\| \int_{c_k}^x q_k(x) dx - \sum_{n=1}^{n=N_q} \left\| \int_{c_n}^x q_n(x) dx + \sum_{i=1}^{i=N_p} \left\| P_i + A; \right. \right. \\
 M(x) &= EI(x)v''(x) = \sum_{k=1}^{k=N_q} \left\| \int_{c_k}^x \int_{c_k}^x q_k(x) dx^2 - \sum_{n=1}^{n=N_q} \left\| \int_{c_n}^x \int_{c_n}^x q_n(x) dx^2 + \right. \\
 &+ \sum_{i=1}^{i=N_p} \left\| P_i(x - c_i) - \sum_{j=1}^{j=N_M} \left\| M_j + Ax + B; \\
 \alpha(x) &= v'(x) = \sum_{k=1}^{k=N_q} \left\| \int_{c_k}^x \frac{\int_{c_k}^x q_k(x) dx^2}{EI(x)} dx - \sum_{n=1}^{n=N_q} \left\| \int_{c_n}^x \frac{\int_{c_n}^x q_n(x) dx^2}{EI(x)} dx + \right. \\
 &+ \sum_{i=1}^{i=N_p} \left\| \int_{c_i}^x \frac{P_i \cdot (x - c_i)}{EI(x)} dx - \sum_{j=1}^{j=N_M} \left\| \int_{c_j}^x \frac{M_j}{EI(x)} dx + \int_0^x \frac{Ax}{EI(x)} dx + \int_0^x \frac{B}{EI(x)} dx + C; \\
 v(x) &= \sum_{k=1}^{k=N_q} \left\| \int_{c_k}^x \int_{c_k}^x \frac{\int_{c_k}^x q_k(x) dx^2}{EI(x)} dx^2 - \sum_{n=1}^{n=N_q} \left\| \int_{c_n}^x \int_{c_n}^x \frac{\int_{c_n}^x q_n(x) dx^2}{EI(x)} dx^2 + \right. \\
 &+ \sum_{i=1}^{i=N_p} \left\| \int_{c_i}^x \int_{c_i}^x \frac{P_i \cdot (x - c_i)}{EI(x)} dx^2 - \sum_{j=1}^{j=N_M} \left\| \int_{c_j}^x \int_{c_j}^x \frac{M_j}{EI(x)} dx^2 + \right. \\
 &+ \int_0^x \int_0^x \frac{Ax}{EI(x)} dx^2 + \int_0^x \int_0^x \frac{B}{EI(x)} dx^2 + Cx + D.
 \end{aligned} \right\} \quad (7.1.6)$$

Usually for typical calculation schemes, which appear very often on practice, there are ready expressions for elements of bending (for shear forces, bending moments, rotation angles, deflections) got using start parameter method and presented in numerous reference books, for example in [6].

§ 7.2. Superposition Principle. Usage of Reference Books for Calculations of Elements of Bending. Strength Condition. The Coefficient of "Support Pair"

Superposition principle. Using the principle of independency of force action it is possible to formulate the *superposition principle*, which is often used in calculation of beams. The superposition principle can be formulated in the next way. If there is a system of external loadings, which consist of several forces acting on the beam simultaneously, the deflection (rotation angle, bending moment, shear force) in some point caused by all external forces is equal to the sum of deflections (rotation angles, bending moments, shear forces) caused by every external force acting separately.

Mathematically this principle can be formulated in the next way

$$\left. \begin{aligned} Q_A(P_1, P_2, \dots, P_n) &= Q_A(P_1) + Q_A(P_2) + \dots + Q_A(P_n); \\ M_A(P_1, P_2, \dots, P_n) &= M_A(P_1) + M_A(P_2) + \dots + M_A(P_n); \\ \alpha_A(P_1, P_2, \dots, P_n) &= \alpha_A(P_1) + \alpha_A(P_2) + \dots + \alpha_A(P_n); \\ v_A(P_1, P_2, \dots, P_n) &= v_A(P_1) + v_A(P_2) + \dots + v_A(P_n). \end{aligned} \right\} (7.2.1)$$

where $P_1, P_2, \dots, P_n$ are external forces both concentrated and distributed ones; Q_A, M_A, α_A, v_A – are shear forces, bending moments, rotation angles, deflections accordingly in some point A .

The superposition method is true also for functions of elements of bending.

Usage of reference books for calculations of elements of bending. The superposition principle is very useful for practical calculations when we have beams loaded by many kinds of external loadings, that these occasions are not reviewed in reference book. Of course if there is no reference book close by, you may apply the start parameters method, but to save the time for calculation it's necessary to present the beam loaded by complex loading as several beams loaded by simple loadings, parameters for which can be found in

reference books. Applying (7.2.1) we can find the elements of bending. The geometry (sizes) of those several beams have to be as the geometry of the initial beam. Also the sum of simple loadings equals to initial complex loading.

For example we have beam loaded by system of external loadings (see fig. 7.2.1,*a*). This scheme can't be found in widely known reference books. To find out elements of bending for this beam it's necessary to review separate beams loaded by simple loadings, presented on fig. 7.2.1,*b*. The elements of bending for those three beams on fig. 7.2.1,*b* can be found in [6].

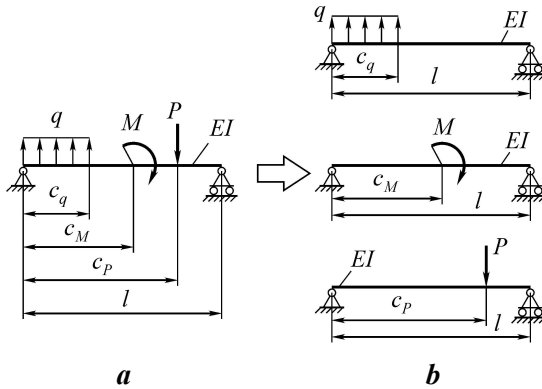


Fig. 7.2.1. For explanation of superposition principle

Elements of bending for beam presented on the fig. 7.2.1 can be found applying principle of superposition of acting forces using expressions

$$\left. \begin{aligned} Q(q, M, P) &= Q(q) + Q(M) + Q(P); \\ M(q, M, P) &= M(q) + M(M) + M(P); \\ \alpha(q, M, P) &= \alpha(q) + \alpha(M) + \alpha(P); \\ v(q, M, P) &= v(q) + v(M) + v(P). \end{aligned} \right\} \quad (7.2.2)$$

Strength condition. The strength of beam is considered to be provided if for the dangerous cross section the strength condition is satisfied. To define accurately dangerous cross section for beam under flat orthogonal bending is quite laborious problem even for simple occasions of loading. For practical calculations it's enough to define dangerous cross section using the law of bending moments $M(x)$. So the dangerous cross section is the cross section where the bending moments have the largest value on modulus. The bending moment $|M_{\max}|$ in dangerous cross section can be defined using expression

$$|M_{\max}| = \max \{|M(x)|\}, \quad (7.2.3)$$

where $|M(x)|$ – is taken on modulus function of distribution of internal bending moments along length of beam.

The coordinate of dangerous cross section is designed as x_d . For dangerous cross section it's necessary to define the real values $M(x_d)$ and $Q(x_d)$. After this it's necessary to define functions of normal and shear stresses, using (6.4.1) and (6.4.2)

$$\sigma(y) = -\frac{M(x_d) \cdot y}{I(x_d)}, \quad \tau(y) = \frac{Q(x_d) \cdot S^*(y)}{I(x_d) \cdot t}, \quad (7.2.4)$$

where $M(x_d)$, $Q(x_d)$, $I(x_d)$ – is the values of bending moment, shear force and inertia moment accordantly in dangerous cross section, see explanations for (6.4.1) and (6.4.2).

Then it's necessary to define one of the functions of equivalent stresses

$$\left. \begin{aligned} \sigma_{eq}^{III}(y) &= \sqrt{\sigma^2(y) + 4\tau^2(y)}, \\ \sigma_{eq}^{IV}(y) &= \sqrt{\sigma^2(y) + 3\tau^2(y)}, \end{aligned} \right\} \quad (7.2.5)$$

where σ_{eq}^{III} , σ_{eq}^{IV} – are equivalent stresses for 3-rd and 4-th strength theories. The functions $\sigma(y)$ and $\tau(y)$ have to be taken from (7.2.4).

The maximum equivalent stresses $\sigma_{eq}^{\max}$ can be found using one of expressions

$$\sigma_{eq}^{\max} = \max \{ \sigma_{eq}^{III}(y) \} \quad \text{or} \quad \sigma_{eq}^{\max} = \max \{ \sigma_{eq}^{IV}(y) \}. \quad (7.2.6)$$

The strength condition can be written

$$\sigma_{eq}^{\max} \leq [\sigma], \quad (7.2.7)$$

where $[\sigma]$ – are allowable stresses.

The strength of beam is considered to be provided if the strength condition (7.2.7) is satisfied. Usually for practical calculations functions $\sigma(y)$ and $\tau(y)$ in analytical view are not used, but they define these functions is several points of the dangerous cross section.

The coefficient of "support pair". Sometimes in practical calculations, when we operate with elastic constraints it's convenient to use not absolute values of yield coefficients or stiffness coefficients of constraint, but to use so called the coefficient of "support pair", which can be defined using expression

$$\kappa = \frac{M_{ec}}{M_{rc}}, \quad (7.2.8)$$

where M_{ec} – is the reactive bending moment in the real elastic constraint we operate with; M_{rc} – is the reactive bending moment in considering if our constraint would be rigid for the same beam with the same loading.

The coefficient of "support pair" is defined within range $0 \leq \kappa \leq 1$.

***Lecture 8. Methods of Resolving of Static Uncertainties.
Bending of Beams with Many Spans. Considering of Shear
Strains for Definition of Kinematic Parameters of Bending***

§ 8.1. Methods of Resolving of Static Uncertainties

There are two common methods of resolving of static uncertainty (redundancy): the method of displacements and the method of forces (fig. 8.1.1).

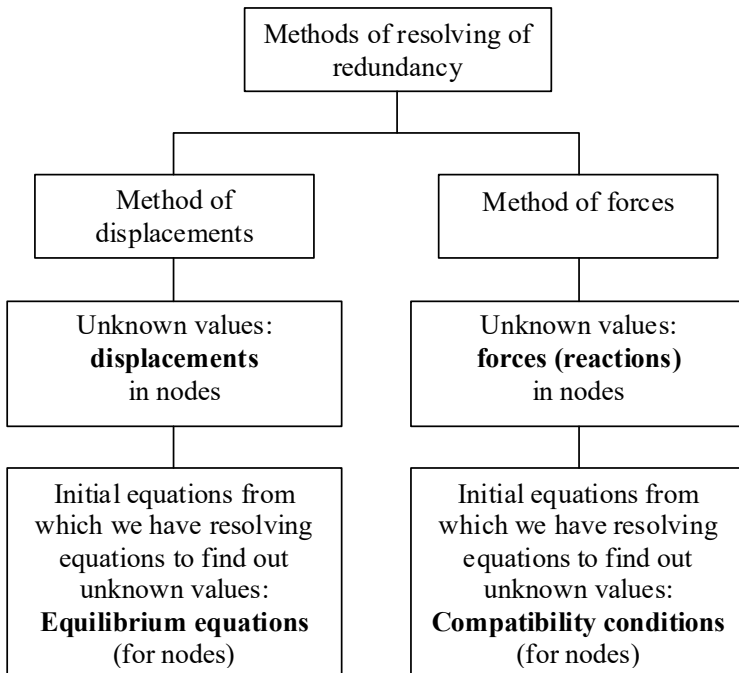


Fig. 8.1.1. Methods of resolving of redundancy of the construction

For the method of displacements there are some displacements are unknown values in some defined points (nodes). To find those displacements it's necessary to compose equilibrium equations using

reactions for every node, after to express reactions in composition of equilibrium equations through unknown displacements. Finally it's necessary to find out the unknown displacements from the system we have got.

For the method of forces there are some forces are unknown values in some defined points (nodes) of construction. To find out those forces it's necessary to form compatibility conditions for displacements for every node of construction, after to express displacements in composition of compatibility conditions through unknown forces. Finally it's necessary to find out the unknown forces from the system we have got.

§ 8.2. Bending of Beams with Many Spans

As it was told above start parameter method can be applied for calculations of beams with many spans, but that procedure is quite laborious. There are more unified methods to do that. Some of these methods are so called methods: "*method of five moments*" and "*method of three moments*". These methods are methods of forces for resolving of redundancy.

We shall use the term "**node**". So the node is place of connection of two or more beams in one point. Nodes can be put in any place of construction to provide convenience, precision and simplicity of calculations.

So called method of five moments is used for resolving of redundancy of beams with many spans on elastic supports. Below this method is explained more carefully.

The methodology of the method of five moments consists of several steps.

For the 1-st step it's necessary to present the whole beam as the system of separate beams, which all are situated on hinges, without paying attention what constraints we have at ends (see fig. 8.2.1); to give names for nodes and for spans. For further explanations we shall

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use the system of numeration presented on the fig. 8.2.1: numbers of nodes and separate beams start from zero. Every separate beam have to be reviewed independently, the beginning of coordinate system for every beam is situated at left end, so every beam have its local coordinates. The laws of distributed loadings have to be rewritten relatively local coordinate systems of every beam. If just at hinge there is concentrated bending moment or concentrated force, it have to be related to one of two neighboring spans (see fig. 8.2.1). Also on the fig. 8.2.1 $A_i, i = 0, 1, 2, 3$, are suppleness coefficients of elastic hinges.

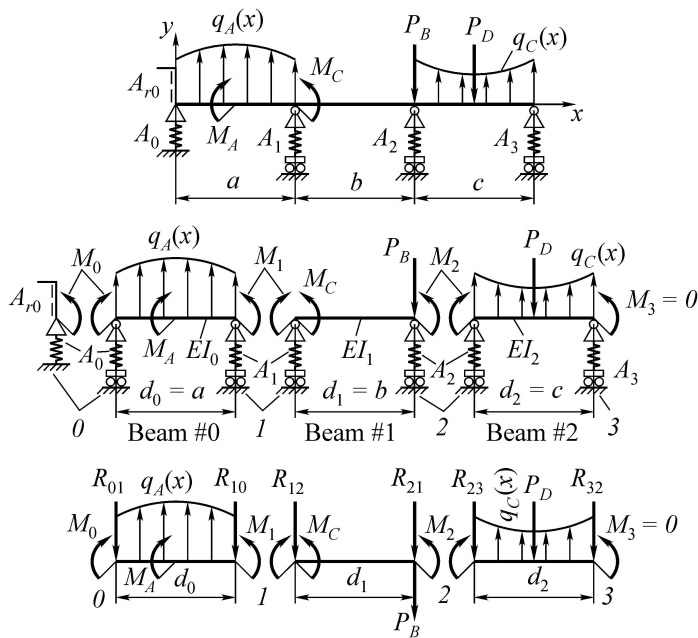


Fig. 8.2.1. The calculation scheme for the method of five moments

For the 2-nd step it's necessary to put reactive moments at ends of those beams. These reactive moments are M_0, M_1, M_2 , and M_3 on the fig. 8.2.1. For different nodes there may be different directions of

reactive bending moments, but within one node directions have to be opposite to each other. For our case all reactive bending moments have the same directions for every node. It's recommended to accept directions of reactive bending moments as it's shown on the fig. 8.2.1.

For the 3-rd step it's necessary to compose compatibility expressions for every node. For nodes situated inside of the beam, with numbers $1 \div (n - 1)$, where n – is the last number in nodes, the compatibility conditions have to be composed using expression

$$\alpha_{i,i-1} = \alpha_{i,i+1}, \quad i = 1 \div (n - 1), \quad (8.2.1)$$

where $\alpha_{i,i-1}, \alpha_{i,i+1}$ – are rotation angles at hinges (see fig. 8.2.2).

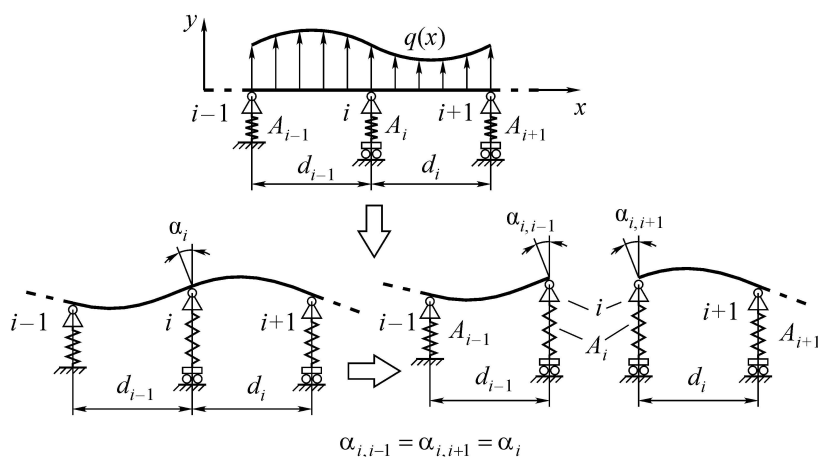


Fig. 8.2.2. For explanations of compatibility conditions

To compose the compatibility conditions for end nodes with numbers 0 and n it's necessary to analyze constraints at ends applying equilibrium equations and properties of constraints there. For example if at left end there is rigid constraint, the compatibility condition is $\alpha_{01} = 0$, etc.

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For the case presented on the fig. 8. 2.1 the compatibility conditions are these

$$\left. \begin{array}{l} \text{for node \#0: } A_{r0}M_0 = \alpha_{01}, \\ \text{for node \#1: } \alpha_{10} = \alpha_{12}, \\ \text{for node \#2: } \alpha_{21} = \alpha_{23}, \end{array} \right\} \quad (8.2.2)$$

where A_{r0} – is the suppleness coefficient for elastic constraint.

For the node #3 it's not need to compose compatibility condition, because the value of reactive moment is know at that node: $M_3 = 0$. We have to compose compatibility conditions only for nodes where reactive moments are unknown.

For the 4-th step it's necessary to express the rotation angles $\alpha_{i,i-1}, \alpha_{i,i+1}$ in (8.2.1) through external loadings, reactive bending moments at ends, and deflections on hinges. For this case superposition principle can be applied. To express angles in compatibility conditions for all nodes next common relations can be used

$$\left. \begin{array}{l} \alpha_{i,i-1} = \alpha_{i,i-1}(M_i, M_{i-1}, q_{i-1}, v_i, v_{i-1}, v_i^a, v_{i-1}^a) = \\ = \alpha_{i,i-1}(M_i) + \alpha_{i,i-1}(M_{i-1}) + \alpha_{i,i-1}(q_{i-1}) + \\ + \alpha_{i,i-1}(v_i, v_{i-1}) + \alpha_{i,i-1}(v_i^a, v_{i-1}^a), \\ i = 1, 2, \dots, n; \\ \alpha_{i,i+1} = \alpha_{i,i+1}(M_i, M_{i+1}, q_i, v_i, v_{i+1}, v_i^a, v_{i+1}^a) = \\ = \alpha_{i,i+1}(M_i) + \alpha_{i,i+1}(M_{i+1}) + \alpha_{i,i+1}(q_i) + \\ + \alpha_{i,i+1}(v_i, v_{i+1}) + \alpha_{i,i+1}(v_i^a, v_{i+1}^a), \\ i = 0, 1, \dots, n-1, \end{array} \right\} \quad (8.2.3)$$

where

i – is number of node;

n – the last number in nodes;

q_{i-1}, q_i – are denotations for all external loadings (distributed and concentrated ones taken together) for accordingly $(i - 1)$ -th and i -th beams;

$\alpha_{i,i-1}(M_i), \alpha_{i,i-1}(M_{i-1}), \alpha_{i,i+1}(M_i), \alpha_{i,i+1}(M_{i+1})$ – are rotation angles caused only by reactive bending moments at ends, considering that our beam is at rigid hinges;

$\alpha_{i,i-1}(q_{i-1}), \alpha_{i,i+1}(q_i)$ – are rotation angles caused only by external loadings, including distributed loadings, both concentrated forces and concentrated moments, considering that our beam is at rigid hinges; to find these values reference books have to be used;

$\alpha_{i,i-1}(v_i, v_{i-1}), \alpha_{i,i+1}(v_i, v_{i+1})$ – are rotation angles caused by only deflections, but considering that our hinges are elastic for this case;

v_{i-1}, v_i, v_{i+1} – are elastic deflections caused by external loadings, they are not known for now, but can be found using reactions R_{ij} (see fig. 8.2.1);

$\alpha_{i,i-1}(v_i^a, v_{i-1}^a), \alpha_{i,i+1}(v_i^a, v_{i+1}^a)$ – are rotation angles caused by only so called assemblage initial deflections;

$v_{i-1}^a, v_i^a, v_{i+1}^a$ – are so called assemblage initial deflections, which appears for reason of inaccuracy in assembling of beam when they install supports; values $v_{i-1}^a, v_i^a, v_{i+1}^a$ for elastic hinges have to be defined relatively some horizontal level when there is no beam on them and no external loadings for them; values $v_{i-1}^a, v_i^a, v_{i+1}^a$ are known in advance.

The 1-st expression in (8.2.3) have to be used to express angles at right ends of beams, but the 2-nd expression in (8.2.3) have to be used to express angles at left ends of beams.

Finding values of $\alpha_{i,i-1}(q_{i-1}), \alpha_{i,i+1}(q_i)$ superposition principle can also be applied. For example using fig. 8.2.1 we can write down expression for $\alpha_{10}(q_0)$

$$\alpha_{10}(q_0) = \alpha_{10}(q_A) + \alpha_{10}(M_A). \quad (8.2.4)$$

It was told above that we can accept the directions for reactive bending moments in a free way (of course following some rules mentioned above). For this case to find $\alpha_{i,i-1}(M_i)$, etc. it's necessary to use reference book (for example [6]) for given directions of M_i you chose. But if directions of M_i are chosen as it's shown on the fig. 8.2.1, there are next template formulas for $\alpha_{i,i-1}(M_i)$, etc. presented below

$$\left. \begin{aligned} \alpha_{i,i-1}(M_i) &= \frac{M_i d_{i-1}}{3EI_{i-1}}, & \alpha_{i,i-1}(M_{i-1}) &= \frac{M_{i-1} d_{i-1}}{6EI_{i-1}}, \\ i &= 1, 2, \dots, n; \\ \alpha_{i,i+1}(M_i) &= -\frac{M_i d_i}{3EI_i}, & \alpha_{i,i+1}(M_{i+1}) &= -\frac{M_{i+1} d_i}{6EI_i}, \\ i &= 0, 1, \dots, n-1. \end{aligned} \right\} \quad (8.2.5)$$

Finding angles $\alpha_{i,i-1}(v_i, v_{i-1})$, $\alpha_{i,i+1}(v_i, v_{i+1})$, we consider that our separate beams are direct, because there are no external loadings there, but these angles are caused only by deflections v_{i-1} , v_i , v_{i+1} (fig. 8.2.3).

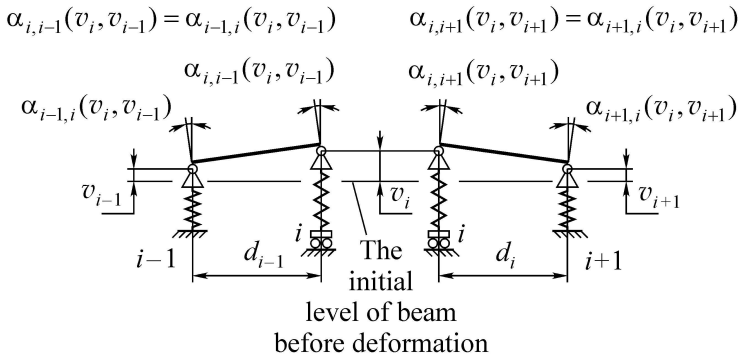


Fig. 8.2.3. Rotation angles caused by deflections of elastic hinges

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The values $\alpha_{i,i-1}(v_i, v_{i-1})$, $\alpha_{i,i+1}(v_i, v_{i+1})$, $\alpha_{i,i-1}(v_i^a, v_{i-1}^a)$, $\alpha_{i,i+1}(v_i^a, v_{i+1}^a)$ can be found using next expressions

$$\left. \begin{aligned} \alpha_{i,i-1}(v_i, v_{i-1}) &= \alpha_{i-1,i}(v_i, v_{i-1}) = \frac{v_i - v_{i-1}}{d_{i-1}}, \\ \alpha_{i,i-1}(v_i^a, v_{i-1}^a) &= \alpha_{i-1,i}(v_i^a, v_{i-1}^a) = \frac{v_i^a - v_{i-1}^a}{d_{i-1}}, \\ i &= 1, 2, \dots, n. \\ \alpha_{i,i+1}(v_i, v_{i+1}) &= \alpha_{i+1,i}(v_i, v_{i+1}) = \frac{v_{i+1} - v_i}{d_i}, \\ \alpha_{i,i+1}(v_i^a, v_{i+1}^a) &= \alpha_{i+1,i}(v_i^a, v_{i+1}^a) = \frac{v_{i+1}^a - v_i^a}{d_i}, \\ i &= 0, 1, \dots, n-1. \end{aligned} \right\} \quad (8.2.6)$$

There is additional task, how to find v_{i-1} , v_i , v_{i+1} . To find elastic deflections v_{i-1} , v_i , v_{i+1} we have to throw away elastic supports and to replace their action by reactions (see fig. 8.2.1). It's necessary to remember the next if for beam the direction of reaction is one, the direction of reaction on support is opposite (following the Newton law). It's recommended to put reactions as it's shown on the fig. 8.2.1.

Deflections on supports v_i are connected with accordant reactions R_i on those supports by next relations

$$v_i = A_i \cdot R_i, \quad (8.2.7)$$

where reactions R_i at supports can be found using formula

$$R_i = R_{i,i-1} + R_{i,i+1}. \quad (8.2.8)$$

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If the index in formulas presented in § 8.2 have the value less than zero or more than the largest number of support or separate beam, we consider that that quantities with "wrong" indexes equals zero. For example the reaction of 0-th support (i.e. $i = 0$) can be found using expression (applying expression (8.2.8))

$$R_0 = R_{0,-1} + R_{0,1}. \quad (8.2.9)$$

But in (8.2.9) we may see that there is summand $R_{0,-1}$ in which one of indexes equals "-1". It means that $R_{0,-1} = 0$, so $R_0 = R_{0,1}$. This principle have to be applied to all formulas in § 8.2. Reactions for beam presented on the fig. 8.2.1, caused by hinges can be found using next expressions

$$\left. \begin{aligned} R_0 &= R_{01}, \\ R_1 &= R_{10} + R_{12}, \\ R_2 &= R_{21} + R_{23}, \\ R_3 &= R_{31}. \end{aligned} \right\} \quad (8.2.10)$$

Reactions R_{ij} within every separate beam can be found using equilibrium equations, because every beam for this case is statically defined. For example to find R_{10} it's necessary to compose next equilibrium equation

$$\left. \begin{aligned} \Sigma \text{ Moments}_{z,1} = 0: & \quad -M_0 - M_A + M_1 - R_{10} d_0 + \\ & + \int_0^{d_0} q_A(x) dx \cdot \left\{ \frac{\int_0^{d_0} q_A(x) x dx}{\int_0^{d_0} q_A(x) dx} \right\} = 0 \Rightarrow R_{10} = \dots \end{aligned} \right\} \quad (8.2.11)$$

In the formula (8.2.11) the expression $\Sigma \text{ Moments}_{z,1} = 0$ means "the sum of moments in projection of axis z relatively point I ".

Inserting expressions for R_{ij} we have got using expressions like (8.2.11) into (8.2.8), after the result we have got inserting into (8.2.7), after the result we have got inserting into (8.2.6), after the result we have got inserting into (8.2.3), i.e. $(8.2.11) \rightarrow (8.2.8) \rightarrow (8.2.7) \rightarrow (8.2.6) \rightarrow (8.2.3)$.

Finally we shall have that expressions for $\alpha_{i,i-1}$ and $\alpha_{i,i+1}$ will contain unknown reactive bending moments M_j .

Every compatibility expression like (8.2.2) contains in the most common occasion five unknown reactive bending moments. This is the reason why this method is called the "method of five moments". Inserting expressions for $\alpha_{i,i-1}$ and $\alpha_{i,i+1}$ into compatibility conditions like (8.2.2) we shall have the system of linear algebraic equations from which we can find unknown reactive bending moments M_j . The number of compatibility equations equals to the number of unknown reactive bending moments.

If elastic supports are completely rigid, we shall not have the summands $\alpha_{i,i-1}(v_i, v_{i-1})$, $\alpha_{i,i+1}(v_i, v_{i+1})$ in (8.2.3). For this case the method of resolving of static redundancy is called the "method of three moments".

After reactive bending moments have been found it's necessary to find the laws for shear forces $Q(x)$ and bending moments $M(x)$ for every separate beam, and to do projective or verify calculations for strength or for rigidity of given beam with many spans. Every beam has to be reviewed separately for this case.

§ 8.3. Consideration of Shear Strains for Deflections and Rotation Angles in Beams

As it was told above together with normal stresses in cross sections of beams there are also shear stresses. Shear stresses cause shears of cross sections of beams. Shear strains cause appearance of additional deflections and rotation angles of cross sections of beams.

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For explanations of influence of shear for deflections and rotation angles we shall denote deflections and rotation angles caused by bending accordingly as $v_b(x)$, $u_b(x)$ and $\alpha_b(x)$, but deflections and rotation angles caused by shear accordingly as $v_s(x)$, $u_s(x)$ and $\alpha_s(x)$.

Total displacements can be found using next relations

$$\left. \begin{aligned} v(x) &= v_b(x) + v_s(x), \\ v'(x) &= v'_b(x) + v'_s(x), \\ \alpha(x) &= \alpha_b(x) + \alpha_s(x) = v'_b(x) + \alpha_s, \\ u(x, y) &= u_b(x, y) + u_s(x, y). \end{aligned} \right\} \quad (8.3.1)$$

Developing the approximate method of definition of displacements caused by shear we consider that there is vertical wall in the plane of bending, which takes up all shear loading.

Influence of shear for displacements take place very much in ship beams with developed girdles (stringers, floors, bottom grillages with double bottoms), because such beams have enhanced bending stiffness EI , having relatively small stiffness for shear GF , where F – is area of orthogonal cross section. For such beams we may consider that distribution of shear stresses τ is almost uniform, so we assume that the value of shear stresses equals to the middle value of shear stresses, i.e.

$$\tau_{xy} = \tau_{mid} = -\frac{Q(x)}{F(x)}, \quad \gamma_{xy} = \gamma_{mid} = \frac{\tau_{xy}}{G} = -\frac{Q(x)}{GF(x)}, \quad (8.3.2)$$

where $Q(x)$ is the value of shear force; $F(x)$ is area of orthogonal cross section; G is shear modulus.

Formulas (8.3.2) are basic expressions for the approximate method of considering of influence of shear for displacements.

The real distribution of shear stresses is nonuniform, but if to consider the real law of distribution of shear stresses, the consideration of shear will be quite difficult.

Displacement $u_s(x)$, which appears as the result of additional rotation of the cross section of beam for angle $\alpha_s(x)$, for shear can be found using evident relation $u_s(x, y) = -z \cdot \alpha_s(x)$. Because the displacement $u_s(x)$ is caused only by shear stresses τ_{xy} , the appropriate linear strain ε_x have to equal to zero

$$\begin{aligned}\varepsilon_x &= \frac{\partial u_s(x)}{\partial x} = -z \frac{d\alpha_s(x)}{dx} = 0 \quad \Rightarrow \\ \Rightarrow \quad \alpha_s(x) &= \alpha_s = \text{const.}\end{aligned}\tag{8.3.3}$$

Thus the expression (8.3.3) means that all cross sections for shear rotate for the same angle.

There is expression for calculation of additional deflections caused by shear

$$v_s(x) = v_s(0) + \alpha_s x - \int_0^x \frac{Q(x)}{GF(x)} dx.\tag{8.3.4}$$

When we calculate statically defined beams, the consideration of displacements caused by beams don't influence for reactions of constraints, and as result for shear forces and bending moments. That's why the elements of bending $v_b(x)$, $\alpha_b = v'_b(x)$, $M(x) = EI(x)v''_b(x)$, $Q(x) = [EI(x)v''_b(x)]'$ can be defined using relations for example (7.1.6) or other ones, in which there is no accounting of shear. The shear force $Q(x)$ after can be used in (8.3.4) to find deflections caused by shear.

For statically uncertain beams the accounting of shear deformations cause change of reactions at constraints and as result it cause change of bending moments and shear forces, and also it cause change of displacement at constraint cross sections for elastic restraints and hinges. For this case the elements of bending have to be defined again using (7.1.6) or other ones (inserting $v_b(x)$ instead $v(x)$), in which

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there is no accounting of shear, but factors A, B, C, D have to be found using boundary conditions in which there are additional displacements at end cross sections caused by shear are accounted.

Also we consider $v_b(0) = v(0)$, $v_b(l) = v(l)$, what means that

$$v_s(0) = v_s(l) = 0. \quad (8.3.5)$$

Using table 6.1.1 and (8.3.5) the formulas for the most common boundary conditions can be written

$$\left. \begin{aligned} -K_r^{\text{left}}[v_b'(0) + \alpha_s] + EI(0)v_b''(0) &= 0, \\ -K_s^{\text{left}}v_b(0) - (EI(0)v_b''(0))' &= 0, \\ -K_r^{\text{right}}[v_b'(l) + \alpha_s] - EI(l)v_b''(l) &= 0, \\ -K_s^{\text{right}}v_b(l) + (EI(l)v_b''(l))' &= 0. \end{aligned} \right\} \quad (8.3.6)$$

Subduing (8.3.4) to conditions (8.3.5) we shall have

$$\left. \begin{aligned} \alpha_s &= \frac{1}{l} \int_0^l \frac{Q(x)}{GF(x)} dx, \\ v_s(x) &= \frac{x}{l} \int_0^l \frac{Q(x)}{GF(x)} dx - \int_0^x \frac{Q(x)}{GF(x)} dx. \end{aligned} \right\} \quad (8.3.7)$$

If $F(x) = F = \text{const}$ the expressions (8.3.7) can be simplified. As the result we have next relations for $v_s(x)$ and α_s , when there is $F(x) = F = \text{const}$

$$\left. \begin{aligned} v_s(x) &= -\frac{M_s(x)}{GF(x)}, \\ \alpha_s &= \frac{1}{lGF} \sum_i M_i^{\text{ext}}, \end{aligned} \right\} \quad (8.3.8)$$

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where $M_s(x)$ – is the law of internal bending moments caused only by orthogonal external loadings: by concentrated forces P_j and distributed loadings $q_j(x)$, assuming rigid hinges at ends of beam (without considering of concentrated and distributed bending moments within span and at constraints); $\sum_i M_i^{\text{ext}}$ – is the algebraic sum of all external

concentrated bending moments, including reactive moments at constraints.

We always consider that we have rigid hinges at ends of beam for definition of $M_s(x)$, no matter what real constraints are there. For welded and rolled profiles welded to hull sheeting, the influence of shear for their deflection is not large, that's why it (shear) is not considered in practical calculations. For such profiles the influence of shear for deflections is well estimated by quantity of relation of beam length to beam height (l/h). With enhancing of the relation (l/h) the influence of shear decreases. When we have the relation $l/h \geq 10$, the influence of shear can be neglected. The shear has to be considered for calculations of short and high beams, which can be met in constructions of different fundaments and reinforcements.

Lecture 9. Bending of Frames of Ship Hull

§ 9.1. Classification of Frames in Composition of Ship Hull

Frame is system of rods loaded by external loadings. There are different kinds of frames: volume and flat ones, frames with movable nodes and with immovable ones, frames with rigid nodes and with pliable ones, etc. Also rods in composition of frames may be prismatic and unprismatic, rods may be direct and curvilinear. In the most common occasion rods of frames accept three fundamental kinds of deformation: axial loading (tension-compression), torsion, bending. In some cases we may not to consider some of those deformations.

Volume frame is system of rods situated and connected with each other in a free way in space. The external loadings for volume frames can also be applied freely in space. Flat frame is frame which has all its rods situated in one plane, and external loadings are also act in that plane, i.e. rods and loadings are situated in one plane.

Frames with movable nodes are frames for which we accept that their nodes able to move.

Frames with immovable nodes are frames for which we accept that nodes don't move during deformation, because those displacements of nodes are quite small, that they (displacements) can be neglected at. Assumption of immovable nodes simplifies the calculation model of frames much, but has to be justified, to avoid inaccuracies in calculations.

Frames with rigid nodes are frames with rigid nodes. Rigid nodes are nodes which move as rigid undeformable body during deformation. The angles between rods for this rigid node don't change during deformation. Assumption of rigid nodes also simplifies much the calculation model for frames, but applying of this model has also to be substantiated.

Frames with pliable nodes are frames which have pliable nodes. For pliable nodes angles between rods change during deformation.

The simplest model for frames is flat frame with rigid immovable nodes. As tests show this model is quite acceptable for ship frames, consisted of ship frames for given orthogonal cross section, because there is symmetry external loading, at nodes there are knees, which make to rotate nodes as rigid unit.

For simple kinds of flat frames, when for every node there are not more than two rods, the method of three moments can be applied. But for complicated occasions it's better to use the method of angular displacements. This method fits for all flat frames with rigid immovable nodes. Below this method is explained in particulars.

§ 9.2. The Method of Angular Displacements

The method of angular displacements belongs to the common method of displacements for resolving of static redundancy. Calculation of flat frames using the method of angular displacements consists of several steps described below. We shall review these steps at example of frame shown on the fig. 9.2.1.

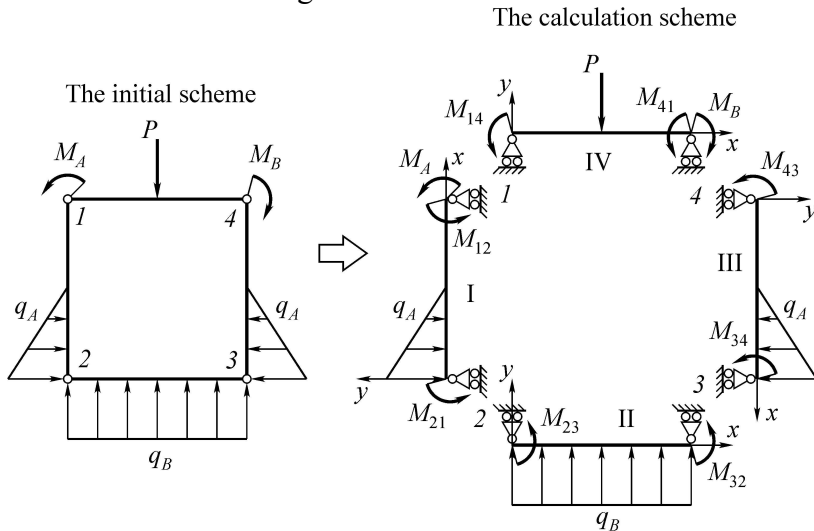


Fig. 9.2.1. The calculation scheme for the method of angular displacements

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For the 1-st step it's necessary to compose the calculation scheme for given frame (see fig. 9.2.1). We present our given frame as system of separate beams. Separate beams are defined between nodes. For the occasion of calculation of flat frames the nodes have to be shown at initial scheme explicitly. Ends of these beams are always situated on rigid hinges. At ends of those separate beams there is reactive bending moments, which all have the same direction. For our case that direction is anticlockwise. Further we shall always keep the direction for reactive bending moments to be anticlockwise. If directly at node there is external bending moment (as moment M_A or M_B on the fig. 9.2.1), it have to be related to one of neighboring separate beams.

For the 2-nd step it's necessary to give names to nodes and optionally to separate beams, and to write down equilibrium equations for bending moments for every node. For equilibrium equations all concentrated bending moments both external and reactive ones have to be accounted. For the occasion presented on the fig. 9.2.1 the equilibrium equations for nodes can be written

$$\left. \begin{array}{l} \text{node \#1: } M_{12} + M_{14} + M_A = 0, \\ \text{node \#2: } M_{21} + M_{23} = 0, \\ \text{node \#3: } M_{32} + M_{34} = 0, \\ \text{node \#4: } M_{43} + M_{41} + M_B = 0. \end{array} \right\} \quad (9.2.1)$$

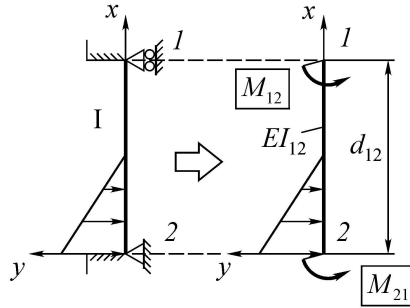
For the 3-rd step it's necessary to express reactive moments M_{ij} through rotation angles of nodes. It can be down using expression

$$M_{ij} = M_{ij}^r + \frac{2EI_{ij}}{d_{ij}}(2\alpha_i + \alpha_j), \quad (9.2.2)$$

where i and j are numbers of nodes within one separate beam; I_{ij} , d_{ij} – are accordantly the inertia moment and the length of the beam between nodes i and j ; α_i , α_j – are rotation angles of i -th and j -th nodes; M_{ij}^r – is the reactive bending moment at end caused by external

loadings within beam, accounting that our beam have rigid constraints and supports at ends (fig. 9.2.2).

Fig. 9.2.2. For explanation of moments M_{ij}^r for the first beam



The directions of reactive bending moments have to be the same as directions of reactive bending moments M_{ij} on hinges (see fig. 9.2.1) when we have just created the calculation scheme. Reactions M_{ij}^r can be found using reference books or applying the method of start parameters.

Next statements are true in expression (9.2.2)

$$\left. \begin{aligned} M_{ij} &\neq M_{ji}, & M_{ij}^r &\neq M_{ji}^r, \\ I_{ij} &= I_{ji}, & d_{ij} &= d_{ji}. \end{aligned} \right\} \quad (9.2.3)$$

Inserting expressions (9.2.2) for every M_{ij} into (9.2.1) we shall have the system of equations in which the unknown values are rotation angles of nodes: $\alpha_1, \alpha_2, \alpha_3, \alpha_4$. After rotation angles have been found they have to be substituted into expressions like (9.2.2) to find the numeric values of all M_{ij} .

After reactive bending moments have been found it's necessary to find the laws for shear forces $Q(x)$ and bending moments $M(x)$ for every separate beam, and to do projective or verify calculations for strength or for rigidity of given frame. Every beam has to be reviewed separately for this case.

Lecture 10. Bending of Beams on Elastic Foundation

§ 10.1. Practical Tasks Which Provide Using the Model of Beam on Elastic Foundation

For example we have the beam on numerous of elastic supports. Of course we may apply the method of five moments for this case, but the labor content for this case is high enough. It's better to apply for such beam the model of beam on elastic foundation.

For the model of beam on elastic foundation we consider that for every orthogonal cross section of the beam there is linear reaction $r(x)$, [N/m], caused by resisting of elastic foundation

$$r(x) = k(x) \cdot v(x), \quad (10.1.1)$$

where $k(x)$ – is the linear stiffness of elastic foundation, [N/m²]; $v(x)$ – is the deflection of the beam.

By the relation (10.1.1) the term *continuous elastic foundation* of variable stiffness is defined. It has the property that its reaction depends linearly on deflection for any orthogonal cross section of the beam. The expression (10.1.1) defines so called the Winkler hypothesis, that's why the elastic foundation for which the relation (10.1.1) is true is called the Winkler foundation. The conditional designation for elastic foundation is shown on the fig. 10.1.1.

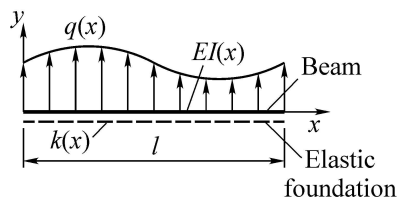


Fig. 10.1.1. Designation of beam on elastic foundation

The model of elastic foundation is widely used in technique. There are examples of such foundations: building slipways, decks of railroad dry docks and decks of launching docks for ship hulls, ground

foundation for long buildings, etc. The ship on numerous of keel blocks can be bring to the model of elastic foundation.

The differential equation for unprismatic beam on elastic foundation can be written

$$[EI(x)v''(x)]'' + k(x) v(x) = q(x). \quad (10.1.2)$$

There is no accurate analytic solution for the (10.1.2), so approximate numeric methods have to be used.

For prismatic beams ($EI = \text{const}$) laying on elastic foundation of constant stiffness ($k = \text{const}$) the differential equation (10.1.2) can be simplified and written

$$EIv^{IV}(x) + k \cdot v(x) = q(x). \quad (10.1.3)$$

There are accurate analytic solutions for the (10.1.3). The solution for (10.1.3) can be presented as the sum of uniform solution v_u , which fits for the uniform equation of (10.1.3), when $q(x) = 0$, and partial solution v_p , which depends on $q(x)$. Mathematically the solution for (10.1.3) can be written

$$v = v_u + v_p. \quad (10.1.4)$$

The uniform solution v_u can be got solving the uniform differential equation

$$EIv^{IV}(x) + k \cdot v(x) = 0. \quad (10.1.5)$$

Inserting the Eulerian substitution $v_u = A \cdot e^{sx}$ (where A and s are constant values which have to be defined, e – is base of natural logarithms) into (10.1.5) we can find the uniform solution v_u .

The uniform solution v_u can be found using expression

$$v_u = e^{-\alpha x} (A_1 e^{i\alpha x} + A_2 e^{-i\alpha x}) + e^{\alpha x} (A_3 e^{i\alpha x} + A_4 e^{-i\alpha x}), \quad (10.1.6)$$

where i – is imaginary one; A_1, A_2, A_3, A_4 – are constants;

$$\alpha = \sqrt[4]{\frac{k}{4EI}}. \quad (10.1.7)$$

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Using different formulas for complex numbers in (10.1.6) we can get other forms of uniform solution v_u , which are written below

$$v_u = e^{-\alpha x} (B_1 \cos \alpha x + B_2 \sin \alpha x) + e^{\alpha x} (B_3 \cos \alpha x + B_4 \sin \alpha x), \quad (10.1.8)$$

$$v_u = C_1 \operatorname{ch} \alpha x \cos \alpha x + C_2 \operatorname{sh} \alpha x \cos \alpha x + C_3 \operatorname{ch} \alpha x \sin \alpha x + C_4 \operatorname{sh} \alpha x \sin \alpha x \quad \left. \vphantom{v_u} \right\}, \quad (10.1.9)$$

where $B_1, B_2, B_3, B_4, C_1, C_2, C_3, C_4$ – are constants.

Every summand in (10.1.6), (10.1.8), (10.1.9) is the partial solution of the uniform differential equation (10.1.5). Any linear combination of these partial solutions with fixed coefficients is also partial solution. That's why we can to compose a lot of different partial solutions.

One of forms which is quite often used in practical calculations can be written

$$v_u = D_0 V_0(\alpha x) + D_1 V_1(\alpha x) + D_2 V_2(\alpha x) + D_3 V_3(\alpha x), \quad (10.1.10)$$

where D_0, D_1, D_2, D_3 – are constants; $V_0(\alpha x), V_1(\alpha x), V_2(\alpha x), V_3(\alpha x)$ – are independent linear combinations of partial solutions of the uniform equation, containing in (10.1.9), which can be found using expressions

$$\left. \begin{aligned} V_0(\alpha x) &= \operatorname{ch} \alpha x \cos \alpha x, \\ V_1(\alpha x) &= \frac{1}{\sqrt{2}} (\operatorname{ch} \alpha x \sin \alpha x + \operatorname{sh} \alpha x \cos \alpha x), \\ V_2(\alpha x) &= \operatorname{sh} \alpha x \sin \alpha x, \\ V_3(\alpha x) &= \frac{1}{\sqrt{2}} (\operatorname{ch} \alpha x \sin \alpha x - \operatorname{sh} \alpha x \cos \alpha x). \end{aligned} \right\} \quad (10.1.11)$$

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For functions $V_n(\alpha x)$, $n = 0, 1, 2, 3$ there is the next rule for derivatives

$$\left. \begin{aligned} V_1'(\alpha x) &= \alpha\sqrt{2} V_0(\alpha x), \\ V_2'(\alpha x) &= \alpha\sqrt{2} V_1(\alpha x), \\ V_3'(\alpha x) &= \alpha\sqrt{2} V_2(\alpha x), \\ V_0'(\alpha x) &= -\alpha\sqrt{2} V_3(\alpha x). \end{aligned} \right\} \quad (10.1.12)$$

At the beginning of coordinates, when $x = 0$, functions $V_n(\alpha x)$ satisfies to the next relations

$$\begin{aligned} & \begin{bmatrix} V_0(0) & V_0'(0) & V_0''(0) & V_0'''(0) \\ V_1(0) & V_1'(0) & V_1''(0) & V_1'''(0) \\ V_2(0) & V_2'(0) & V_2''(0) & V_2'''(0) \\ V_3(0) & V_3'(0) & V_3''(0) & V_3'''(0) \end{bmatrix} = \\ & = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \alpha\sqrt{2} & 0 & 0 \\ 0 & 0 & 2\alpha^2 & 0 \\ 0 & 0 & 0 & 2\sqrt{2}\alpha^3 \end{bmatrix}. \end{aligned} \quad (10.1.13)$$

The partial solution v_p for heterogeneous differential equation (10.1.3) can be easily got if within span there is distributed external loading, the law of which can be defined by the polynomial, the order of which is not more than 3-rd

$$q(x) = a + bx + cx^2 + dx^3, \quad (10.1.14)$$

where a, b, c, d – are free real factors.

The partial solution v_p can be found using the expression

$$v_p = \frac{q(x)}{k} = \frac{1}{k}(a + bx + cx^2 + dx^3). \quad (10.1.15)$$

The common solution v for differential heterogeneous equation (10.1.3), for the occasion if the external loading is set by the relation (10.1.14) can be written

$$v = v_u + \frac{1}{k}q(x), \quad (10.1.16)$$

where v_u can be accepted in forms (10.1.6), (10.1.8), (10.1.9) or (10.1.10). Four unknown factors A_p , B_p , C_i or D_n have to be found from four boundary conditions at ends of beam. These boundary conditions are the same as for the beams without elastic foundation.

On reality for beams on elastic foundation there are complicated external loadings: concentrated forces, concentrated bending moments, several distributed loadings which may interrupt, and also there are several elastic foundations which also may interrupt (see fig. 10.1.2).

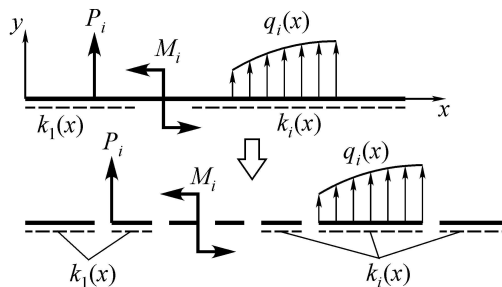


Fig. 10.1.2. Beam on elastic foundation with complicated loadings

There are several ways to solve this task, from simple ones but with a lot of work there to complicated ones but less laborious (for example start parameters method, etc.).

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The most universal and the simplest way consists of several steps. For the 1-st step it's necessary to divide the beam into separate beams, taking into consideration boundaries of external loadings, elastic foundations and boundaries of laws of changing of the orthogonal cross section of the beam along its length (see fig. 10.1.2). For the 2-nd step it's necessary at ends of those separate beams to show unknown displacements (if we are going to use the displacements method to resolve the static redundancy), or to show unknown reactions (if we are going to use the force method to resolve the static redundancy). For the 3-rd step it's necessary for every beam to compose boundary conditions and to solve differential equations (10.1.2) or (10.1.3), expressing constants in (10.1.6), (10.1.8), (10.1.9) or (10.1.10) through unknown displacements or reactions (see step 2-nd). For every separate beam we shall have four unknown values. For the 4-th step it's necessary to resolve the static redundancy using described above procedures for resolving of static redundancies applying the method of displacements or the method of forces.

Lecture 11. Bending of Flat Grillages

§ 11.1. Classification of Grillages as Rod Systems

Grillage is the system of rods situated in one plane, but the external loadings act in direction perpendicular for this plane. There are different kinds of grillages to which different models and calculation methods can be applied, but there is no common classification of grillages. At ends of beams there are different constraints. To resolve the static redundancy the method of displacements and the method of forces can be used.

Ship hull grillages usually consist of two systems of beams: beams of major direction (BMD) and cross beams (CB) (see fig. 11.1.1). Usually these systems of beams are mutually decussate, but this is not the rule. Also these systems are regular for geometry and external loadings.

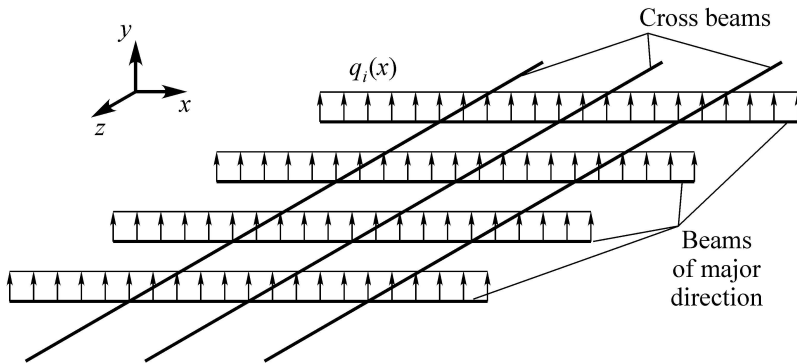


Fig. 11.1.1. Typical ship hull grillage

Historically ship hull grillages usually are classified in next way: grillages with little number of BMDs and CBs, grillages with numerous of BMDs and one CB, grillages with numerous of BMDs and several CBs.

Grillage with little number of BMDs and CBs are usually calculated using both method of displacements or method of forces for resolving

of static redundancy. Within this lecture we shall review the Navier method for such kinds of grillages.

Grillage with numerous of BMDs and one CB are usually calculated if to present this grillages as a beam on elastic foundation. The cross beam is the main beam which have to be calculated but the BMDs create the elastic foundation. The linear stiffness k of elastic foundation is constant, the value of which can be calculated using next expression

$$k = \frac{K}{a}, \quad (11.1.1)$$

where K – is the stiffness of BMD in point of connection two beams for concentrated force applied in this point: given BMD and CB;
 a – is the sum of distances to neighboring BMDs divided by two.

The stiffness K of any beam for given point for concentrated force can be calculated in next way:

1) it's necessary to find out the deflection in given point caused by concentrated force applied in this point, but the value of this force have to equal one;

2) the value K equals to the contrary value of the deflection found in p.1 (see above).

Regular grillage with numerous of BMDs and several CBs can be calculated using accurate analytical solutions offered by Rostovtsev D.M., Nicolai B.A., etc.

Offered methods for different kinds of grillages are not the rule. Other universal methods can be used to calculate the grillages (for example the Navier method, method of finite elements, etc).

§ 11.2. The Navier Method for Calculation of Grillages

Within this paragraph the Navier method will be explained. The Navier method belongs to the method of forces for resolving of static redundancy. It also can be applied for calculation of beams on elastic supports.

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The basic steps for this method will be explained for grillage presented on the fig. 11.2.1.

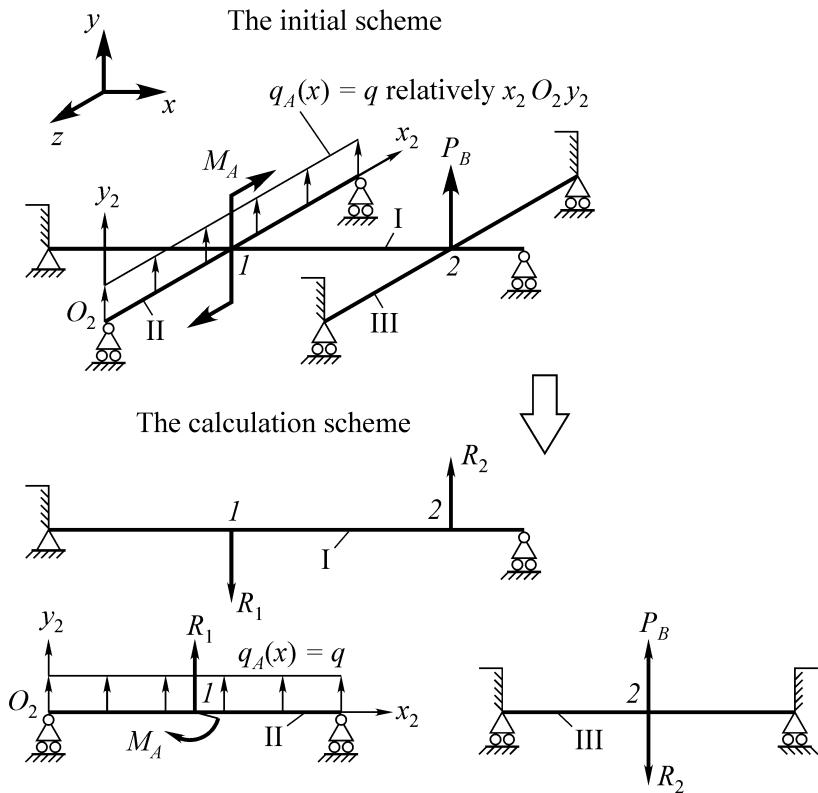


Fig. 11.2.1. The calculation scheme for Navier method

For the 1-st step it's necessary to present the grillage as the system of separate beams (see fig. 11.2.1), to give names to separate beams and to nodes. If at some node there is concentrated external force (as P_B on the fig. 11.2.1) it has to be related to one of beams to provide convenience in calculations. If there is concentrated moment

just at node (as M_A on the fig. 11.2.1), it has to be related to beam for which this moment cause bending. Because for we neglect with the stiffness for torsion considering it to be much less that stiffness for bending.

For the 2-nd step it's necessary to show reactions at nodes relatively global coordinate system for the whole grillage. Reactions for now are not known. These reactions for the given node have to have opposite directions, but the same modulus (following the 3-rd Newton law).

For the 3-rd step it's necessary to compose the compatibility conditions for deflections in all nodes using the common expression

$$v_i^a = v_i^b, \quad i = 1, \dots, n, \quad (11.2.1)$$

where i – is number (name) of given node; a and b – are numbers (names) of beams intersecting in given node; n – total number of all nodes.

The total number of compatibility conditions equal to the total number of nodes in grillage. For the occasion shown on the fig. 11.2.1 the expressions for the compatibility conditions can be written

$$v_1^I = v_1^{II}; \quad v_2^I = v_2^{III}. \quad (11.2.2)$$

For the 4-th step it's necessary to express the deflections in (11.2.1) through unknown reactions R_j and external loads, using common relation based on superposition principle

$$v_i^a = \sum_j v_i^a(R_j) + v_i^a(q), \quad (11.2.3)$$

where $v_i^a(R_j)$ – is the deflection for a -th beam in i -th node caused only by reaction R_j situated in j -th node; j has node numbers, which belong to a -th beam; $v_i^a(q)$ – is the deflection for a -th beam in i -th node caused only by all external loadings.

The deflection $v_i^a(q)$ can also be written using the superposition principle. For example for the II-th beam the deflection caused by all external loadings can be written

$$v_1^{\text{II}}(q) = v_1^{\text{II}}(M_A) + v_1^{\text{II}}(q_A). \quad (11.2.4)$$

At last for the occasion presented on the fig.11.2.1 deflections in (11.2.2) can be written using next formulas on base of principle of independency of acting forces

$$\left. \begin{aligned} v_1^{\text{I}}(q) &= v_1^{\text{I}}(R_1) + v_1^{\text{I}}(R_2), \\ v_1^{\text{II}}(q) &= v_1^{\text{II}}(R_1) + v_1^{\text{II}}(M_A) + v_1^{\text{II}}(q_A), \\ v_2^{\text{I}}(q) &= v_2^{\text{I}}(R_1) + v_2^{\text{I}}(R_2), \\ v_2^{\text{III}}(q) &= v_2^{\text{III}}(R_2) + v_2^{\text{III}}(P_B). \end{aligned} \right\} \quad (11.2.5)$$

Substituting (11.2.5) into (11.2.2) we shall have the system of equations from which we can find unknown reactions R_j .

After reactive forces R_j have been found it's necessary to find the laws for shear forces $Q(x)$ and bending moments $M(x)$ for every separate beam, and to do projective or verify calculations for strength or for rigidity of given grillage. Every beam has to be reviewed separately for this case.

PART III. BUCKLING (BENDING WITH AXIAL FORCE) AND STABILITY OF RODS AND ROD SYSTEMS

Lecture 12. Orthogonal Bending with Axial Force of Rods and Rod Systems

§ 12.1. General Relations for Orthogonal Bending with Axial Force

Very often there are cases when there are additional axial forces for rods under orthogonal bending. There are different approaches for considering of axial forces. For the simplest and quite inaccurate models we consider that bending and tension-compression are independent, what is widely used in finite elements method. But on reality axial force influence for values of shear forces and bending moments, and it have to be taken into account. The differential equations offered within this lectures take into account influence of axial forces for bending, that's why these differential equations are quite accurate and more precise than other models.

On the fig. 12.1.1 the accepted designations and positive directions for bending with axial forces are shown.

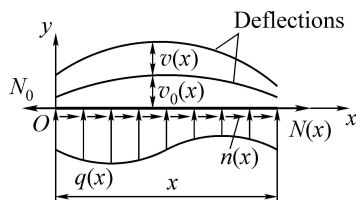


Fig. 12.1.1. Accepted denotations for bending with axial forces

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On fig. 12.1.1 it's shown:

N_0 – the value of external axial force at left end, [N];

$n(x)$ – distributed external axial forces, [N/m];

$q(x)$ – distributed orthogonal external forces, [N/m];

$N(x)$ – internal axial force for given cross section with coordinate x , [N];

$v_0(x)$ – initial deflection of the undeformable beam, [m];

$v(x)$ – the deflection caused by external forces, [m].

The initial deflection $v_0(x)$ is defined for beam which is not loaded by external forces. For pure direct beams $v_0(x) = 0$.

On the fig. 12.1.2 the deviated state of elementary volume is shown. It's defined between two orthogonal planes with coordinates x and $x + dx$.

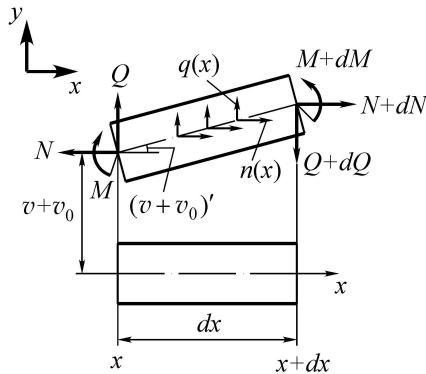


Fig. 12.1.2. The elementary volume of beam under orthogonal bending with axial force

Composing equilibrium equations for deviated elementary volume we may get general relations for bending with axial force

$$\frac{dN}{dx} = -n, \quad \frac{dQ}{dx} = q, \quad Q = \frac{dM}{dx} - N(v_0 + v)', \quad (12.1.1)$$

where N – are axial internal forces, [N]; Q – internal shear forces, [N]; M – internal bending moments, [N·m]; $(v_0 + v)'$ – the rotation angle of the element dx , [rad].

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It's obvious that the relation for internal bending moments is the same as for usual orthogonal bending

$$M(x) = EIv''(x). \quad (12.1.2)$$

Considering third equation in (12.1.1) and (12.1.2) we may write down the expression for shear forces

$$Q(x) = (EIv'')' - N(v_0 + v). \quad (12.1.3)$$

Taking into account (12.1.1), (12.1.2) and (12.1.3) we may compose the differential equation for orthogonal bending with axial force, which can be written

$$[EI(x)v''(x)]'' - [N(x)v'(x)]' = q(x) + [N(x)v'_0(x)]. \quad (12.1.4)$$

If there is prismatic beam ($EI = \text{const}$), with constant axial force along beam length ($N = \text{const}$), the differential equation (12.1.4) can be simplified

$$EIv^{IV}(x) - Nv''(x) = q(x) + Nv''_0(x). \quad (12.1.5)$$

There is no common analytic solution for (12.1.4), but for simplified differential equation (12.1.5) there are analytic solutions.

Integrating first equation in (12.1.1) we can find the expression for axial internal forces for given cross section, which can be written

$$N(x) = N_0 - \int_0^x n(x) dx. \quad (12.1.6)$$

The boundary conditions for bending with axial forces for deflections, rotation angles and bending moments are the same as for usual orthogonal bending. Boundary conditions for bending with axial force, containing shear forces, can be got from boundary conditions for usual orthogonal bending but replacing the member (EIv'') on member $[(EIv'')' - N(v_0 + v)]$.

Thus, for given external loadings $q(x)$ and $T(x)$ the task is narrowed down to definition of deflection $v(x)$ solving the differential equation (12.1.4) or (12.1.5) for given boundary conditions. After this internal forces $Q(x)$ and $M(x)$ have to be defined, using expressions (12.1.2) and (12.1.3). Normal and shear stresses can be defined using expressions

$$\sigma = \frac{N}{F} - \frac{M \cdot y}{I}, \quad \tau = \frac{Q \cdot S^*}{I \cdot t}, \quad (12.1.7)$$

where F – is the cross section area, $[m^2]$, but the rest designations in (12.1.6) have the same meaning as for usual orthogonal bending.

The verify calculations can be done as for usual orthogonal bending (see § 7.2 partition "Strength conditions").

§ 12.2. Analytic Solutions for Prismatic Beams. Superposition Principle

Analytic solutions for prismatic beams. As it was told above there are accurate analytic solutions for prismatic beams when there is constant axial force. The differential equation for this case is defined by (12.1.5). The common solution $v(x)$ for (12.1.5), when there is tensile axial force can be written

$$v(x) = v_p(x) + A_0 + A_1 \beta x + A_2 \operatorname{ch} \beta x + A_3 \operatorname{sh} \beta x, \quad (12.2.1)$$

where $v_p(x)$ – is partial solution which depends on the right part of (12.1.5); A_0, A_1, A_2, A_3 – are free constants;

$$\beta = \sqrt{\frac{N}{EI}}. \quad (12.2.2)$$

If the axial force is compressive, the β appears to be imaginary basing on expression (12.2.2). Denoting for this case

$$\beta^* = \sqrt{\frac{|N|}{EI}}, \quad (12.2.3)$$

the common integral of (12.1.5) can be written

$$\begin{aligned} v(x) = v_p(x) + B_0 + B_1 \beta^* x + \\ + B_2 \cos \beta^* x + B_3 \sin \beta^* x, \end{aligned} \quad (12.2.4)$$

where B_0, B_1, B_2, B_3 – are free constants.

If to consider that on the basis of (12.2.2) and (12.2.3) there is $\beta = i\beta^*$ (for compressive axial force), also taking into account known Eulerian formulas

$$\operatorname{sh} i\beta^* x = i \sin \beta^* x, \quad \operatorname{ch} i\beta^* x = \cos \beta^* x, \quad (12.2.5)$$

it's not difficult to get the solution (12.2.4), in case of compressive axial force, from solution (12.2.1) for axial tensile force.

The partial solution $v_p(x)$ can be got using known methods from mathematics. But for the simplest occasions when the orthogonal external loading $q(x)$ and initial deflection $v_0(x)$ are set by relations

$$q(x) = q_0 + q_1 \left(\frac{x}{l} \right), \quad v_0(x) = a_0 \sin \left(\frac{\pi x}{l} \right), \quad (12.2.6)$$

the partial solution $v_p(x)$ can be found using next expression

$$v_p(x) = -\frac{q_0 x^2}{2N} - \frac{q_1 x^3}{6Nl} - \frac{Na_0 \sin \left(\frac{\pi x}{l} \right)}{N + \frac{\pi^2 EI}{l^2}}, \quad (12.2.7)$$

where l – is beam length; N – constant external axial force at ends, which is positive if it cause tension and negative if it cause compression;

Superposition principle. Analyzing the differential equation (12.1.4) or (12.1.5) and boundary conditions, we may see that deflection $v(x)$ depends linearly from orthogonal external loadings and nonlinearly from axial force N . That's why the superposition principle is true only for orthogonal external loadings, i.e.

$$\begin{aligned} v(N, q_1, q_2, \dots, q_n) &= \\ &= v_1(N, q_1) + v_2(N, q_2) + \dots + v_n(N, q_n), \end{aligned} \quad (12.2.8)$$

where $v(x)$ – is the deflection of beam caused by simultaneous action of axial force N and orthogonal loadings; $q_1, q_2, \dots, q_n$; $v_1, v_2, \dots, v_n$ – are deflections of beam caused by orthogonal loadings; $q_1, q_2, \dots, q_n$, when there is action of every of it together with axial force N .

§ 12.3. Estimation of Influence of Axial Force for Bending. Calculation of Beam under Orthogonal Bending With Axial Forces for Complicated Loadings

Estimation of influence of axial force for bending. For convenience of calculations there are assisting functions depending of axial forces N . Let's explain their meanings. For example we have some prismatic beam loaded by orthogonal external loadings and constant axial forces at ends. It's necessary to calculate this beam as for usual orthogonal bending without paying attention for axial forces. Further it's necessary to find out the values of assisting functions for given value of axial force. Finally it's necessary to multiply elements of orthogonal bending we have got for appropriate values of those functions. These functions are called the corrective functions for elements of orthogonal bending (for shear forces, for deflections, for rotation angles and for deflections), taking into consideration axial force. These functions were offered by Bubnov I.G. and can be found in reference books.

Sometimes these corrective functions can be very helpful. For example calculating the beam on elastic supports but with additional axial forces and ends of that beam, we have to write down the expressions for the method of five moments as for usual orthogonal bending, but rotation angles caused by external loadings and reactive moments have to be multiplied by these appropriate corrective functions. The same principle is for frames and for grillages. For now methods of calculation of rod systems, based on these corrective functions are considered to be obsolete, because unified numeric methods are mostly used for today.

Calculation of beam under orthogonal bending with axial forces for complicated loadings. If for the beam under orthogonal bending with axial force together with several kinds of orthogonal distributed loadings, which may interrupt there are also concentrated forces and bending moments, this beam can be calculated following the same methodology as beam on elastic foundation with complicated loadings. So we have to divide it for separate beams, using concentrated loadings and beginnings and ends of distributed loadings as boundaries. At end of beams we have to show unknown displacements (rotation angles, deflections) or reactions; to solve differential equations (12.1.4) or (12.1.5) for every separate beam, expressing displacements or reactions at ends through parameters of bending. After following to the common algorithms of the method of displacements or the method of forces for resolving of static redundancy, we may find elements of bending.

Lecture 13. Stability of Rods and Its Systems

§ 13.1. Basic Concepts and Statements of Stability of Elastic Systems

From the course of theoretical mechanics it's known that mechanical system may stay in one of three possible equilibrium states: in stable state, in unstable state and in indifferent state (fig. 13.1.1).

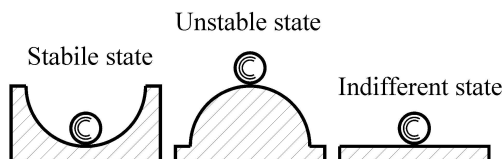


Fig. 13.1.1. Equilibrium states of the system at example of material sphere laying on different surfaces

The equilibrium state of system is called to be *stable* if for any possible little deviation of the system from given initial position the system being available for itself return into this initial position.

The equilibrium state of system is called to be *unstable* if for any possible little deviation of the system from given initial position the system being available for itself don't return into this initial position but continuing deviation from it.

The equilibrium state of system is called to be *indifferent* if for any possible little deviation of the system from given initial position the system being available for itself stay in deviated position.

For researching of stability of elastic systems and bodies relatively infinitely small deviations the simplified geometrical linear relations can be used. But in case of exploration of stability relatively finite deviations it's necessary to apply amended nonlinear equations, which are true in the whole diapason of exploring strains. Using of nonlinear equations complicate the problem much.

For explorations of stability of elastic systems the deviated equilibrium states are usually explored. For one elastic system there may be several equilibrium states. For deviated states there are equations (algebraic, differential, integral ones). There may be several or one solutions for one equation. Several solutions match for the occasion when we have several equilibrium states for one system.

The *lower critical loading* N_{low} is the value, for levels of loadings less than it the system appears to be stable not only for small deviations but also for finite ones. If the values of real external loadings are less than values of lower critical loadings the initial position of the system is the only possible stable state of the system.

The *upper critical loading* N_{up} is the value, for the levels of loadings higher than it the system appears to be unstable not only for finite deviations but also for infinitely small ones.

The large influence for stability may have initial deviations of the system from the correct geometrical form. One important factor which influences for stability much is physical nonlinearity (i.e. deviation on linear Hook's law).

The loadings causing loss of stability which are defined theoretically in assumption of validity of linear Hook's law are called Eulerian loadings. But the loadings causing loss of stability which are defined theoretically or experimentally taking into account deviation on linear Hook's law are called critical collapse stresses.

There are theoretical and experimental methods of exploring of stability. Also there are static and dynamic methods of exploration of stability. For static method we don't consider inertia forces, but for dynamic method we also consider inertial forces.

In our lectures we shall explore static stability of rods and plates applying theoretical methods.

§13.2. Stability of Rods with One Span.

The Common Algorithm of Definition of Eulerian Collapse Forces for Beams with One Span

For solution of the problem of stability of rods we use the same hypothesis that for orthogonal bending with axial forces. Additionally we consider that there is no orthogonal loadings ($q(x) = 0$) and initial deflections equals to zero ($v_0(x) = 0$). For the problem of stability it's convenient to consider the compressive forces to be positive. On the fig. 13.2.1 the explored rod for stability is shown.

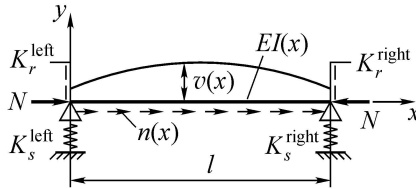


Fig. 13.2.1. For explanation of the stability of the direct rod for the most common occasion

Taking into consideration presented above statements we may compose the differential equation for stability of direct rod with one span, which can be written

$$[EI(x)v''(x)]'' + [N(x)v'(x)]' = 0, \quad (13.2.1)$$

where $EI(x)$ – is bending stiffness in plane yOx ;

$$N(x) = N_0 + \int_0^x n(x) dx - \text{is the value of internal compressive force}$$

in the cross section with coordinate x .

For prismatic rods ($EI = \text{const}$) having constant axial force ($N = \text{const}$) at ends, the differential equation (13.2.1) can be simplified and written

$$EIv^{IV}(x) + Nv''(x) = 0. \quad (13.2.2)$$

There is no common analytic solution for differential equation (13.2.1), but there are analytic solutions for (13.2.2), which will be reviewed below.

Boundary conditions for differential equations (13.2.1) and (13.2.2) are the same as for orthogonal bending with axial force, but considering that compressive axial force N is positive. For the most common occasion presented on the fig. 13.2.1 there are next boundary conditions

$$\left. \begin{aligned} -K_r^{\text{left}} v'(0) + EI(0)v''(0) &= 0, \\ -K_s^{\text{left}} v(0) - [(EI(0)v''(0))' + Nv'(0)] &= 0, \\ -K_r^{\text{right}} v'(l) - EI(l)v''(l) &= 0, \\ -K_s^{\text{right}} v(l) + [(EI(l)v''(l))' + Nv'(l)] &= 0. \end{aligned} \right\} \quad (13.2.3)$$

Both differential equations (13.2.1), (13.2.2) and boundary conditions (13.2.3) are uniform relatively deflection $v(x)$ and its derivatives. That's why those differential equations have the trivial solution $v(x) = 0$. It's necessary to look for nonzero solutions.

The common solution of simplified equation (13.2.2) on basis of (12.2.4) can be written

$$v(x) = B_0 + B_1 \beta x + B_2 \cos \beta x + B_3 \sin \beta x, \quad (13.2.4)$$

where

$$\beta = \sqrt{\frac{N}{EI}}. \quad (13.2.5)$$

Free constants B_0, B_1, B_2, B_3 have to be defined from boundary conditions.

Let's review how to calculate stability of prismatic rod on rigid hinges (see fig. 13.2.2), when there are constant compressive axial forces at ends.

Boundary conditions at ends for this case are

$$\left. \begin{aligned} v(0) &= 0, \\ EIv''(0) &= 0 \Rightarrow v''(0), \\ v(l) &= 0, \\ EIv''(l) &= 0 \Rightarrow v''(l). \end{aligned} \right\} \quad (13.2.6)$$

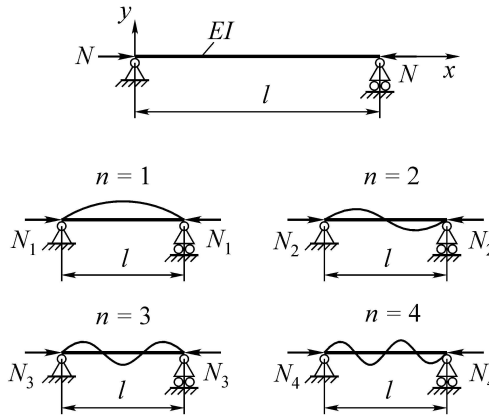


Fig. 13.2.2. For explanation of stability of the rod on rigid hinges

Obeying the common solution (13.2.4) to boundary conditions (13.2.6) we shall have next

$$\left. \begin{aligned} B_0 + B_2 &= 0, \\ -B_2\beta^2 &= 0, \\ B_0 + B_1\beta l + B_2\cos\beta l + B_3\sin\beta l &= 0, \\ -B_2\beta^2\cos\beta l - B_3\beta^2\sin\beta l &= 0. \end{aligned} \right\} \quad (13.2.7)$$

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From the second and first equations of (13.2.7) we have that $B_0 = B_2 = 0$. That's why the (13.2.7) may be simplified and can be written

$$\left. \begin{aligned} B_1 \beta l + B_3 \sin \beta l &= 0, \\ -B_3 \beta^2 \sin \beta l &= 0. \end{aligned} \right\} \quad (13.2.8)$$

The system has obvious solution $B_1 = B_3 = 0$ which fits for undeformed (direct) state of the rod. Nonzero solution $B_1 \neq 0, B_3 \neq 0$ the given system (13.2.8) may have if its determinant equals zero

$$\begin{vmatrix} \beta l & \sin \beta l \\ 0 & -\beta^2 \sin \beta l \end{vmatrix} = -\beta^3 l \sin \beta l = 0. \quad (13.2.9)$$

As multiplier $-\beta^3 l \neq 0$, the determinant equals to zero if $\sin \beta l = 0$. That's why the solution of the characteristic equation (13.2.9) can be written

$$\beta l = n\pi, \quad (13.2.10)$$

where n – is any integer number.

For this case the system (13.2.8) has the solution $B_1 = 0, B_3 \neq 0$, and additionally the quantity B_3 can be free. Thus, when $\beta l = n\pi$ the law of deflections for deviated equilibrated state can be defined by the expression

$$v(x) = B_3 \sin\left(\frac{n\pi x}{l}\right), \quad (13.2.11)$$

where B_3 has undefined expression.

Substituting (13.2.5) into (13.2.10) we may write down the expression for axial compressive Eulerian force which cause collapse of the rod

$$\frac{N}{EI} l^2 = n^2 \pi^2 \Rightarrow N = \frac{n^2 \pi^2 EI}{l^2}. \quad (13.2.12)$$

For $n = 1$ we have (see fig. 13.2.2)

$$N = N_1 = \frac{\pi^2 EI}{l^2}, \quad (13.2.13)$$

$$v_1 = B_3 \sin\left(\frac{\pi x}{l}\right). \quad (13.2.14)$$

As B_3 is free there is infinite number of equilibrium states for the form v_1 defined by (13.2.14). Hence the rod jumps into indifferent equilibrium state for the form v_1 . For $n = 2, 3, 4 \dots$ we have accordant values of Eulerian collapse forces

$$N_2 = 4N_1, \quad N_3 = 9N_1, \quad N_4 = 16N_1. \quad (13.2.15)$$

But appropriate deflections (see fig. 13.2.2) for $n = 2, 3, 4 \dots$ are

$$\begin{aligned} v_2 &= B_3 \sin\left(\frac{\pi x}{l}\right), \quad v_3 = B_3 \sin\left(\frac{3\pi x}{l}\right), \\ v_4 &= B_3 \sin\left(\frac{4\pi x}{l}\right). \end{aligned} \quad (13.2.16)$$

The values of axial forces N_2, N_3, N_4 don't take an interest, because when $N \geq N_1$ the rod jump into instable equilibrium state relatively buckling of the 1-st form v_1 . That's why the value of force N_1 defined by (13.2.13) is the Eulerian force of the prismatic rod situated on rigid hinges and compressed by constant axial forces applied to mass centers of orthogonal cross sections at ends.

For calculations of stability they usually operate with stresses which cause collapse. The common formula for definition of Eulerian collapse stresses can be written

$$\sigma_{eu} = \frac{N_{eu}}{F} = \frac{\pi^2 EI}{(\mu l)^2 F} = \frac{\pi^2 E}{\lambda^2}, \quad (13.2.17)$$

where N_{eu} – is Eulerian collapse force; μ – is special factor which depends on kinds of constraints at ends of the rod; λ – is flexibility which is defined by expression

$$\lambda = \frac{\mu l}{r}, \quad (13.2.18)$$

r – is inertia radius of the cross section which is defined by expression

$$r = \sqrt{\frac{I}{F}}. \quad (13.2.19)$$

For isolated beam the inertia moment I used in formulas presented below fits for the minimal central principle inertia moment. But if the we define our beam from complex grillages or framing connected with ship shell the inertia moment I have to be taken the same as for bending, despite this is may be maximum inertia moment.

The common algorithm of definition of Eulerian collapse forces for beams with one span. The common algorithm for calculation of prismatic rods for stability, when there are constant axial forces at ends consists of five steps:

1) to write down boundary conditions for given rod (for example using (13.2.3));

2) to substitute the expression for deflections $v(x)$ defined by (13.2.4) into boundary conditions and to get the system of homogenous linear algebraic equations for definition of constants B_0, B_1, B_2, B_3 ;

3) to equal to zero the determinant of the system we have got in p. 2 and to find out the smallest root β_1 of characteristic equation created from that determinant;

4) to substitute the smallest root β_1 into expression (13.2.5) and to find out the Eulerian collapse force;

5) to find out the form of buckling it's necessary to substitute the smallest root β_1 into homogenous system got in p. 2, after to express all factors B_i through one of factors and to insert the result into expression for deflections (13.2.4).

§ 13.3. Consideration of Deviation on Linear Hook's Law

The formula (13.2.15) for definition of collapse stresses in rods can be applied in cases if flexibility λ is more than limit flexibility λ_0 defined by the expression

$$\lambda_0 = \pi \sqrt{\frac{E}{\sigma_{pp}}}, \quad (13.3.1)$$

where σ_{pp} – is the limit of proportionality (see mechanical properties of materials);

If the real flexibility of the rod is less than the limit flexibility λ_0 defined by (13.3.1) other expressions for definition of collapse stresses have to be applied. Because if $\lambda < \lambda_0$ it's necessary to consider the deviation on Hook's law, and as it was told above collapse stresses in this case are named critical collapse stresses. Usually those expressions are empirical, but also there are theoretical formulas for critical collapse stresses.

For example there are expressions for critical collapse stresses which depend on flexibility and can be defined by expressions

$$\left. \begin{aligned} \sigma_{cr} &= a - \lambda b, \\ \sigma_{cr} &= a - \lambda b + c\lambda^2, \end{aligned} \right\} \quad (13.3.2)$$

where a, b, c – are empirical factors, which depends on kind of material, and can be found in [1].

The 1-st expression in (13.3.2) may be used for steels, but the 2-nd one for cast irons. There is another way of considering of deviation on Hook's law. It's based on using of reductive factor φ which reduces the values of Eulerian collapse stresses. For this way there is connection between critical and Eulerian collapse stresses

$$\sigma_{cr} = \varphi \cdot \sigma_{eu}, \quad (13.3.3)$$

where $0 < \varphi \leq 1$ – is special reductive factor which can be defined empirically and depends on material, kind of construction (rod, plate,

shell) and constraints. The approximate connection between Eulerian and critical collapse stresses is shown on the fig. 13.3.1.

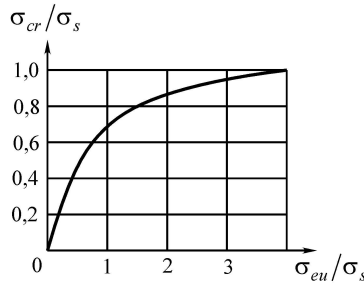


Fig. 13.3.1. The connection between Eulerian and critical collapse stresses

Graphs like on the fig. 13.3.1 are usually approximated by analytical relations. There are numerous of analytical relations between Eulerian and collapse stresses like $\sigma_{cr} = f(\sigma_{eu})$, or contrary relations like $\sigma_{eu} = \psi(\sigma_{cr})$. For example one of the used expressions $\sigma_{cr} = f(\sigma_{eu})$ for steels are

$$\left. \begin{array}{l} \text{if } \frac{\sigma_{eu}}{\sigma_s} < 0,6 \quad \text{then } \sigma_{cr} = \sigma_{eu}, \\ \text{if } 0,6 \leq \frac{\sigma_{eu}}{\sigma_s} \leq 2,6 \quad \text{then } \sigma_{cr} = \frac{\sigma_{eu}}{\sqrt[3]{1 + (\sigma_{eu}/\sigma_s)^3}}, \\ \text{if } \frac{\sigma_{eu}}{\sigma_s} > 2,6 \quad \text{then } \sigma_{cr} = \sigma_s. \end{array} \right\} \quad (13.3.4)$$

The reductive factor φ for this case can be defined using next expression

$$\varphi = \frac{\sigma_{cr}}{\sigma_{eu}}. \quad (13.3.5)$$

If the graph like presented on the fig.13.3.1 is used the reductive factor φ can be found using expression

$$\varphi = \frac{(\sigma_{cr}/\sigma_s)}{(\sigma_{eu}/\sigma_s)}. \quad (13.3.6)$$

The critical collapse stresses can't be more than the yield limit, because of the statement formulation, i.e. it is always $\sigma_{cr} \leq \sigma_s$.

§ 13.4. Stability of Grillages

Stability of grillages takes place when the orthogonal grillage is compressed by axial forces along longitudinals or cross beams, without taking into account external orthogonal loadings (see fig. 13.4.1).

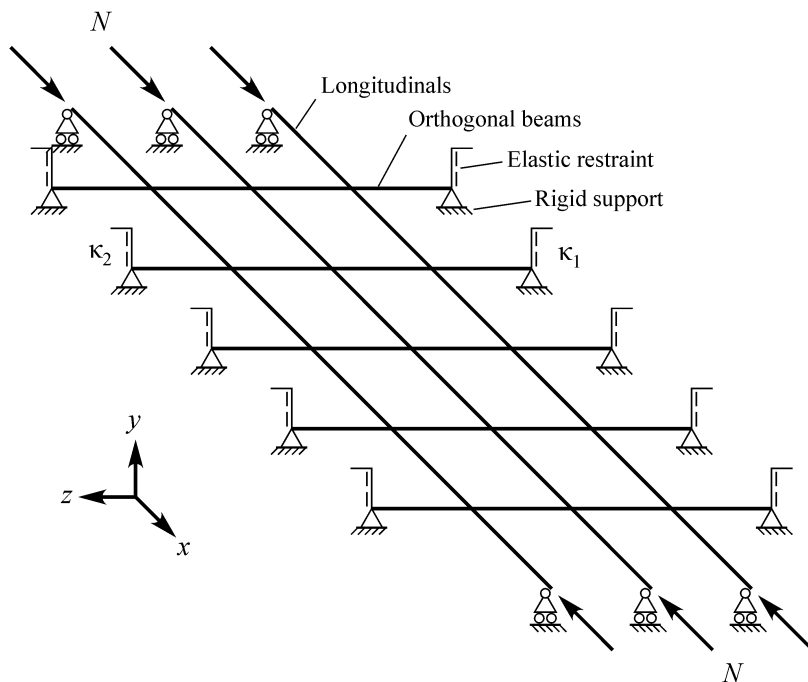


Fig. 13.4.1. Calculation scheme for grillages

For exploring of stability of grillages we consider that cross beams (which are called for this case longitudinals) are always situated on rigid hinges, but orthogonal beams (beams of major direction) are situated on rigid supports with elastic restraints. This model is confirmed by nature experiments.

There are different geometrical models of grillages, with different distances between longitudinals and orthogonal beams. Also rods of the grillage may have different geometrical properties. But boundary conditions at ends are always as presented on the fig. 13.4.1 and described above. For example for top deck grillages the orthogonal beams are beams of ship hull connected with top deck sheeting.

Within our course we shall review the model of the simplest grillage. The common denotations for the simplest grillage are presented on the fig. 13.4.2. We consider the simplest grillage to be regular along directions z and x . There are differential equations for grillages which are not reviewed here.

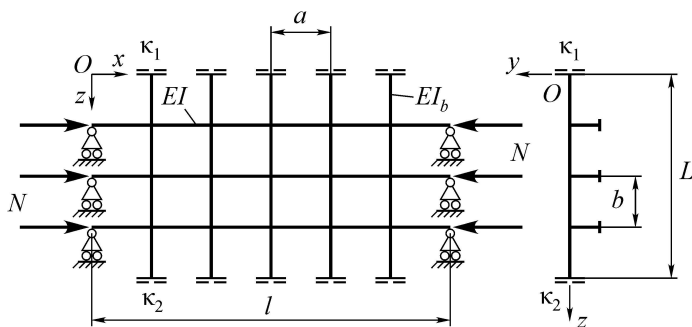


Fig. 13.4.2. The calculation scheme for the simplest grillage

For calculation of stability of grillages the next algebraic equation is used

$$I_b = \left(\frac{\pi}{\mu_1} \right)^4 \left(\frac{L}{a} \right)^3 \left(\frac{L}{b} \right) \Phi I \chi_{j \max}(\lambda), \quad (13.4.1)$$

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where I_b – is the central principal moment of inertia of cross beams in plane yOz ; $(\pi/\mu_1)_4$ – the factor which depends on coefficients of "support pair" κ_1, κ_2 and can be found in [8]; φ – is the reductive factor defined by (13.3.5) or (13.3.6); I – is the central principal moment of inertia of longitudinals in plane yOx ; $\chi_{j_{\max}}(\lambda)$ – is the special function which depends on flexibility λ and number of cross beams and can be found in [8]; λ – is flexibility which can be found within range $0 \leq \lambda \leq 1$.

If $\lambda = 0$ we consider that there is no cross beams at all, but if $\lambda = 1$ we have that our cross beams serve as completely rigid hinges for longitudinals. But if λ lies within range $\lambda \in (0; 1)$ we have that longitudinals and cross beams have some finite deflections when grillage collapses.

For design or projective calculations of grillages they usually define the critical stiffness of beams, when cross beams behave as rigid hinges for longitudinals (when $\lambda = 1$). The value of critical inertia moment of cross beams (which fits for critical stiffness of beams) can be found using expression

$$I_{b.cr} = \left(\frac{\pi}{\mu_1} \right)^4 \left(\frac{L}{a} \right)^3 \left(\frac{L}{b} \right) \varphi I \chi_{j_{\max}}(1). \quad (13.4.2)$$

There are two kinds of problems which can be solved using expression (13.4.1).

The Problem 1. For this problem there are given next quantities: the sizes l, L ; the number of cross beams n , and the number of longitudinals m ; the inertia moment I and the area f for longitudinals, including joined girdles; the coefficients of "support pair" κ_1, κ_2 (see expression (7.2.8)); mechanical properties of material $E, \sigma_s, \sigma_{eu} = \psi(\sigma_{cr})$; and critical collapse stresses for the grillage σ_{cr} .

It's necessary to find the value of inertia moment of cross beams I_b , for which the stability of grillage is provided for given σ_{cr} .

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The solution can be found in several steps using the scheme presented on the fig. 13.4.3.

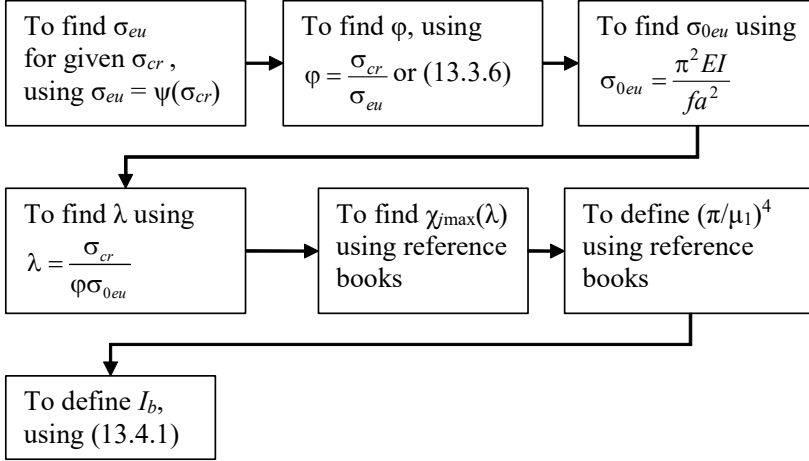


Fig. 13.4.3. The sequence of the solution of the problem #1 for stability of grillages

The *Problem 2*. For the 2-nd problem there are given quantities: $l, L, a, b, I, I_b, \kappa_1, \kappa_2, E, \sigma_s, \sigma_{cr} = f(\sigma_{eu}), m, n$. It's necessary to find σ_{cr} . The solution of this problem can be provided using next expression

$$\varphi \chi_{j\max}(\lambda) = \frac{I_b}{\left(\frac{\pi}{\mu_1}\right)^4 \left(\frac{L}{a}\right)^3 \left(\frac{L}{b}\right) I}. \quad (13.4.3)$$

The solution of the problem #2 can be found if to find out the root of the next equation $F(\lambda)$ depending on flexibility and created on basis on (13.4.3)

$$F(\lambda) = \varphi \chi_{j\max}(\lambda) - \frac{I_b}{\left(\frac{\pi}{\mu_1}\right)^4 \left(\frac{L}{a}\right)^3 \left(\frac{L}{b}\right) I}, \quad (13.4.4)$$

where

$$\left. \begin{aligned} \varphi &= \frac{\sigma_{cr}}{\sigma_{eu}} \quad \text{or} \quad \varphi = \frac{(\sigma_{cr}/\sigma_s)}{(\sigma_{eu}/\sigma_s)}, \\ \sigma_{eu} &= \lambda \sigma_{0eu}, \quad \sigma_{0eu} = \frac{\pi^2 EI}{fa^2}, \quad \sigma_{cr} = f(\sigma_{eu}). \end{aligned} \right\} \quad (13.4.5)$$

After the root of (13.4.4) have been found, it's necessary finally to define σ_{cr} using (13.4.5).

PART IV. BENDING AND STABILITY OF PLATES. NUMERIC METHODS

Lecture 14. Bending of Thin Rectangular Plates

§ 14.1. Basic Concepts for Plates

Plate is a prism for which the sizes of foundation are much larger than the prism height. There are different kinds of plates that are why there are different classification systems for plates. There are thin and thick plates. Thin plates are plates for which the smallest size of foundation is in 5 (five) or more times larger than the height size. The contrary definition is for thick plates. Plates can be classified for the geometry of the foundation, there are plates: rectangular, square, round, elliptic, etc. Also there are plates closed one time or several times (so they have holes). There are plates with constant thickness and plates with varied thickness. There is classification of plates for their strain-stress state presented below.

The middle plane (or surface for plates with varying thickness) is geometrical place of points which are equidistant from outer foundations (see fig. 14.1.1). The beginning of coordinate axes lies in the middle plane for plates with constant thickness. For plates with varied thickness the beginning of coordinate axes is specified. External loadings act in orthogonal directions to the middle plane of plate.

On the fig. 14.1.1 there are: d_{min} – is the minimal size of plate foundations, [m]; t – thickness, [m]; P – external concentrated force, [N]; M – external concentrated bending moment, [N·m]; $q(x, y)$ – external distributed loading, [N/m²].

As for bending of beams we don't use the distributed bending moments reviewing bending of plates. Further we shall review only thin isotropic plates with constant thickness.

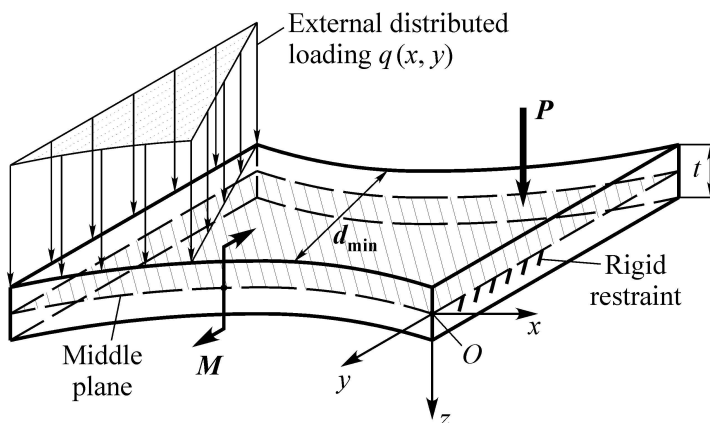


Fig. 14.1.1. Major denotations for thin plates

§ 14.2. Hypothesis of Technical Theory of Bending of Thin Plates. Linear Internal Forces. Karman Differential Equations. Boundary Conditions for Rectangular Plates. Classification of Plates

Hypothesis of technical theory of bending of thin plates.

There are five common assumptions for the theory of bending of thin plates, which are called Kirchhoff assumptions. These assumptions are:

- 1) the displacement w (the deflection of plate) is constant within thickness of plate and considered to be small quantity;
- 2) displacements u and v are much less than the displacement w ;

3) the hypothesis of direct normal lines is true: direct linear elements of plate orthogonal to middle plane before deformation still to be direct and orthogonal to middle plane after deformation;

4) every layer parallel to middle plane of plate stay in flat strain-stress state, but pressure of these parallel layers for each other can be neglected at;

5) the material of plate is isotropic obeying to linear Hook's law.

Linear internal forces. At cross sections orthogonal to the middle plane of plate there are linear internal forces act. Positive directions of these internal forces acting at faces of elementary volume of plate are presented on the fig. 14.2.1.

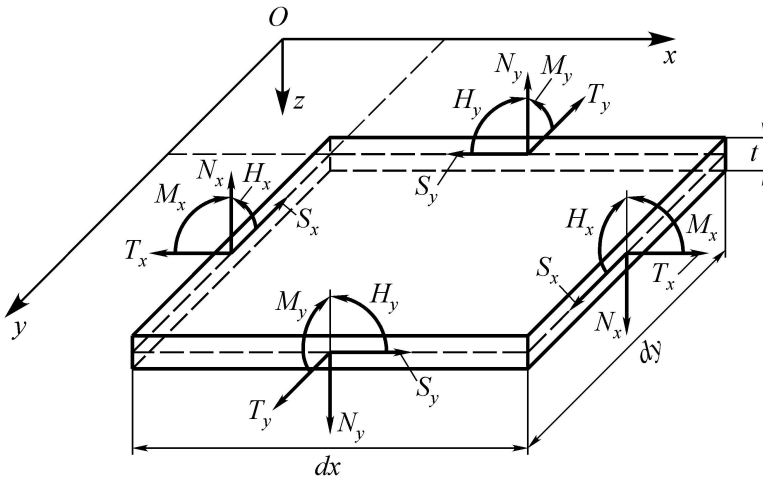


Fig. 14.2.1. Internal forces on the plate elementary volume

The names of internal forces shown on the fig. 14.2.1 are: T – normal forces, $[N/m]$; S – tangent forces, $[N/m]$; N – shear forces, $[N/m]$; M – bending moments, $[N]$; H – torques, $[N]$. Quantities T and S are sometimes called chain forces.

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Internal linear forces acting at cross sections of plate can be found using next expressions

$$\left. \begin{aligned} T_x &= \int_{-0,5t}^{0,5t} \sigma_x dz, & T_x &= \int_{-0,5t}^{0,5t} \sigma_y dz, \\ N_x &= \int_{-0,5t}^{0,5t} \tau_{xz} dz, & N_y &= \int_{-0,5t}^{0,5t} \tau_{yz} dz, \\ S &= S_x = S_y = \int_{-0,5t}^{0,5t} \tau_{xy} dz, \\ M_x &= \int_{-0,5t}^{0,5t} \sigma_x z dz, & M_y &= \int_{-0,5t}^{0,5t} \sigma_y z dz, \\ H &= H_x = H_y = \int_{-0,5t}^{0,5t} \tau_{xy} z dz. \end{aligned} \right\} \quad (14.2.1)$$

As usually they do in structural mechanics we have to define the elementary volume of plate in deviated state after deformation and to compose equilibrium equations for that in deviated state. The procedure of deduction of equilibrium equations is quite complex that's why it is not presented here, for the reason of limitation of our course. Internal forces are not independent but they influence for each other.

In the theory of bending and stability of plates the stress function $F(x, y)$ for flat strain stress state is used which is connected with other physical quantities by the next relations

$$T_x = t \frac{\partial^2 F}{\partial y^2}, \quad T_y = t \frac{\partial^2 F}{\partial x^2}, \quad S = -t \frac{\partial^2 F}{\partial x \partial y}. \quad (14.2.2)$$

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Stresses caused by internal forces can be found using next formulas

$$\left. \begin{aligned} \sigma_x &= \frac{T_x}{t} + \frac{12M_x}{t^3} z, & \sigma_y &= \frac{T_y}{t} + \frac{12M_y}{t^3} z, \\ \tau_{xy} &= \frac{S}{t} + \frac{12H}{t^3} z, & \tau_{xz} &= \frac{6N_x}{t^3} \left(\frac{t^2}{4} - z^2 \right), \\ \tau_{yz} &= \frac{6N_y}{t^3} \left(\frac{t^2}{4} - z^2 \right). \end{aligned} \right\} \quad (14.2.3)$$

The maximum values of stresses caused by bending moments M , torque H and shear forces N can be defined using next expressions

$$\left. \begin{aligned} \left| \sigma_x^b \right|_{\max} &= \frac{6|M_x|}{t^2}, & \left| \sigma_y^b \right|_{\max} &= \frac{6|M_y|}{t^2}, \\ \left| \tau_{xy}^{tor} \right|_{\max} &= \frac{6|H|}{t^2}, & \text{when } z &= \pm t/2 \\ \left| \tau_{xz} \right|_{\max} &= \frac{3|N_x|}{2t}, & \left| \tau_{yz} \right|_{\max} &= \frac{3N_y}{2t}, \quad \text{when } z = 0, \end{aligned} \right\} \quad (14.2.4)$$

where $\left| \sigma_x^b \right|_{\max}$, $\left| \sigma_y^b \right|_{\max}$ – are normal stresses caused by accordant bending moments M_x and M_y ; $\left| \tau_{xy}^{tor} \right|_{\max}$ – are shear stresses caused by torque H .

Average shear stresses can be found using next expressions

$$\tau_{xz}^{av} = \frac{N_x}{t}, \quad \tau_{yz}^{av} = \frac{N_y}{t}. \quad (14.2.5)$$

Between deflection w and internal forces there are next relations

$$\left. \begin{aligned} M_x &= -D \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right), \\ M_y &= -D \left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right), \\ H &= H_x = H_y = -D(1-\mu) \frac{\partial^2 w}{\partial x \partial y}, \\ N_x &= -D \frac{\partial}{\partial x} \nabla^2 w, \quad N_y = -D \frac{\partial}{\partial y} \nabla^2 w, \\ D &= \frac{Et^3}{12(1-\mu^2)}, \quad \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \end{aligned} \right\} \quad (14.2.6)$$

where D – is so called cylindrical stiffness of plate: the stiffness of beam under bending the width of which equals to one but the height equals to t .

Karman differential equations. The system of differential equations which define the behavior of thin isotropic plates can be written

$$\left. \begin{aligned}
 \nabla^2 \nabla^2 F &= E \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right], \\
 D \nabla^2 \nabla^2 w &= \\
 &= q + t \left(\frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right), \\
 \nabla^2 \nabla^2 &\equiv \\
 &= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) = \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4},
 \end{aligned} \right\} (14.2.7)$$

where E – is modulus of elasticity in tension; $q = q(x, y)$ – is the law of external distributed loadings; t – is thickness; F – is the stress function defined by (14.2.2).

The system of equations (14.2.7) is called the major system of solving equations in the theory of thin isotropic plates of Karman.

Boundary conditions for rectangular plates. For the contour of plate there may be different boundary conditions composed which are expressed through the desired function $w(x, y)$. These boundary conditions are: deflections, rotation angles, bending moments, shear forces, normal forces, torques or relations between these quantities. In the table 14.2.1 there are boundary conditions for some constraints at edges of rectangular plates are presented.

Table 14.2.1. Boundary conditions for different constraints of edges of plates

#	The name and the graphic denotation of the plate boundary condition	The expression for the boundary condition
1	Rigid hinge 	$\left. \begin{aligned} w = 0, \quad \frac{\partial^2 w}{\partial x^2} = 0 \\ \text{for } x = \text{const}; \\ w = 0, \quad \frac{\partial^2 w}{\partial y^2} = 0 \\ \text{for } y = \text{const.} \end{aligned} \right\}$
2	Rigid restraint 	$\left. \begin{aligned} w = 0, \quad \frac{\partial w}{\partial x} = 0 \\ \text{for } x = \text{const}; \\ w = 0, \quad \frac{\partial w}{\partial y} = 0 \\ \text{for } y = \text{const.} \end{aligned} \right\}$
3	Free end 	$\left. \begin{aligned} M_x = 0, \quad N_x = 0, \quad H = 0 \\ \text{for } x = \text{const}; \\ M_y = 0, \quad N_y = 0, \quad H = 0 \\ \text{for } y = \text{const.} \end{aligned} \right\}$

Classification of plates. There is no accurate common analytic solution for Karman equations. Karman equations can be simplified for some occasions. There are next kinds of plates which depend on strain-stress state of plates.

Rigid plates. The deflection of these plates is so small that influence of chain forces T_x, T_y, S can be disregarded ($T_x = T_y = S = 0$). The full system of Karman differential equations (14.2.7) for this case can be simplified

$$D\nabla^2\nabla^2 w = q, \quad (14.2.8)$$

i.e. the deflection w can be defined independently from stress function $F(x, y)$. Model of rigid plates is true for cargo ship hull sheeting.

Bendable plates (or plates of finite stiffness) of small deflection. For the reason of smallness of deflection w we can neglect with nonlinear members in the right part of the first equation (14.2.7). Karman equations for this case can be written

$$\left. \begin{aligned} \nabla^2\nabla^2 F &= 0, \\ D\nabla^2\nabla^2 w &= q + t \times \\ &\times \left(\frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right). \end{aligned} \right\} \quad (14.2.9)$$

Equations (14.2.9) are little used in usual calculation practice but these equations are widely used in the linear theory of stability of plates.

Bendable plates (or plates of finite stiffness) of large deflection. For this case there are no simplifications. The behavior of these plates is described by full system of Karman differential equations (14.2.7).

Absolutely flexible plates (membranes). For these plates bending stiffness D is very small, so it can be neglected at (i.e. $D=0$). These plates can receive only chain forces T_x, T_y, S . For this case differential equations of Karman can be written

$$\left. \begin{aligned} \nabla^2\nabla^2 F &= E \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right], \\ \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} &= -\frac{q}{t}. \end{aligned} \right\} \quad (14.2.10)$$

**§ 14.3. Bending of Rigid Rectangular Plates.
Usage of Reference Books**

The model of rigid plate is the simplest model of plate and widely used in practical calculations despite for some limitations for this model. The sheeting defined within rigid contour for cargo ships as a rule is always considered as the rigid plate. Differential equations for rigid plates were got above and can be written

$$D\nabla^2\nabla^2w = q \Rightarrow \frac{\partial^4w}{\partial x^4} + 2\frac{\partial^4w}{\partial x^2\partial y^2} + \frac{\partial^4w}{\partial y^4} = \frac{q}{D}. \quad (14.3.1)$$

For rectangular plates there are common accurate analytic solutions for any boundary conditions and for any law of external distributed loadings $q(x, y)$. The common form of the solution of (14.3.1) can be presented as

$$w(x, y) = w_u(x, y) + w_p(x, y), \quad (14.3.2)$$

where w_u – is the solution of the uniform equation $D\nabla^2\nabla^2w = 0$, but w_p – is the partial solution which depends on external distributed loading $q(x, y)$.

Usage of reference books. For rectangular rigid plates there are common accurate analytic solutions for any boundary conditions and for any law of external distributed loadings $q(x, y)$. The formulas for analytic solutions are quite heavy for practical manual calculations, because as a rule they contain rows. That's why there are special reference books (for example [7]) and tables where necessary factors are already calculated for different calculation schemes of plates.

Example. Check the strength of rectangular plate under uniform external loading $q(x, y) = q = \text{const}$, presented on the fig. 14.3.1. There are next sizes: $a = 2,4$ m; $b = 1,2$ m; $t = 0,01$ m. The material is steel with $\sigma_s = 235$ MPa. The value of external loading $q = 10^4$ Pa.

Solution. At the fig. 14.3.1 the initial scheme (on the left) and the appropriate calculation scheme (on the right) for the given reference book [7] are presented.

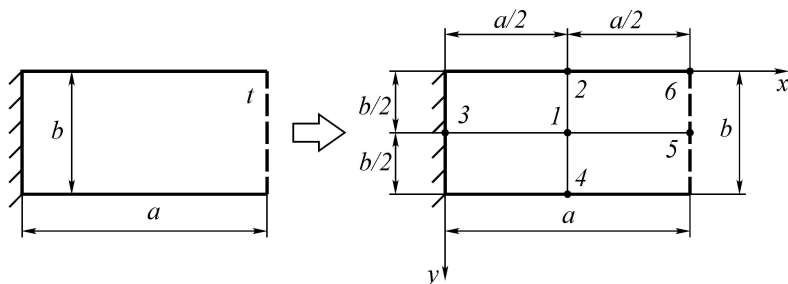


Fig. 14.3.1. The plate to be verified for example

For rigid plates bending moments are major factors which cause strain stress state of the plate. It's necessary to find the values of bending moments in special points 1, 2, ... 6. For our occasion it's enough to find the values of bending moments in points 1, 3. The expressions for bending moments for points 1, 3 can be found using next expressions

$$M_{x3} = v_{13}qb^2, \quad M_{y5} = v_{25}qb^2, \quad (14.3.3)$$

where factors $v_{13} = 0,125$ and $v_{25} = 0,131$ are found from special tables in [7] for $\gamma = a/b = 2$.

Calculating bending moments using (14.3.3) we have

$$M_{x3} = 0,125 \cdot 10^4 \cdot 1,2^2 = 1800,0 \text{ N};$$

$$M_{y5} = 0,131 \cdot 10^4 \cdot 1,2^2 = 1886,4 \text{ N}.$$

It's necessary to choose the dangerous point. The dangerous point is there where the equivalent stresses σ_{eq} have the maximum value. Without considering of shear stresses the value of equivalent stresses according to the 4-th strength theory can be found using expression

$$\sigma_{eq} = \frac{6}{t^2 \sqrt{2}} \sqrt{2M_{xi}^2 + 2M_{yi}^2 - 2M_{xi} M_{yi}}, \quad (14.3.4)$$

where i – is the number of point.

Analyzing our points on basis of (14.3.4) we have that the most dangerous point is point 5. The value of equivalent stresses σ_{eq} for the point 5 is

$$\left. \begin{aligned} \sigma_{eq}^{\max} = \sigma_{eq} &= \frac{6}{t^2 \sqrt{2}} \sqrt{2M_{x5}^2 + 2M_{y5}^2 - 2M_{x5} M_{y5}} = \\ &= \frac{6}{0,01^2 \sqrt{2}} \sqrt{2 \cdot 1886,4^2} = 1,13 \cdot 10^8 \text{ Pa.} \end{aligned} \right\} \quad (14.3.5)$$

The condition of providing of strength can be written

$$\sigma_{eq}^{\max} \leq [\sigma], \quad (14.3.6)$$

where $\sigma_{eq}^{\max}$ – are maximum equivalent stresses in construction; $[\sigma]$ are allowable stresses which can be found using expression

$$[\sigma] = \frac{\sigma_s}{n_s} = \frac{235}{1,5} = 157 \text{ MPa}, \quad (14.3.7)$$

$n_s = 1,5$ – is safety factor for plastic materials in conditions of static load.

Substituting (14.3.7) and (14.3.5) into (14.3.6) we may see that the strength condition (14.3.6) is provided.

Answer. The strength of plate is provided for given sizes, load and material.

Lecture 15. Stability of Thin Rectangular Plates

§ 15.1. Problem Formulation, Differential Equations and Boundary Conditions for the Stability of Plates. Partial Solutions

Problem formulation, differential equations and boundary conditions for the stability of plates. The problem of stability takes place if plate is loaded by compressive axial or/and tangent forces (i.e. loaded only by T_x, T_y, S) without considering of other internal and external forces (i.e. $M_x = M_y = N_x = N_y = H = 0, q(x, y) = 0$). For some values of those loadings the collapse (losing of stability) may take place for plates and after collapse plate doesn't participate fully in work of construction.

To explore the stability of plates it's necessary to have equilibrium equations in deviated state. Such equations were got in lecture 14. Reviewing stability for little deviations there are differential equations for stability of plates which were got on basis of equations for bendable plates (or plates of finite stiffness) of small deflection (14.2.9) and (14.2.2) considering $T_x < 0$ and $T_y < 0$, which can be written

$$D\nabla^2\nabla^2 w + T_x \frac{\partial^2 w}{\partial x^2} - 2S \frac{\partial^2 w}{\partial x \partial y} + T_y \frac{\partial^2 w}{\partial y^2} = 0. \quad (15.1.1)$$

The boundary conditions for (15.1.1) are the same as for bending of plates. Chain forces T_x, T_y, S can be found after solution of flat problem of the theory of elasticity (see lecture 5). The problem of stability of plates can be narrowed down to finding of nonzero solutions of (15.1.1) for given boundary conditions and laws of T_x, T_y, S . There are some accurate analytic solutions for (15.1.1) for rectangular plates.

Partial solutions. In the table 15.1.1 some partial solutions of differential equation of stability of plates (15.1.1) for some laws of chain forces T_x, T_y, S and boundary conditions are presented.

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Table 15.1.1. Some partial solutions of differential equation of stability of plates (15.1.1) for rectangular plates

#	The calculation scheme	The expression for Eulerian collapse stresses
1		$\left. \begin{aligned} \sigma_{eu} &= \frac{4\pi^2 D}{b^2 t} \\ \text{for } \gamma &\geq 1; \\ \sigma_{eu} &= \frac{\pi^2 D(1 + \gamma^2)^2}{a^2 t} \\ \text{for } \gamma &< 1. \end{aligned} \right\}$
2		$\sigma_{eu} = \frac{k\pi^2 D}{b^2 t},$ where $k = f(\gamma)$ can be found in reference books
3	$\sigma_x = \sigma_0 \left(1 - \frac{\beta y}{b} \right),$ where β – is set factor.	$\sigma_{eu} = \frac{k_1 \pi^2 D}{b^2 t},$ where $k_1 = f_1(\beta, \gamma)$ can be found in reference books
4		$\tau_{eu} = \frac{k_\tau \pi^2 D}{b^2 t},$ where $k_\tau = f_\tau(\gamma)$ can be found in reference books

Lecture 15

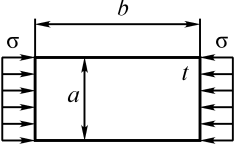
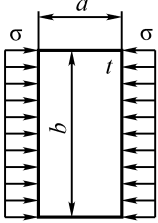
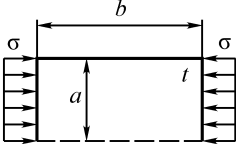
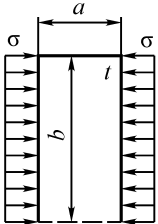
In the table 15.1.1 the parameter γ can be found using expression

$$\gamma = \frac{a}{b}. \quad (15.1.2)$$

In the table 15.1.1 the quantity D is cylindrical stiffness of plate defined by (14.2.6).

For steel plates ($E = 2 \cdot 10^{11}$, $\mu = 0,3$) expressions in the table 15.1.1 can be simplified. Simplified expressions for Eulerian collapse stresses are presented in the table 15.1.2.

Table 15.1.2. Expressions for Eulerian collapse stresses for steel plates

#	The calculation scheme	The expression for Eulerian collapse stresses
1		$\sigma_{eu} = 80 \left(\frac{100t}{a} \right)^2$
2		$\sigma_{eu} = 20 \left(\frac{100t}{a} \right)^2 \left(1 + \left(\frac{a}{b} \right)^2 \right)^2$
3		$\sigma_{eu} = 8,4 \left(\frac{100t}{a} \right)^2 \text{ for } b \gg a$
4		$\sigma_{eu} = 20 \left(\frac{100t}{a} \right)^2 \times \left[1 + 0,426 \left(\frac{a}{b} \right)^2 - \frac{0,143a}{b(4 + (a/b)^2)} \right]$

The expressions in the table 15.1.2 are used in practice to estimate the stability of ship hull sheeting.

§ 15.2. Consideration of Deviation on Hook's Law For Exploring Of Stability of Plates

The formulas for collapse stresses are true only within of applicability of linear Hook's law. The formulas in tables 15.1.1 and 15.1.2 are true if stresses in plate don't exceed the proportionality limit σ_{pp} , i.e. $\sigma_{eu} \leq \sigma_{pp}$. To provide the condition $\sigma_{eu} \leq \sigma_{pp}$ for plate on the 1-st scheme in the table 15.1.2 the next relation has to be satisfied

$$\left(\frac{b}{t}\right) \geq 100 \sqrt{\frac{80}{\sigma_{pp}}}. \quad (15.2.1)$$

To provide the condition $\sigma_{eu} \leq \sigma_{pp}$ for plate on the 2-nd scheme in the table 15.1.2 the next relation has to be satisfied (for $b \gg a$)

$$\left(\frac{a}{t}\right) \geq 100 \sqrt{\frac{20b}{\sigma_{pp}}}. \quad (15.2.2)$$

For orthogonal hull structural arrangement the condition (15.2.2) as a rule is satisfied, but for longitudinal structural arrangement the condition (15.2.1) is not always provided.

Let's show the way of consideration of deviation on Hook's law for exploration of stability of plates. All layers parallel to the middle cross section are in its unique flat strain-stress states. So every layer parallel to the middle cross section stay is flat strain-stress state which differs from strain-stress states of neighboring parallel layers.

Accurate investigations of stability of plates in elastoplastic field may be provided by methods of the theory of elasticity applied for flat strain-stress state. Instead of linear Hook's law in the theory of bending of plates it's necessary to substitute nonlinear equations which connect

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stresses and strains. As result we shall have the possibility to get equations for exploration of stability of plates for stresses higher than the proportionality limit σ_{pp} , and to define critical collapse stresses from those stability equations. It's necessary to note that for this way the investigation of stability of isotropic plates in elastoplastic field is narrowed down to the problem of stability in elastic zone of orthotropic plates with different stiffness for bending in two directions and for torsion. The relations between rigidities of orthotropic plate depend on the level of enlarging of the proportionality limit by collapse stresses.

***Lecture 16. Modern Numeric Methods of Solutions for
the Problems of the Theory of Elasticity
and Ship Structural Mechanics***

**§16.1. The General Description of Structures
Strength Analysis Methods**

The mathematical model describes behavior of the physical model that is affected by some external loads. Choosing or development of the efficient method of the mathematical model investigation is one of the missions of the strength analysis.

The typical physical models of the theory of elasticity and the structural mechanics (the massive body, rod, plate) have been described in the previous lectures. Mathematical models for these models have been described using differential equations. But we should note that other forms of mathematical description of physical models exist, such as functional equations, integral equations. In the theory of elasticity and structural mechanics variational principle is widely used. It is based on the energy theorems and principles. This form is usable for development of approximate methods of mathematical model investigation (we do not show it in the present course). For now methods based on variational principles are one of the most efficient methods which provide high accuracy of results and relatively low labor-consuming and good algorithmization. That's why variational methods are widely used for now.

We may categorize all the mathematical methods into three groups regardless the type of the mathematical model. These are *analytical*, *semi-analytical* and *discrete* groups of methods. The methods of the first group allow to obtain the accurate precise solution for given equation in the function form. The methods of the last category are the most universal for all equations and allow to obtain the approximate solution for certain body points or for discrete volumes of the body.

consists on. Discrete group of methods divide into two common kinds: net methods and so called discrete volume methods. In net methods (collocation method, finite difference method (FDM), minimal squares method (MSM), etc.) we deal with certain body points and can know the function values only in those points. For discrete volume methods (Ritz method, Bubnov-Galerkin method, finite element method (FEM), boundary element method (BEM), finite volume method (FVM), etc.) we deal with discrete volumes, which together form the whole body. For every discrete volume the analytic solution of the required function is known. Usually net methods are simpler, less labor-consuming but less accurate in comparison with discrete volume methods.

Semi-analytical methods hold an intermediate position. There are a lot of semi-analytical methods so it's hard to classify them. These methods sometimes can be very effective, but sometimes it's impossible to apply them, because they require some analytic solutions of the given equations, which can be provided not always. One of the prominent semi-analytical methods is for example one of variants of boundary elements methods.

The development of the analytical solution is difficult in many cases due to variable coefficients of differential equations and complicated boundary conditions. This fact has been shown by analysis of mathematical models in the previous lections. Accurate analytic solutions are possible only for very simple models and boundary conditions. But analytic solutions are important to check numeric results. Therefore, discrete numerical methods are mainly used for solution obtaining. Such methods allow to obtain approximate solution with required level of accuracy and to get round mentioned difficulties.

Numerical methods should be used in practice together with computer and appropriate software. For lack of place here below only the simplest discrete methods are explained such as finite difference method and finite element method.

§ 16.2. The Finite-Difference Method

Equations describing behavior of physical models may contain derivatives. There are different approximate formulas (so called-difference expressions) for derivatives, which define the derivative value of given function and for given order in defined point through function values defined in neighboring points. Substituting these approximate formulas into describing equations we shall have so called finite difference model or finite difference equations. Writing finite-difference equations for every body point we shall have the system of equations relatively function values for every point. The accuracy of FDM method depends much on number of body points and point's regularity and other factors.

We may obtain mentioned finite-difference expressions in a variety of ways. But expansion in a Taylor series is most often used way. The central finite-difference approximation (FDA) is mostly used in practice, but other approximations (left FDA, right FDA) are also used.

For example one of formulas for central FDA for the function depending on one variable can be written

$$\left. \begin{aligned} f'(x_i) &= \frac{f(x_i + h) - f(x_i - h)}{2h}; \\ f''(x_i) &= \frac{f(x_i + h) - 2f(x_i) + f(x_i - h)}{h^2}; \\ f'''(x_i) &= \frac{f(x_i + 2h) - 2f(x_i + h) + 2f(x_i - h) - f(x_i - 2h)}{2h^3}; \\ f^{IV}(x_i) &= \frac{f(x_i + 2h) - 4f(x_i + h) + 6f(x_i) - 4f(x_i - h) + f(x_i - 2h)}{h^4}, \end{aligned} \right\} (16.2.1)$$

where f – is function depending on one variable; x_i – is fixed abscissa; h – is step.

The accuracy of expressions (16.2.1) is about h^2 . For regular mesh expressions (16.2.1) can be written

$$\left. \begin{aligned} f'_i &= \frac{1}{2h} (f_{i+1} - f_{i-1}); \\ f''_i &= \frac{1}{h^2} (f_{i+1} - 2f_i + f_{i-1}); \\ f'''_i &= \frac{1}{2h^3} (f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2}); \\ f^{IV}_i &= \frac{1}{h^4} (f_{i+2} - 4f_{i+1} + 6f_i - 4f_{i-1} + f_{i-2}), \end{aligned} \right\} \quad (16.2.2)$$

where v_i – is the deflection in point i .

Explanations for expressions (16.2.1) and (16.2.2) are presented on the fig. 16.2.1,*a*.

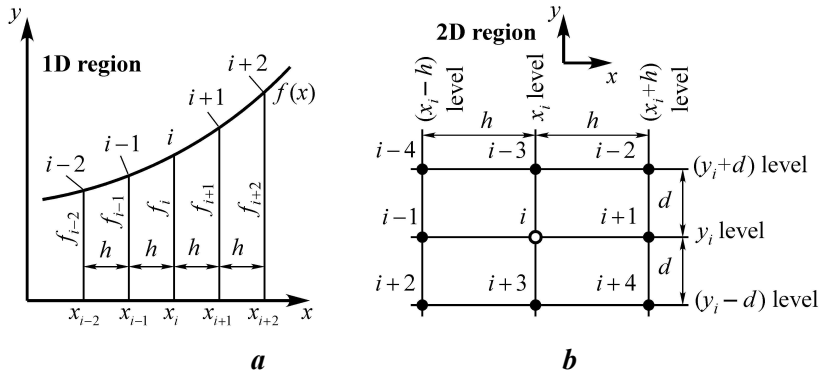


Fig. 16.2.1. For explanation of the finite-difference approximation

Finite-difference approximation also can be used for functions depending on several variables. Expressions (16.2.1) or/and (16.2.2) are also can be used to find derivative value in certain i -th point for function of several variables (see fig. 16.2.1,*b*).

Let's review on example how to find the mixed derivative $\partial^2 f / \partial x \partial y$ in i -th point (see fig. 16.2.1,*b*), where $f = f(x, y)$ is some

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function depending on two variables, but x_i and y_i are coordinates of i -th point. The given mixed derivative $\partial^2 f / \partial x \partial y$ can be written

$$\frac{\partial^2 f(x, y)}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial f(x, y)}{\partial x} \right). \quad (16.2.3)$$

So first it's necessary to take derivative in x -direction $\partial f / \partial x$, and after it's necessary to take derivative in y -direction $\partial (\partial f / \partial x) / \partial y$ according to (16.2.3). To take derivative in x -direction means to take derivative for all fixed y -levels (see fig. 16.2.1, **b**). For some y -level y coordinate in $f(x, y)$ is fixed, but x coordinate is variable, so we deal with function depending only on one variable x , and 1-st equation of (16.2.1) or/and (16.2.2) can be applied for this function of one variable. To find derivative $\partial f / \partial x$ for y_i level means to take derivative for i -th point. Analogously to take derivative $\partial f / \partial x$ for $(y_i - d)$ and for $(y_i + d)$ levels means to take this derivative for points $i + 3$ and $i - 3$ accordingly. On base of the 1-st expression of (16.2.1) and (16.2.2) derivative $\partial f / \partial x$ for levels $(y_i - d), y_i, (y_i + d)$ can be written

$$\left. \begin{aligned} \frac{\partial}{\partial x} f(x_i, y_i - d) &= \frac{f(x_i + h, y_i - d) - f(x_i - h, y_i - d)}{2h} = \\ &= \frac{1}{2h} (f_{i+4} - f_{i+2}) = Y_{j-1} \\ \frac{\partial}{\partial x} f(x_i, y_i) &= \frac{f(x_i + h, y_i) - f(x_i - h, y_i)}{2h} = \\ &= \frac{1}{2h} (f_{i+1} - f_{i-1}) = Y_j \\ \frac{\partial}{\partial x} f(x_i, y_i + d) &= \frac{f(x_i + h, y_i + d) - f(x_i - h, y_i + d)}{2h} = \\ &= \frac{1}{2h} (f_{i-2} - f_{i-4}) = Y_{j+1}, \end{aligned} \right\} \quad (16.2.4)$$

where f_{i+4} – is the function value $f(x, y)$ in point $i + 4$, etc.; Y_{j-1} – is derivative value in point $i + 3$ for $(y_i - d)$ level, etc.

To take derivative in y -direction we consider y -levels to be as "points". For every fixed y -level the derivative value $\partial f / \partial x$ is already known. These values are Y_{j-1}, Y_j, Y_{j+1} for levels $(y_i - d), y_i, (y_i + d)$ accordingly. Using derivative values for y -levels as values for the function of one variable y on base of 1-st expression of (16.2.1) or/and (16.2.2) we can write down the formula to find the demanded derivative $\partial (\partial f / \partial x) / \partial y$

$$\begin{aligned} \frac{\partial^2 f(x, y)}{\partial x \partial y} &= \frac{\partial}{\partial y} \left(\frac{\partial f(x, y)}{\partial x} \right) = \\ &= \frac{1}{2d} (Y_{j+1} - Y_{j-1}). \end{aligned} \quad (16.2.5)$$

Substituting (16.2.4) into (16.2.5) we shall have the finite-difference approximation to find mixed derivative $\partial^2 f / \partial x \partial y$ in point for the function $f = f(x, y)$, depending on two variables, which can be written

$$\left. \begin{aligned} \frac{\partial^2 f(x, y)}{\partial x \partial y} &= \frac{\partial}{\partial y} \left(\frac{\partial f(x, y)}{\partial x} \right) = \\ &= \frac{1}{4dh} [f(x_i + h, y_i + d) - f(x_i - h, y_i + d) - \\ &\quad - f(x_i + h, y_i - d) + f(x_i - h, y_i - d)] = \\ &= \frac{1}{4dh} (f_{i-2} - f_{i-4} - f_{i+4} + f_{i+2}). \end{aligned} \right\} \quad (16.2.6)$$

Analogously other FDA can be reached for other mixed derivatives.

§ 16.3. The Finite Element Method

Finite element method (FEM) is based on the suppositive representation of a solid body (a continuum) as an assemblage of discrete finite elements (FE). These elements interact with each other in nodal points. Usually the body consists on one type of FEs. The FEs we approximate a body have set numbers of degrees of freedom. For every degree of freedom there is the common nodal displacement q_i and the common appropriate nodal reaction R_i fit (see fig. 16.3.1).

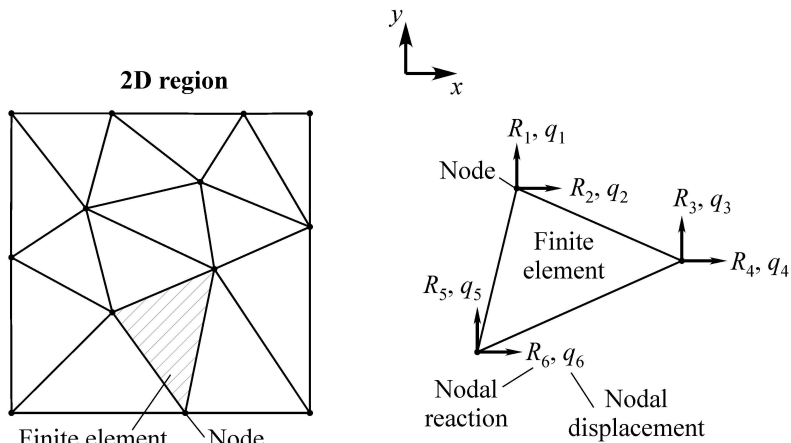


Fig. 16.3.1. Approximation of the plate for the flat (2D) problem of the theory of elasticity by FE net

The required function within FE and other accompany functions can be defined through nodal displacements $\{q\}$ or nodal reactions $\{R\}$, where $\{q\} = \{q_1, q_2, \dots, q_n\}$ – is vector of nodal displacements for all n number of nodes, but $\{R\} = \{R_1, R_2, \dots, R_n\}$ – is vector of nodal reactions. The required function usually is defined in local coordinate system connected with EF.

Between nodal displacements $\{q\}$ and nodal reactions $\{R\}$ there is connection. In the most common occasion $\{q\}$ and $\{R\}$ are connected with some functions

$$\left. \begin{array}{l} q_1 = f_1(R_1, R_2, \dots, R_n); \\ q_2 = f_2(R_1, R_2, \dots, R_n); \\ \dots \\ q_n = f_n(R_1, R_2, \dots, R_n); \end{array} \right\}; \quad \left. \begin{array}{l} R_1 = \varphi_1(q_1, q_2, \dots, q_n); \\ R_2 = \varphi_2(q_1, q_2, \dots, q_n); \\ \dots \\ R_n = \varphi_n(q_1, q_2, \dots, q_n). \end{array} \right\} \quad (16.3.1)$$

where n – is the number of degrees of freedom of the FE.

For physical models based on the classic linear theory of elasticity relations (16.3.1) can be much simplified and can be written

$$\{q\} = [A]\{R\}, \quad \{R\} = [K]\{q\}, \quad (16.3.2)$$

where $\{q\} = \{q_1, q_2, \dots, q_n\}$ – is a vector of nodal displacements within given FE; $\{R\} = \{R_1, R_2, \dots, R_n\}$ – is a vector of nodal reactions within one given FE; $[A]$ – is a suppleness square matrix; $[K]$ – is a stiffness square matrix.

Using 1-st equation in (16.3.2) we shall have FEM in so called "force method" performance, but using 2-nd equation in (16.3.2) we shall have FEM in "displacement method" performance. As practice show the "force method" for FEM realization is much less suitable for realization and algorithmization than "displacement method". That's why FEM in "displacement method" performance has always been using up till now.

The general steps for FEM in "displacement method" performance are presented below.

For the 1-st step it's necessary to develop the stiffness matrix $[K]$ for given type of FE using mainly variational and other (if possibly) principles. Factors for $[K]$ depend on geometrical sizes and physical (stiffness) properties of given FE. If the stiffness matrix for given FE is created in FE local coordinate system it's necessary to develop additionally so called transform coordinate matrix $[T]$.

For the 2-nd step it's necessary to explode the construction into FEs getting information about position of nodes and FEs. Node and

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FE numbers have to be determined in global enumeration. Physical properties for every FE have to be defined.

For the 3-rd step it's necessary to compute stiffness matrix for every FE in global coordinate system, using expression

$$[K'] = [T]^T [K] [T], \quad (16.3.3)$$

where $[T]$ – is a transform coordinate matrix; $[K]$ – is a stiffness matrix written in local FE coordinate system; $[K']$ – is a stiffness matrix written in global coordinate system of the whole construction; top index T means transposition operation.

For the 4-nd step it's necessary to compose equilibrium equations for every node. After we have to express nodal reactions $\{R\}$ through nodal displacements $\{q\}$ applying 2-nd equation of (16.3.2) to FE stiffness matrices we've got in 3-rd step. Within this step we form so called *total stiffness matrix* for the whole construction and *total vector of nodal loads* (the right part of the system) which don't consider kinematic boundary conditions using FE stiffness matrices of the 3-rd step and node positions. This step is voluminous and has several ways of realization. One of the most simple and clear methods to get total stiffness matrix and vector of nodal loads is based on so called *index matrix* described in [2, 4, 5].

The 5-th step consists on creation of so called *cut total stiffness matrix* for the whole construction taking into account kinematic boundary conditions. Within this procedure we eliminate some rows and columns in total stiffness matrix we've got in 4-th step. Numbers of eliminated rows and columns fit for the numbers of the known nodal displacements in global enumeration. Elimination concerns the right part too. Only elastic displacements are taken into account.

For the 6-th step we solve the system of linear algebraic equations, we've got in 5-th step to get unknown displacements. Having known nodal displacements for every FE we can find the required function and other accompany functions within separate FE.

FEM can be applied for all physic models (rods, plates, shells, massive bodies) in structural mechanics. Within some simplifying model the "quality" of the finite elements can be different. Usually the more nodes FE has the more accurate results can be achieved. On the fig. 16.3.2 some kinds of FEs for different physical models are presented, where the right coordinate system is applied.

On the fig. 16.3.2 there are next FEs are presented:

- a) FE for direct rods under axial strain-stress state;
- b) FE for direct rod under all kinds of loads: axial, bending, torsion;
- c) triangle precised FE for the flat (2D) problem of the theory of elasticity;
- d) quadrangle FE for the flat (2D) problem of the theory of elasticity;
- e) tetrahedral FE for the three-dimensional (3D) problem of the theory of elasticity;
- f) brick FE for the three-dimensional (3D) problem of the theory of elasticity;
- g) quadrangle FE for rigid plates under orthogonal bending;
- h) triangle FE for rigid plates under orthogonal bending.

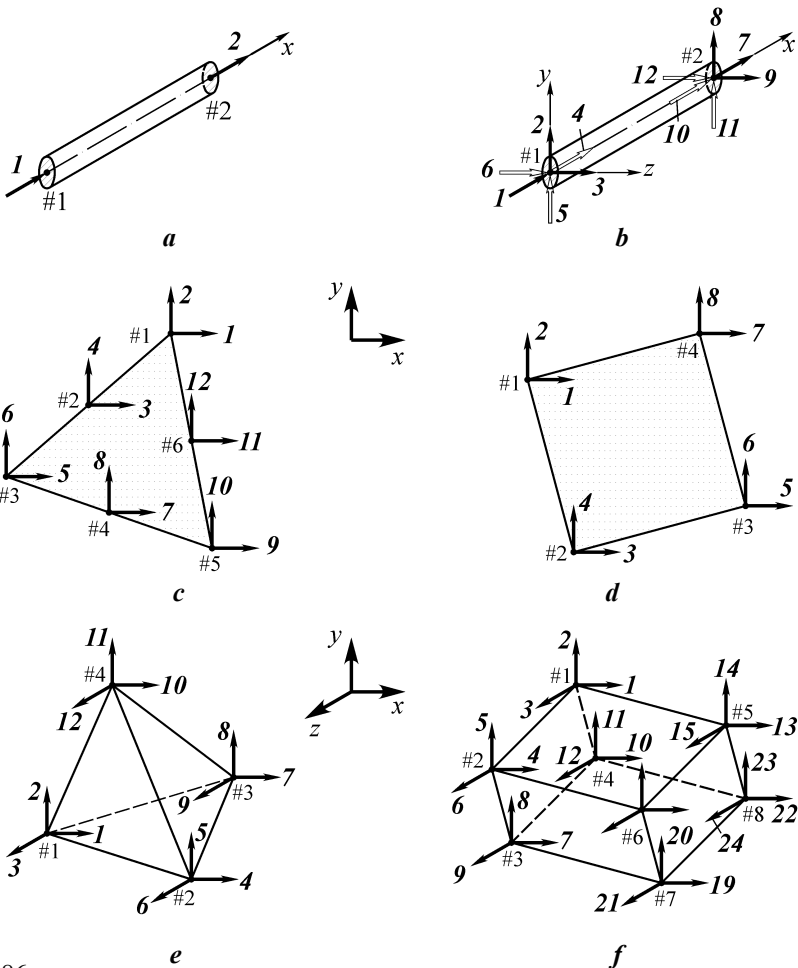
There are FEs for which its stiffness matrix $[K]$ is written in the local FE coordinate system (see fig. 16.3.2: **a, b, g, h**). But sometimes FE stiffness matrix $[K]$ can be written directly in global coordinate system (these FEs are shown on fig. 16.3.2: **c, d, e, f**). For the last case the next statement is true

$$[K'] = [K]. \quad (16.3.4)$$

FEM can be also used to solve physical and geometrical nonlinear problems using the linear FEM model as a "superstructure". Using FEM other problems of structural mechanics can be solves such as rigidity, stability, vibration, shock, plastic flow, etc. FEM can be used for mutual connected problems for example simultaneous action of temperature, external loads, magnetic field etc. for the body.

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FEM has more advantages than imperfections. This method allows to achieve fine results using its quite simple realization, it has quite simple uniform universal algorithms and don't use computer resources (memory, processor time execution) much in comparison with other numerical methods. But result got on base of FEM depends on mesh quality much. That's why good meshing is a guaranty of accurate results.



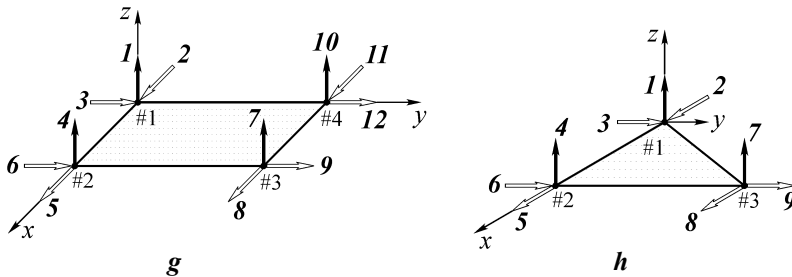


Fig. 16.3.2. Finite elements for different physical models:

→ – concentrated force; ⇨ – concentrated moment

For now FEM is standard in structural engineering calculations. FEM is realized in such well-known engineering simulation software as ANSYS, Abaqus, Nastran, NX, etc.

TASKS

Below there are tasks and its solutions are presented. These practical tasks envelop all important engineering tasks within this course. These tasks are simple for initial acquaintance with structural mechanics and the theory of elasticity. There are 11 tasks are reviewed. Task#1 and task#2 are dedicated for the theory of elasticity; next six tasks (tasks #3 ÷ #8) are dedicated for checkout strength calculations of rods and rod systems; tasks #9, #10 are dedicated for calculations of stability of rods and plates; in those tasks (#9, #10) there are engineering methodologies are presented to find geometrical parameters (area, inertia moment, plate thickness) from conditions of strength and stability. Task#11 is dedicated to basis of numeral methods used in structural mechanics and the theory of elasticity; in that task (#11) the simplest finite difference method (FDM) applying for beam is reviewed.

Here below the particular topic list of tasks is presented:

- 1) **Task #1** is dedicated to solution of the contrary task of the theory of elasticity using known displacements (lecture #3);
- 2) **Task #2** is dedicated to exploring of strain-stress state in point (lecture #4);
- 3) **Task #3** is dedicated to strength calculation of beams under bending using the start parameters method (lecture #7);

TASKS

4) **Task #4** is dedicated to calculation of beams with many spans under bending for strength using the method of five moments (lecture #8);

5) **Task #5** is dedicated to strength calculation of flat frames with rigid immovable nodes using the method of angular displacements (lecture #9);

6) **Task #6** is dedicated to calculation of beams on elastic foundation under bending for strength (lecture #10);

7) **Task #7** is dedicated to strength calculation of grillages under bending using the Navier method (lecture #11);

8) **Task #8** is dedicated to calculation of beams under bending with axial forces for strength (lecture #12);

9) **Task #9** is dedicated to calculation of beams for stability considering the deviation on Hook's law, when we define cross section sizes using strength and stability conditions (lecture #13);

10) **Task #10** is dedicated to calculation of plates for strength and stability considering the deviation on Hook's law, when we define plate size using strength and stability conditions (lecture #14, #15);

11) **Task #11** is dedicated to calculation of beam on elastic foundation under bending (the same beam as in task #6) but using the numeric method called finite difference method (FDM) and to comparison analytic and numeral results.

Solving these tasks 15–17 sings in float point numbers were kept, but here only significant 3÷4 numbers are presented. For convenience initial schemes and some plots are presented in different scales. Total number of hours for these tasks is estimated about 32 hours.

Task #1 (4–6 hours). Solution of the contrary problem of the theory of elasticity. Solve the contrary task of the theory of elasticity for known displacements for given body presented on the fig. 17.1. Not to take into account temperature.

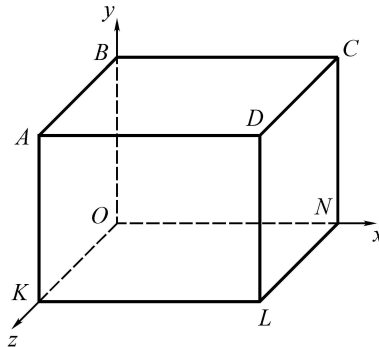


Fig. 17.1. Given elastic body

Given: $ON = 0,5 \text{ m}$; $OB = 0,05 \text{ m}$; $OK = 0,005 \text{ m}$; material is steel 3; displacements are set by laws:

$$\left. \begin{aligned} u &= (7x + 8y + 9z) \cdot 10^{-4}, \text{ m}; \\ v &= (10x^2 + 11y^2 + 12z^2) \cdot 10^{-4}, \text{ m}; \\ w &= (13x^3 + 14y^3 + 15z^3) \cdot 10^{-3}, \text{ m}. \end{aligned} \right\} \quad (17.1)$$

Find:

- 1) the laws of strains, stresses, external surface and volume loads;
- 2) checkout the compatibility conditions for stresses and strains;
- 3) principle vector and moment for surface forces for plane $ABCD$;

Task #1

4) principle vector and moment for volume loads for the whole body;

5) show graphs of external surface forces acting on plane $ABCD$;

6) show graphs of stresses σ_x, τ_{xy} and displacements u, v, w in line parallel to axis x coming through the mass centre of the body;

Solution

1) Using Cauchy relations strains can be written as

$$\left. \begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = 700 \cdot 10^{-6}; \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 2 \cdot 10^{-3} x + 800 \cdot 10^{-6} \\ \varepsilon_y &= \frac{\partial v}{\partial y} = 2,2 \cdot 10^{-3} y; \\ \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 2,4 \cdot 10^{-3} z + 42 \cdot 10^{-3} y^2 \\ \varepsilon_z &= \frac{\partial w}{\partial z} = 45 \cdot 10^{-3} z^2; \\ \gamma_{zx} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 39 \cdot 10^{-3} x^2 + 900 \cdot 10^{-6} \end{aligned} \right\} \quad (17.2)$$

Using Hook's law we can find stresses for known strains. Not to consider temperature means, that $T = 0$. Expressions for stresses can be written as

$$\left. \begin{aligned}
 \sigma_x &= 2G \left(\varepsilon_x + \frac{\mu\theta}{1-2\mu} - \frac{1+\mu}{1-2\mu} \alpha T \right) = \\
 &= 253,846 \cdot 10^6 y + 5,192 \cdot 10^9 z^2 + 188,462 \cdot 10^6, \text{ Pa}; \\
 \sigma_y &= 2G \left(\varepsilon_y + \frac{\mu\theta}{1-2\mu} - \frac{1+\mu}{1-2\mu} \alpha T \right) = \\
 &= 592,308 \cdot 10^6 y + 5,192 \cdot 10^9 z^2 + 80,769 \cdot 10^6, \text{ Pa}; \\
 \sigma_z &= 2G \left(\varepsilon_z + \frac{\mu\theta}{1-2\mu} - \frac{1+\mu}{1-2\mu} \alpha T \right) = \\
 &= 253,846 \cdot 10^6 y + 12,115 \cdot 10^9 z^2 + 80,769 \cdot 10^6, \text{ Pa}; \\
 \tau_{xy} &= G\gamma_{xy} = 153,846 \cdot 10^6 x + 61,538 \cdot 10^6, \text{ Pa}; \\
 \tau_{yz} &= G\gamma_{yz} = 184,615 \cdot 10^6 z + 3,231 \cdot 10^9 y^2, \text{ Pa}; \\
 \tau_{zx} &= G\gamma_{zx} = 3 \cdot 10^9 x^2 + 69,231 \cdot 10^6, \text{ Pa},
 \end{aligned} \right\} \quad (17.3)$$

where $\theta = \varepsilon_x + \varepsilon_y + \varepsilon_z$.

Using equilibrium equations for elementary parallelepiped and known expressions for stresses (17.3) we can define volume external forces. Expressions for volume external forces can be written as

$$\left. \begin{aligned}
 X &= - \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) = 0, \text{ N/m}^3; \\
 Y &= - \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) = -930,769 \cdot 10^6, \text{ N/m}^3; \\
 Z &= - \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} \right) = \\
 &= -6 \cdot 10^9 x - 24,231 \cdot 10^9 z - 6,462 \cdot 10^9 y, \text{ N/m}^3.
 \end{aligned} \right\} \quad (17.4)$$

Task #1

Using equilibrium equations for elementary tetrahedron and known expressions for stresses (17.3) we can define surface external forces. Expressions for surface external forces can be written as

$$\left. \begin{aligned}
 p_{vx} &= \sigma_x l + \tau_{yx} m + \tau_{zx} n = (253,846 \cdot 10^6 y + \\
 &+ 5,192 \cdot 10^9 z^2 + 188,462 \cdot 10^6) l + \\
 &+ (153,846 \cdot 10^6 x + 61,538 \cdot 10^6) m + \\
 &+ (3 \cdot 10^9 x^2 + 69,231 \cdot 10^6) n, \text{ N/m}^2; \\
 p_{vy} &= \tau_{xy} l + \sigma_y m + \tau_{zy} n = \\
 &= (153,846 \cdot 10^6 x + 61,538 \cdot 10^6) l + \\
 &+ (592,308 \cdot 10^6 y + 5,192 \cdot 10^9 z^2 + 80,769 \cdot 10^6) m + \\
 &+ (184,615 \cdot 10^6 z + 3,231 \cdot 10^9 y^2) n, \text{ N/m}^2; \\
 p_{vz} &= \tau_{xz} l + \tau_{yz} m + \sigma_z n = (3 \cdot 10^9 x^2 + 69,231 \cdot 10^6) l + \\
 &+ (184,615 \cdot 10^6 z + 3,231 \cdot 10^9 y^2) m + \\
 &+ (253,846 \cdot 10^6 y + 12,115 \cdot 10^9 z^2 + 80,769 \cdot 10^6) n, \text{ N/m}^2.
 \end{aligned} \right\} \quad (17.5)$$

2) Despite it is option, but we shall check compatibility conditions for strains and stresses to be sure that our expressions for strains and stresses are correct and also to have assurance that expressions connected with strains and stresses are correct.

First we shall check compatibility conditions for strains using Cauchy compatibility conditions. Checkouts for strain compatibility conditions are presented below

$$\left. \begin{aligned}
 \frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} &= \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \Rightarrow 0 = 0; \\
 \frac{\partial}{\partial x} \left(\frac{\partial \gamma_{xy}}{\partial z} - \frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} \right) &= 2 \frac{\partial^2 \varepsilon_x}{\partial y \partial z} \Rightarrow 0 = 0; \\
 \frac{\partial^2 \varepsilon_y}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial y^2} &= \frac{\partial^2 \gamma_{yz}}{\partial y \partial z} \Rightarrow 0 = 0; \\
 \frac{\partial}{\partial y} \left(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} \right) &= 2 \frac{\partial^2 \varepsilon_y}{\partial z \partial x} \Rightarrow 0 = 0; \\
 \frac{\partial^2 \varepsilon_z}{\partial x^2} + \frac{\partial^2 \varepsilon_x}{\partial z^2} &= \frac{\partial^2 \gamma_{zx}}{\partial z \partial x} \Rightarrow 0 = 0; \\
 \frac{\partial}{\partial z} \left(\frac{\partial \gamma_{zx}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yz}}{\partial x} \right) &= 2 \frac{\partial^2 \varepsilon_z}{\partial x \partial y} \Rightarrow 0 = 0.
 \end{aligned} \right\} \quad (17.6)$$

As we asked not to consider temperature it's enough to use Beltrami-Mitchell compatibility conditions for stresses. Checkouts for stresses compatibility conditions after Beltrami-Mitchell equations are presented below

Task #1

$$\left. \begin{aligned}
 &\nabla^2 \sigma_x + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial x^2} = -2 \frac{\partial X}{\partial x} - \frac{\mu}{1-\mu} \times \\
 &\times \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} \right) \Rightarrow 1,038 \cdot 10^{10} = 1,038 \cdot 10^{10}; \\
 &\nabla^2 \sigma_y + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial y^2} = -2 \frac{\partial Y}{\partial y} - \frac{\mu}{1-\mu} \times \\
 &\times \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} \right) \Rightarrow 1,038 \cdot 10^{10} = 1,038 \cdot 10^{10}; \\
 &\nabla^2 \sigma_z + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial z^2} = -2 \frac{\partial Z}{\partial z} - \frac{\mu}{1-\mu} \times \\
 &\times \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} \right) \Rightarrow 5,885 \cdot 10^{10} = 5,885 \cdot 10^{10}; \\
 &\nabla^2 \tau_{xy} + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial x \partial y} = - \left(\frac{\partial X}{\partial y} + \frac{\partial Y}{\partial x} \right) \Rightarrow 0 = 0; \\
 &\nabla^2 \tau_{yz} + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial y \partial z} = - \left(\frac{\partial Y}{\partial z} + \frac{\partial Z}{\partial y} \right) \Rightarrow 6,462 \cdot 10^9 = 6,462 \cdot 10^9; \\
 &\nabla^2 \tau_{zx} + \frac{1}{1+\mu} \frac{\partial^2 \Theta}{\partial z \partial x} = - \left(\frac{\partial Z}{\partial x} + \frac{\partial X}{\partial z} \right) \Rightarrow 6,0 \cdot 10^9 = 6,0 \cdot 10^9,
 \end{aligned} \right\} (17.7)$$

where $\Theta = \sigma_x + \sigma_y + \sigma_z$.

As we can see from (17.6), (17.7) compatibility conditions for strains and stresses are provided, that's why we have assurance we operate with correct expressions for strains and stresses.

3) To find out principle vector and moment for surface forces for plane $ABCD$ it is necessary to write down the laws of external surface forces for this plane, considering that equation for this plane is

$$y = 0,05. \quad (17.8)$$

Also directions cosines for the plane $ABCD$ are next

$$l_{ABCD} = 0; \quad m_{ABCD} = 1; \quad n_{ABCD} = 0 \quad (17.9)$$

Taking into account (17.8) and (17.9) expressions for external surface forces are

$$\left. \begin{aligned} p_{vx} &= 1,538 \cdot 10^8 x + 6,154 \cdot 10^7, \text{ N/m}^2; \\ p_{vy} &= 5,192 \cdot 10^9 z^2 + 1,104 \cdot 10^8, \text{ N/m}^2; \\ p_{vz} &= 1,846 \cdot 10^8 z + 8,077 \cdot 10^6, \text{ N/m}^2. \end{aligned} \right\} \quad (17.10)$$

Components of principle vector P_F^{ABCD} and the principle moment M_F^{ABCD} of external surface forces for plane $ABCD$ into coordinate axes can be found using next expressions

$$\left. \begin{aligned} P_{Fx}^{ABCD} &= \int_F p_{vx} dF = \iint_{D_{xz}} p_{vx} dx dz = \int_0^{0,5} dx \int_0^{0,005} p_{vx} dz = \\ &= \int_0^{0,5} dx \int_0^{0,005} (1,538 \cdot 10^8 x + 6,154 \cdot 10^7) dz = 2,5 \cdot 10^5 \text{ N}; \\ P_{Fy}^{ABCD} &= \int_F p_{vy} dF = \iint_{D_{xz}} p_{vy} dx dz = \int_0^{0,5} dx \int_0^{0,005} p_{vy} dz = \\ &= \int_0^{0,5} dx \int_0^{0,005} (5,192 \cdot 10^9 z^2 + 1,104 \cdot 10^8) dz = 2,761 \cdot 10^5 \text{ N}; \\ P_{Fz}^{ABCD} &= \int_F p_{vz} dF = \iint_{D_{xz}} p_{vz} dx dz = \int_0^{0,5} dx \int_0^{0,005} p_{vz} dz = \\ &= \int_0^{0,5} dx \int_0^{0,005} (1,846 \cdot 10^8 z + 8,077 \cdot 10^6) dz = 2,135 \cdot 10^4 \text{ N}. \end{aligned} \right\} \quad (17.11)$$

Task #1

$$\begin{aligned}
 M_{F_x}^{ABCD} &= \int_F (p_{vz}y - p_{vy}z) dF = \iint_{D_{xz}} p_{vz} dx dz = \int_0^{0,5} dx \int_0^{0,005} (p_{vz}y - p_{vy}z) dz = \\
 &= \int_0^{0,5} dx \int_0^{0,005} ((1,846 \cdot 10^8 z + 8,077 \cdot 10^6) \cdot 0,05 - (5,192 \cdot 10^9 z^2 + 1,104 \cdot 10^8) z) dz = \\
 &= 3,770 \cdot 10^2 \text{ N} \cdot \text{m}; \\
 M_{F_y}^{ABCD} &= \int_F (p_{vx}z - p_{vz}x) dF = \iint_{D_{xz}} p_{vz} dx dz = \int_0^{0,5} dx \int_0^{0,005} (p_{vx}z - p_{vz}x) dz = \\
 &= \int_0^{0,5} dx \int_0^{0,005} ((1,538 \cdot 10^8 x + 6,154 \cdot 10^7) z - (1,846 \cdot 10^8 z + 8,077 \cdot 10^6) x) dz = \\
 &= -4,712 \cdot 10^3 \text{ N} \cdot \text{m}; \\
 M_{F_z}^{ABCD} &= \int_F (p_{vy}x - p_{vx}y) dF = \iint_{D_{xz}} p_{vz} dx dz = \int_0^{0,5} dx \int_0^{0,005} (p_{vy}x - p_{vx}y) dz = \\
 &= \int_0^{0,5} dx \int_0^{0,005} ((5,192 \cdot 10^9 z^2 + 1,104 \cdot 10^8) x - (1,538 \cdot 10^8 x + 6,154 \cdot 10^7) \cdot 0,05) dz = \\
 &= 5,652 \cdot 10^4 \text{ N} \cdot \text{m}.
 \end{aligned}
 \tag{17.12}$$

Values of principle vector and principle moment of external surface forces acting for plane $ABCD$ can be found using expressions

$$\left. \begin{aligned}
 P_F^{ABCD} &= \sqrt{(P_{F_x}^{ABCD})^2 + (P_{F_y}^{ABCD})^2 + (P_{F_z}^{ABCD})^2} = \\
 &= \sqrt{(2,5 \cdot 10^5)^2 + (2,761 \cdot 10^5)^2 + (2,135 \cdot 10^4)^2} = 3,731 \cdot 10^5 \text{ N}; \\
 M_F^{ABCD} &= \sqrt{(M_{F_x}^{ABCD})^2 + (M_{F_y}^{ABCD})^2 + (M_{F_z}^{ABCD})^2} = \\
 &= \sqrt{(3,770 \cdot 10^2)^2 + (-4,712 \cdot 10^3)^2 + (5,652 \cdot 10^4)^2} = \\
 &= 5,671 \cdot 10^4 \text{ N} \cdot \text{m}.
 \end{aligned} \right\} (17.13)$$

4) Components of principle vector P_V and principle moment M_V of external volume forces for the whole body into coordinate axes can be found using next expressions

$$\left. \begin{aligned}
 P_{V_x} &= \int_V X dV = \iiint_{D_{xyz}} X dx dy dz = \int_0^{0,5} dx \int_0^{0,05} dy \int_0^{0,005} 0 dz = 0 \text{ N}; \\
 P_{V_y} &= \int_V Y dV = \iiint_{D_{xyz}} Y dx dy dz = \int_0^{0,5} dx \int_0^{0,05} dy \int_0^{0,005} -930,769 \cdot 10^6 dz = \\
 &= -1,163 \cdot 10^5 \text{ N}; \\
 P_{V_z} &= \int_V Z dV = \iiint_{D_{xyz}} Z dx dy dz = \\
 &= \int_0^{0,5} dx \int_0^{0,05} dy \int_0^{0,005} (-6 \cdot 10^9 x - 24,231 \cdot 10^9 z - 6,462 \cdot 10^9 y) dz = \\
 &= -2,153 \cdot 10^5 \text{ N}.
 \end{aligned} \right\} (17.14)$$

Task #1

$$\begin{aligned}
 M_{V_x} &= \int_V (Zy - Yz) dV = \iiint_{D_{xyz}} (Zy - Yz) dx dy dz = \\
 &= \int_0^{0,5} dx \int_0^{0,05} dy \int_0^{0,005} [(-6 \cdot 10^9 x - 24,231 \cdot 10^9 z - 6,462 \cdot 10^9 y) y - \\
 &\quad - (-930,769 \cdot 10^6) z] dz = -5,259 \cdot 10^3 \text{ N} \cdot \text{m}; \\
 M_{V_y} &= \int_V (Xz - Zx) dV = \iiint_{D_{xyz}} (Xz - Zx) dx dy dz = \\
 &= \int_0^{0,5} dx \int_0^{0,05} dy \int_0^{0,005} [0 \cdot z - (-6 \cdot 10^9 x - 24,231 \cdot 10^9 z - 6,462 \cdot 10^9 y) x] dz = \\
 &= 6,944 \cdot 10^4 \text{ N} \cdot \text{m}; \\
 M_{V_z} &= \int_V (Yx - Xy) dV = \iiint_{D_{xyz}} (Yx - Xy) dx dy dz = \\
 &= \int_0^{0,5} dx \int_0^{0,05} dy \int_0^{0,005} (-930,769 \cdot 10^6 x - 0 \cdot y) dz = -2,909 \cdot 10^4 \text{ N} \cdot \text{m}.
 \end{aligned}
 \tag{17.15}$$

Values of principle vector and principle moment of external volume forces acting within the whole body can be found using expressions

$$\begin{aligned}
 P_V &= \sqrt{(P_{V_x})^2 + (P_{V_y})^2 + (P_{V_z})^2} = \\
 &= \sqrt{0^2 + (-1,163 \cdot 10^5)^2 + (-2,153 \cdot 10^5)^2} = 2,447 \cdot 10^5 \text{ N}; \\
 M_V &= \sqrt{(M_{V_x})^2 + (M_{V_y})^2 + (M_{V_z})^2} = \\
 &= \sqrt{(-5,259 \cdot 10^3)^2 + (6,944 \cdot 10^4)^2 + (-2,909 \cdot 10^4)^2} = 7,547 \cdot 10^4 \text{ N} \cdot \text{m}.
 \end{aligned}
 \tag{17.16}$$

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5) Having laws of external surface loadings for plane $ABCD$ (17.10) we can show plots of it. These plots are presented on the fig.17.2.

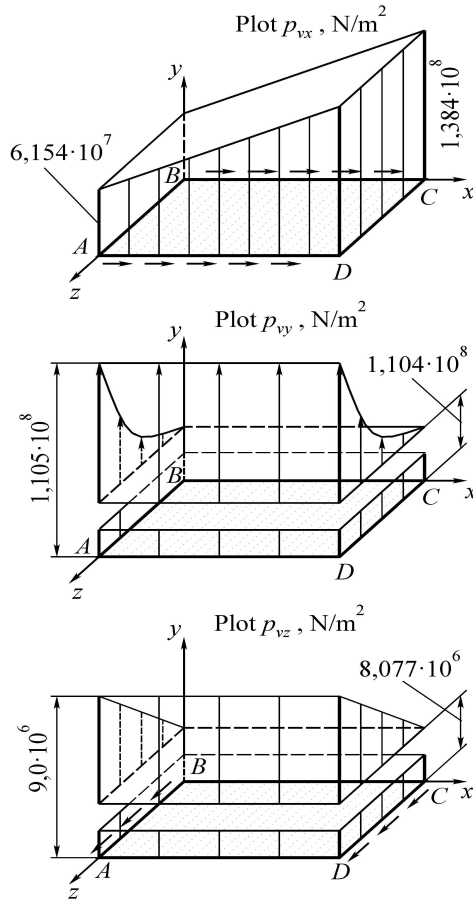


Fig. 17.2. Plots of external surface loadings acting for plane $ABCD$

6) Having laws of σ_x , τ_{xy} (17.3) and displacements u , v , w (17.1) we can show plots of it. To show plots of stresses σ_x , τ_{xy} , and displacements u , v , w it's necessary to compose the laws of these

200

Task #1

values for line which come through the mass centre of the body and parallel to axis x . Mass centre of the body have coordinates

$$x_M = 0,25 \text{ m}; \quad y_M = 0,025 \text{ m}; \quad z_M = 0,0025 \text{ m}. \quad (17.17)$$

Using (17.17) the laws of $\sigma_x, \tau_{xy}, u, v, w$ can be written

$$\left. \begin{aligned} \sigma_x &= 1,948 \cdot 10^8 \text{ Pa}; \\ \tau_{xy} &= 1,538 \cdot 10^8 x + 6,154 \cdot 10^7 \text{ Pa}; \\ u &= 7 \cdot 10^{-4} x + 2,225 \cdot 10^{-5} \text{ m}; \\ v &= 1 \cdot 10^{-3} x^2 + 6,95 \cdot 10^{-7} \text{ m}; \\ w &= 0,013 x^3 + 2,19 \cdot 10^{-7} \text{ m}. \end{aligned} \right\} \quad (17.18)$$

The plots of $\sigma_x, \tau_{xy}, u, v, w$ are presented on the fig.17.3.

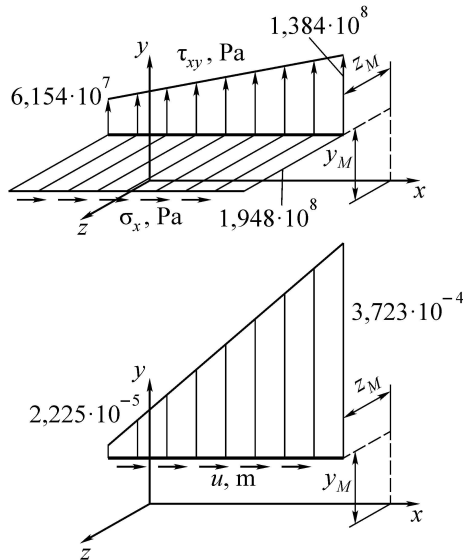


Fig. 17.3 a. Plots of σ_x, τ_{xy} and u
(the line parallel to axis x is presented within the body)

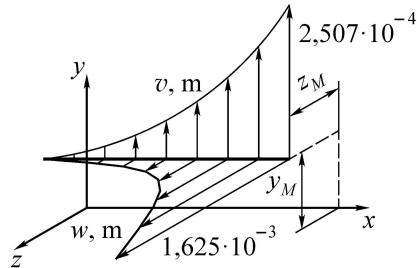


Fig. 17.3 b. Plots of σ_x , τ_{xy} , u , v , w
(the line parallel to axis x is presented within the body)

Task #2 (4 hours). Exploration of the strain-stress state in point. Explore the strain-stress state in the point M situated in the mass centre for given body presented on the fig. 17.1, using results of the task #1. Not to take into account temperature.

Given: material is steel 3; results of previous task #1.

Find:

- 1) Find principle stresses and its directions, principle strains, the maximum shear stresses and strains;
- 2) estimate the strength in the given point M , using 3-rd and 4-th strength theories;
- 3) find full stresses and its direction, full normal stresses and full shear stresses in the point M for plane which normal vector is

$\nu \left(\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}} \right)$; show graphically full normal stresses;

- 4) find full displacement of the point M , and show it graphically.

Solution

- 1) Having laws of stresses from previous task #1, we can find values of stresses for the point M laying in the mass centre, substituting

Task #2

coordinates of this point $M(0,25; 0,025; 0,0025)$. The values of stresses in point M are presented below in tensor view

$$\left. \begin{aligned} \sigma_x &= 1,948 \cdot 10^8 \text{ Pa}; & \sigma_y &= 9,561 \cdot 10^7 \text{ Pa}; \\ \sigma_z &= 8,719 \cdot 10^7 \text{ Pa}; & \tau_{xy} &= 1,0 \cdot 10^8 \text{ Pa}; \\ \tau_{yz} &= 2,481 \cdot 10^6 \text{ Pa}; & \tau_{zx} &= 2,567 \cdot 10^8 \text{ Pa}. \end{aligned} \right\} \quad (18.1)$$

Invariants can be calculated using expressions

$$\left. \begin{aligned} I_1(T_\sigma) &= \sigma_x + \sigma_y + \sigma_z = \\ &= 1,948 \cdot 10^8 + 9,561 \cdot 10^7 + 8,719 \cdot 10^7 = 3,776 \cdot 10^8 \text{ Pa}; \\ I_2(T_\sigma) &= \sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 = \\ &= -3,196 \cdot 10^{16} \text{ Pa}^2; \\ I_3(T_\sigma) &= \sigma_x \sigma_y \sigma_z + 2\tau_{xy} \tau_{yz} \tau_{zx} - \sigma_x \tau_{yz}^2 - \sigma_y \tau_{zx}^2 - \sigma_z \tau_{xy}^2 = \\ &= -5,423 \cdot 10^{24} \text{ Pa}^3. \end{aligned} \right\} \quad (18.2)$$

The equation for definition of principal normal stresses for point M can be written

$$\begin{aligned} \sigma_v^3 - I_1 \sigma_v^2 + I_2 \sigma_v - I_3 &= 0 \Rightarrow \\ \Rightarrow \sigma_v^3 - 3,776 \cdot 10^8 \sigma_v^2 - 3,196 \cdot 10^{16} \sigma_v + 5,423 \cdot 10^{24} &= 0. \end{aligned} \quad (18.3)$$

Solving the equation (18.3) we have three real roots, which are principle stresses. These principle stresses are

$$\begin{aligned} \sigma_1 &= 4,229 \cdot 10^8 \text{ Pa}; \\ \sigma_2 &= 9,285 \cdot 10^7 \text{ Pa}; & \sigma_3 &= -1,381 \cdot 10^8 \text{ Pa}. \end{aligned} \quad (18.4)$$

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To check principle stresses defined by (18.4) it's necessary to find invariants expressed through these stresses

$$\left. \begin{aligned} I_1(T_\sigma) &= \sigma_1 + \sigma_2 + \sigma_3 = \\ &= 4,229 \cdot 10^8 + 9,285 \cdot 10^7 + (-1,381 \cdot 10^8) = 3,776 \cdot 10^8 \text{ Pa}; \\ I_2(T_\sigma) &= \sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1 = 4,229 \cdot 10^8 \cdot 9,285 \cdot 10^7 + \\ &+ 9,285 \cdot 10^7 \cdot (-1,381 \cdot 10^8) + (-1,381 \cdot 10^8) \cdot 4,229 \cdot 10^8 = \\ &= -3,196 \cdot 10^{16} \text{ Pa}; \\ I_3(T_\sigma) &= \sigma_1 \sigma_2 \sigma_3 = 4,229 \cdot 10^8 \cdot 9,285 \cdot 10^7 \cdot (-1,381 \cdot 10^8) = \\ &= -5,423 \cdot 10^{24} \text{ Pa}. \end{aligned} \right\} (18.5)$$

As we see invariants defined by (18.2) and (18.5) are the same. It means that we operate with correct values of principle stresses for point M .

To find directions of principle planes we shall use next system

$$\left. \begin{aligned} l_i &= \pm \frac{1}{\sqrt{1 + a_i^2 + b_i^2}}, \\ m_i &= \pm \frac{a_i}{\sqrt{1 + a_i^2 + b_i^2}}, \quad n_i = \pm \frac{b_i}{\sqrt{1 + a_i^2 + b_i^2}}. \end{aligned} \right\} (18.6)$$

$$\left. \begin{aligned} \tau_{yx} a_i + \tau_{zx} b_i &= \sigma_i - \sigma_x; \\ (\sigma_y - \sigma_i) a_i + \tau_{zy} b_i &= -\tau_{xy}, \end{aligned} \right\} (18.7)$$

where i – is the number of principle stresses.

Substituting into (18.7) instead σ_i sequentially $\sigma_1, \sigma_2, \sigma_3$, we shall have three systems for definition of factors $a_1, b_1, a_2, b_2, a_3, b_3$.

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After substituting these factors into (18.6) we can find direction cosines for normal vectors of principle planes. For example the systems for definition of factors $a_1, b_1, a_2, b_2, a_3, b_3$ can be written

$$\left. \begin{aligned} 1,0 \cdot 10^8 a_1 + 2,567 \cdot 10^8 b_1 &= 2,281 \cdot 10^8; \\ -3,273 \cdot 10^8 a_1 + 2,481 \cdot 10^6 b_1 &= -1,0 \cdot 10^8. \end{aligned} \right\} \quad (18.8)$$

$$\left. \begin{aligned} 1,0 \cdot 10^8 a_2 + 2,567 \cdot 10^8 b_2 &= -1,020 \cdot 10^8; \\ 2,758 \cdot 10^6 a_2 + 2,481 \cdot 10^6 b_2 &= -1,0 \cdot 10^8. \end{aligned} \right\} \quad (18.9)$$

$$\left. \begin{aligned} 1,0 \cdot 10^8 a_3 + 2,567 \cdot 10^8 b_3 &= -3,330 \cdot 10^8; \\ 2,337 \cdot 10^8 a_3 + 2,481 \cdot 10^6 b_3 &= -1,0 \cdot 10^8. \end{aligned} \right\} \quad (18.10)$$

Solving (18.8), (18.9), (18.10) we have next:

$$\left. \begin{aligned} a_1 &= 0,311; & b_1 &= 0,767; \\ a_2 &= -55,271; & b_2 &= 21,132; \\ a_3 &= -0,416; & b_3 &= -1,135. \end{aligned} \right\} \quad (18.11)$$

Substituting known factors (18.11) into (18.6) we can get direction cosines of principle planes. These direction cosines are (only top signs are taken in (18.6))

$$\left. \begin{aligned} l_1 &= 0,770; & m_1 &= 0,240; & n_1 &= 0,591; \\ l_2 &= 0,017; & m_2 &= -0,934; & n_2 &= 0,357; \\ l_3 &= 0,637; & m_3 &= -0,265; & n_3 &= -0,723. \end{aligned} \right\} \quad (18.12)$$

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Direction cosines have to be checked for conditions of mutual orthogonality. Checks of direction cosines defined by (18.12) are presented below

$$\left. \begin{aligned} l_1^2 + l_2^2 + l_3^2 &= 0,770^2 + 0,017^2 + 0,637^2 \approx 1,0; \\ m_1^2 + m_2^2 + m_3^2 &= 0,240^2 + (-0,934)^2 + (-0,265)^2 \approx 1,0; \\ n_1^2 + n_2^2 + n_3^2 &= 0,591^2 + 0,357^2 + (-0,723)^2 \approx 1,0; \\ l_1^2 + m_1^2 + n_1^2 &= 0,770^2 + 0,240^2 + 0,591^2 \approx 1,0; \\ l_2^2 + m_2^2 + n_2^2 &= 0,017^2 + (-0,934)^2 + 0,357^2 \approx 1,0; \\ l_3^2 + m_3^2 + n_3^2 &= 0,637^2 + (-0,265)^2 + (-0,723)^2 \approx 1,0; \\ l_1 m_1 + l_2 m_2 + l_3 m_3 &= \\ &= 0,770 \cdot 0,240 + 0,017 \cdot (-0,934) + 0,637 \cdot (-0,265) \approx 0; \\ m_1 n_1 + m_2 n_2 + m_3 n_3 &= \\ &= 0,240 \cdot 0,591 + (-0,934) \cdot 0,357 + (-0,265) \cdot (-0,723) \approx 0; \\ n_1 l_1 + n_2 l_2 + n_3 l_3 &= \\ &= 0,591 \cdot 0,770 + 0,357 \cdot 0,017 + (-0,723) \cdot 0,637 \approx 0; \\ l_1 l_2 + m_1 m_2 + n_1 n_2 &= \\ &= 0,770 \cdot 0,017 + 0,240 \cdot (-0,934) + 0,591 \cdot 0,357 \approx 0; \\ l_2 l_3 + m_2 m_3 + n_2 n_3 &= \\ &= 0,017 \cdot 0,637 + (-0,934) \cdot (-0,265) + 0,357 \cdot (-0,723) \approx 0; \\ l_3 l_1 + m_3 m_1 + n_3 n_1 &= \\ &= 0,637 \cdot 0,770 + (-0,265) \cdot 0,240 + (-0,723) \cdot 0,591 \approx 0. \end{aligned} \right\} \quad (18.13)$$

As we may see from (18.13) normal vectors of principle planes are mutually orthogonal to each other. So we operate with correct values of directions cosines of normal vectors of principle planes.

Task #2

Definitions of principle strains, the maximum shear stresses and strains are presented below in the table 18.1.

Table 18.1. Definition of elements of strain-stress state for point *M*

#	Name of value calculating	Calculation formula	Numeral value
1	Principle strain ε_1	$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)]$	$2,182 \cdot 10^{-3}$
2	Principle strain ε_2	$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \mu(\sigma_3 + \sigma_1)]$	$3,707 \cdot 10^{-5}$
3	Principle strain ε_3	$\varepsilon_3 = \frac{1}{E}[\sigma_3 - \mu(\sigma_1 + \sigma_2)]$	$-1,464 \cdot 10^{-3}$
4	Maximum shear stresses τ_1 , Pa	$\tau_1 = \frac{\sigma_2 - \sigma_3}{2}$	$1,155 \cdot 10^8$
5	Maximum shear stresses τ_2 , Pa	$\tau_2 = \tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}$	$2,805 \cdot 10^8$
6	Maximum shear stresses τ_3 , Pa	$\tau_3 = \frac{\sigma_1 - \sigma_2}{2}$	$1,650 \cdot 10^8$
7	Maximum shear stresses γ_1 , rad	$\gamma_1 = \frac{\tau_1}{G} = \frac{(\sigma_2 - \sigma_3)}{2G} = \varepsilon_2 - \varepsilon_3$	$1,501 \cdot 10^{-3}$
8	Maximum shear stresses γ_2 , rad	$\gamma_2 = \gamma_{\max} = \frac{\tau_2}{G} = \frac{(\sigma_1 - \sigma_3)}{2G} = \varepsilon_1 - \varepsilon_3$	$3,647 \cdot 10^{-3}$
9	Maximum shear stresses γ_3 , rad	$\gamma_3 = \frac{\tau_3}{G} = \frac{(\sigma_1 - \sigma_2)}{2G} = \varepsilon_1 - \varepsilon_2$	$2,145 \cdot 10^{-3}$

2) Equivalent stresses for 3-rd and 4-th strength theories can be found using next expressions

$$\sigma_{eq}^{\text{III}} = \sigma_1 - \sigma_3. \quad (18.14)$$

$$\sigma_{eq}^{IV} = \frac{1}{\sqrt{2}} \times \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}. \quad (18.15)$$

Substituting numeral values into (18.14) and (18.15) we can write down numeral values for equivalent stresses

$$\sigma_{eq}^{III} = 5,610 \cdot 10^8 \text{ Pa}; \quad \sigma_{eq}^{IV} = 4,884 \cdot 10^8 \text{ Pa}. \quad (18.16)$$

The condition of providing of strength can be written as

$$\sigma_{eq} \leq [\sigma], \quad (18.17)$$

where $[\sigma] = \frac{\sigma_s}{n_s} = \frac{235 \cdot 10^6}{1,5} = 157 \text{ MPa}$ – are allowable stresses;

$n_s = 1,5$ – is the accepted safety factor; $\sigma_s = 235 \text{ MPa}$ – is the yield limit.

We have that the strength condition (18.17) is failed, because $\{\sigma_{eq}^{III} = 561,0 \text{ MPa}\} > \{[\sigma] = 157 \text{ MPa}\}$

and $\{\sigma_{eq}^{IV} = 488,4 \text{ MPa}\} > \{[\sigma] = 157 \text{ MPa}\}$, that's why the strength in point M is failed. To provide strength of the body in point M it's necessary to apply better materials and alloys for our body, for example bridge steels.

3) Having equilibrium equations of elementary tetrahedron we can find projections of full stresses into coordinate axes. Substituting coordinates of point $M(0,25; 0,025; 0,0025)$ and values of directions

cosines $\sqrt{\left(l = \frac{1}{\sqrt{3}}; \quad m = \frac{1}{\sqrt{3}}; \quad n = \frac{1}{\sqrt{3}}\right)}$ into equilibrium equations

of elementary tetrahedron (17.5) we can write down projections of full stresses into coordinate axes

Task #2

$$p_{vx} = 3,184 \cdot 10^8 \text{ Pa};$$

$$p_{vy} = 1,144 \cdot 10^8 \text{ Pa}; \quad p_{vz} = 2,0 \cdot 10^8 \text{ Pa}. \quad (18.18)$$

Full stresses in point M for given plane can be found using expression

$$\left. \begin{aligned} p_v &= \sqrt{p_{vx}^2 + p_{vy}^2 + p_{vz}^2} = \\ &= \sqrt{(3,184 \cdot 10^8)^2 + (1,144 \cdot 10^8)^2 + (2,0 \cdot 10^8)^2} = \\ &= 3,931 \cdot 10^8 \text{ Pa} \end{aligned} \right\} \quad (18.19)$$

Direction cosines of full stresses can be found using expressions

$$\left. \begin{aligned} l_p &= \frac{p_{vx}}{p_v} = \frac{3,184 \cdot 10^8}{3,931 \cdot 10^8} = 0,810; \\ m_p &= \frac{p_{vy}}{p_v} = \frac{1,144 \cdot 10^8}{3,931 \cdot 10^8} = 0,291; \\ n_p &= \frac{p_{vz}}{p_v} = \frac{2,0 \cdot 10^8}{3,931 \cdot 10^8} = 0,509. \end{aligned} \right\} \quad (18.20)$$

Full normal stresses can be found using expression

$$\left. \begin{aligned} \sigma_v &= p_{vx}l + p_{vy}m + p_{vz}n = \\ &= (3,184 \cdot 10^8) \frac{1}{\sqrt{3}} + (1,144 \cdot 10^8) \frac{1}{\sqrt{3}} + \\ &+ (2,0 \cdot 10^8) \frac{1}{\sqrt{3}} = 3,654 \cdot 10^8 \text{ Pa}. \end{aligned} \right\} \quad (18.21)$$

Full shear stresses can be found using expressions

$$\begin{aligned}\tau_v &= \sqrt{p_v^2 - \sigma_v^2} = \\ &= \sqrt{(3,931 \cdot 10^8)^2 - (3,654 \cdot 10^8)^2} = 1,449 \cdot 10^8 \text{ Pa.} \quad (18.22)\end{aligned}$$

Having direction cosines of full stresses we may show it. Full stresses are shown on the fig. 18.1.

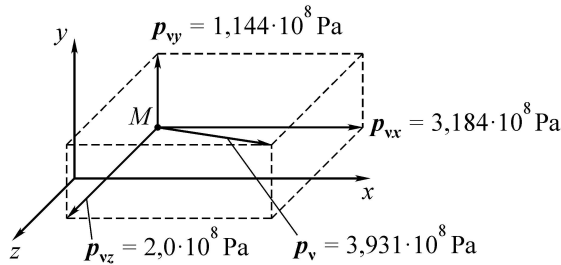


Fig. 18.1. Full stresses vector

4) Substituting coordinates of point $M(0,25; 0,025; 0,0025)$ into the laws of displacements (17.1) we shall have values projections of full displacement into coordinate axes. Displacements u, v, w in point M have next values

$$\begin{aligned}u &= 1,972 \cdot 10^{-4} \text{ m;} \\ v &= 6,319 \cdot 10^{-5} \text{ m;} \quad w = 2,033 \cdot 10^{-4} \text{ m.} \quad (18.23)\end{aligned}$$

Full displacement in point M can be found using next relation

$$\begin{aligned}\rho &= \sqrt{u^2 + v^2 + w^2} = \\ &= \sqrt{(1,972 \cdot 10^{-4})^2 + (6,319 \cdot 10^{-5})^2 + (2,033 \cdot 10^{-4})^2} = \\ &= 2,903 \cdot 10^{-4} \text{ m.}\end{aligned} \quad (18.24)$$

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Direction cosines of full stresses can be found using expressions

$$\left. \begin{aligned} l_p &= \frac{u}{\rho} = \frac{1,972 \cdot 10^{-4}}{2,903 \cdot 10^{-4}} = 0,680; \\ m_p &= \frac{v}{\rho} = \frac{6,319 \cdot 10^{-5}}{2,903 \cdot 10^{-4}} = 0,218; \\ n_p &= \frac{w}{\rho} = \frac{2,033 \cdot 10^{-4}}{2,903 \cdot 10^{-4}} = 0,701. \end{aligned} \right\} \quad (18.25)$$

Having direction cosines of full displacement we may show it. Full displacements are shown on the fig. 18.2.

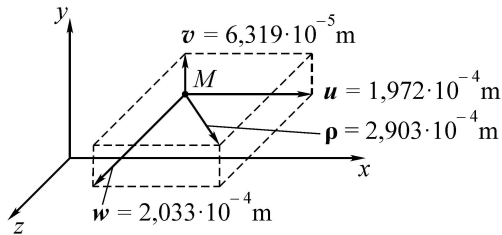


Fig. 18.2. Full displacement vector

Task #3 (4 hours). Calculation of beams under bending for strength using the start parameter method. Check out the strength of the beam in conditions of usual orthogonal bending shown on the fig. 19.1, applying the method of start parameters.

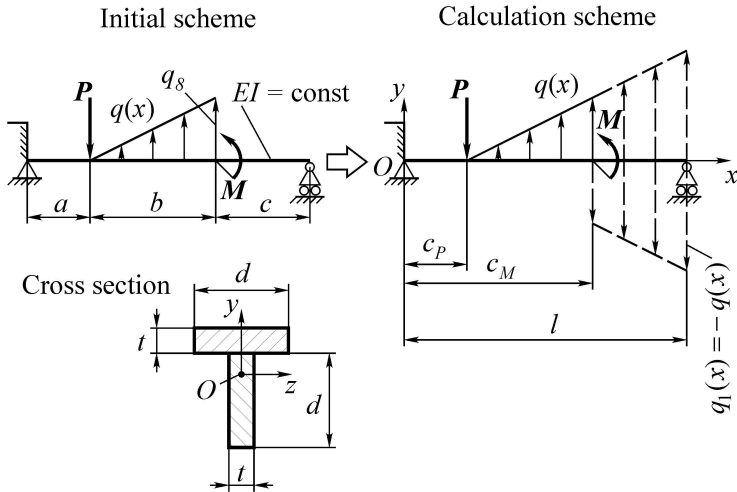


Fig. 19.1. The beam to be verified for strength

Given: $P = 2000 \text{ N}$; $M = 1000 \text{ N} \cdot \text{m}$; $a = 2 \text{ m}$; $b = 4 \text{ m}$; $c = 3 \text{ m}$; $t = 10 \text{ mm}$; $d = 100 \text{ mm}$; $q_8 = 500 \text{ N/m}$; material is steel 3.

Solution

To solve this task it's necessary to define the laws of distributions $q(x)$ and $q_1(x)$. Using the methodology of definition of the equation of the direct line using two points we can find the law of $q(x)$. The laws of $q(x)$ and $q_1(x)$ can be written

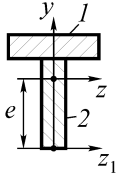
$$q(x) = \frac{q_8(x-2)}{3}, \quad q_1(x) = -q(x) = -\frac{q_8(x-2)}{3}. \quad (19.1)$$

Further it's necessary to compute geometrical properties of the cross section presented on the fig. 19.1. On the fig. 19.1 given axes are central principle, and point O is the mass centre of the cross section. As the given cross section is symmetric and we have to calculate geometrical properties only in plane yOx the computation of geometric properties can be simplified. In the table 19.1 the calculations of

Task #3

the geometric properties of the cross section are presented. Also parameters on the calculation scheme presented on the fig. 19.1 are $c_P = 2$ m, $c_M = 6$ m, $l = 9$ m. Properties of the steel 3 are: modulus of elasticity in tension $E = 2 \cdot 10^{11}$ Pa, yield limit $\sigma_s = 235$ MPa, the safety factor we accept $n_s = 1,5$.

Table 19.1. Computations of geometrical properties of the cross section

The figure for the cross section	No	Name	Sizes, mm	F_i, m^2	z_i, m	$F_i \cdot z_i, \text{m}^3$	$F_i z_i^2, \text{m}^4$	i_i, m^4
	1	Shelf	10×100	0,001	0,105	1,050E-04	1,103E-05	8,333E-09
	2	Wall	10×100	0,001	0,05	5,000E-05	2,500E-06	8,333E-07
Sums				$\Sigma_1 = 2,0\text{E-}3$		$\Sigma_2 = 1,550\text{E-}4$	$\Sigma_3 = 1,437\text{E-}5$	

Properties of the cross section	Calculative formula	Numeric results
The cross section area F, m^2	$F = \Sigma_1$	2,0E-3
Distance from comparison axis to neutral axis e, m	$e = \frac{\Sigma_2}{\Sigma_1}$	0,0775
Central principal moment of inertia I, m^4	$I = \Sigma_3 - e^2 \Sigma_1$	2,354E-06

Further it's necessary to calculate the appropriate calculation scheme for the method of start parameters. As we have the distributed external load $q(x)$ not finished at right end we have to continue this loading up till right end, but at the section we continued this loading we have to set the same load but with opposite sign as it's shown on the fig. 19.1.

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The differential equation for the scheme presented on the fig. 19.1 can be written

$$\left. \begin{aligned} EIv^{IV}(x) &= \left\| \left\| q(x) \right\|_{c_P} + \left\| \left\| q_1(x) \right\|_{c_M} \right\|_{c_P} - \left\| \left\| q(x) \right\|_{c_P} \right\|_{c_M} \right\|_{c_P} \\ &= \left\| \left\| \frac{q_8(x-2)}{3} \right\|_{c_P=2} - \left\| \left\| \frac{q_8(x-2)}{3} \right\|_{c_M=6} \right\|_{c_P=2} \right\|_{c_P=2} \end{aligned} \right\} \quad (19.2)$$

Integrating sequentially (19.2) several times we can get expressions for internal shear forces Q , bending moments M , rotations angles α and deflections v

$$\left. \begin{aligned} EIv'''(x) &= Q(x) = \\ &= \left\| \left\| \int_{c_P=2}^x \frac{q_8(x-2)}{3} dx - \left\| \left\| \int_{c_M=6}^x \frac{q_8(x-2)}{3} dx - \left\| P + A \right\|_{c_P=2} \right\|_{c_M=6} \right\|_{c_P=2} \right\|_{c_P=2} \\ &= \left\| \left\| \frac{q_8(x-2)^2}{6} \right\|_2 - \left\| \left\| \frac{q_8((x-2)^2-16)}{6} \right\|_6 - \left\| P + A \right\|_2 \right\|_2 \end{aligned} \right\} \quad (19.3)$$

$$\left. \begin{aligned} EIv''(x) &= M(x) = \\ &= \left\| \left\| \int_{2,2}^x \frac{q_8(x-2)^2}{6} dx - \left\| \left\| \int_{6,6}^x \frac{q_8((x-2)^2-16)}{6} dx - \left\| \left\| P dx - \left\| M + Ax + B \right\|_{c_M=6} \right\|_{c_P=2} \right\|_{c_P=2} \right\|_{c_P=2} \right\|_{c_P=2} \\ &= \left\| \left\| \frac{q_8(x-2)^3}{18} \right\|_2 - \left\| \left\| q_8 \left[\frac{((x-2)^3-64)}{18} - \frac{8(x-6)}{3} \right] - \left\| P(x-2) - \left\| M + Ax + B \right\|_6 \right\|_6 \right\|_2 \end{aligned} \right\} \quad (19.4)$$

Task #3

$$\begin{aligned}
 v'(x) = \alpha(x) = & \left. \begin{aligned} & \left| \int_2^x \frac{q_8(x-2)^3}{18EI} dx - \left| \int_6^x q_8 \left[\frac{((x-2)^3 - 64)}{18EI} - \frac{8(x-6)}{3EI} \right] dx - \right. \right. \\ & - \left| \int_2^x \frac{P(x-2)}{EI} dx - \left| \int_6^x \frac{M}{EI} dx + \frac{Ax^2}{2EI} + \frac{Bx}{EI} + C = \right. \\ & = \left| \int_2^x \frac{q_8(x-2)^4}{72EI} - \left| \int_6^x q_8 \left[\frac{(x-2)^4 - 256}{72EI} - \frac{32(x-6)}{9EI} - \frac{4(x-6)^2}{3EI} \right] - \right. \\ & - \left| \int_2^x \frac{P(x-2)^2}{2EI} - \left| \int_6^x \frac{M(x-6)}{EI} + \frac{Ax^2}{2EI} + \frac{Bx}{EI} + C. \right. \right. \end{aligned} \right\} \\
 & (19.5)
 \end{aligned}$$

$$\begin{aligned}
 v(x) = & \left. \begin{aligned} & \left| \int_2^x \frac{q_8(x-2)^4}{72EI} dx - \left| \int_6^x q_8 \left[\frac{(x-2)^4 - 256}{72EI} - \frac{32(x-6)}{9EI} - \frac{4(x-6)^2}{3EI} \right] dx - \right. \right. \\ & - \left| \int_2^x \frac{P(x-2)^2}{2EI} dx - \left| \int_6^x \frac{M(x-6)}{EI} dx + \frac{Ax^3}{6EI} + \frac{Bx^2}{2EI} + Cx + D = \right. \\ & = \left| \int_2^x \frac{q_8(x-2)^5}{360EI} - \left| \int_6^x q_8 \left[\frac{(x-2)^5 - 1024}{360EI} - \frac{32(x-6)}{9EI} - \frac{16(x-6)^2}{9EI} - \frac{4(x-6)^3}{9EI} \right] - \right. \\ & - \left| \int_2^x \frac{P(x-2)^3}{6EI} - \left| \int_6^x \frac{M(x-6)^2}{2EI} + \frac{Ax^3}{6EI} + \frac{Bx^2}{2EI} + Cx + D. \right. \right. \end{aligned} \right\} \\
 & (19.6)
 \end{aligned}$$

Boundary conditions can be written as

$$\left. \begin{aligned} v(0) &= 0, \\ v'(0) &= 0, \\ v(l) &= 0, \\ EIv''(l) &= 0. \end{aligned} \right\} \quad (19.7)$$

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Providing boundary conditions (19.7) using (19.3) ÷ (19.6) we shall have next

$$v(0) = 0 \Rightarrow D = 0. \quad (19.8)$$

$$v'(0) = 0 \Rightarrow C = 0. \quad (19.9)$$

$$\left. \begin{aligned} v(l) = v(9) = 0 \Rightarrow & \frac{q_8(9-2)^5}{360EI} - \\ & - q_8 \left[\frac{(9-2)^5 - 1024}{360EI} - \frac{32(9-6)}{9EI} - \frac{16(9-6)^2}{9EI} - \frac{4(9-6)^3}{9EI} \right] - \\ & - \frac{P(9-2)^3}{6EI} - \frac{M(9-6)^2}{2EI} + \frac{A \cdot 9^3}{6EI} + \frac{B \cdot 9^2}{2EI} = 0 \Rightarrow \\ \Rightarrow & 2,58071 \cdot 10^{-4} \cdot A + 8,60238 \cdot 10^{-5} \cdot B - 0,20832 = 0. \end{aligned} \right\} \quad (19.10)$$

$$\left. \begin{aligned} M(l) = & \frac{q_8(9-2)^3}{18} - q_8 \left[\frac{(9-2)^3 - 64}{18} - \frac{8(9-6)}{3} \right] - \\ & - P(9-2) - M + A \cdot 9 + B = 0 \Rightarrow \\ \Rightarrow & 9,0 \cdot A + B - 9,22222 \cdot 10^3 = 0. \end{aligned} \right\} \quad (19.11)$$

Solving together (19.10), (19.11) we may find values of A and B . The values of A and B are next

$$A = 1133,42 \text{ N}; \quad B = -978,60 \text{ N} \cdot \text{m}. \quad (19.12)$$

Knowing parameters A , B , C , D we may show graphs for elements of bending $Q(x)$, $M(x)$, $\sigma(x)$, $v(x)$, which are shown on the fig. 19.2.

Task #3

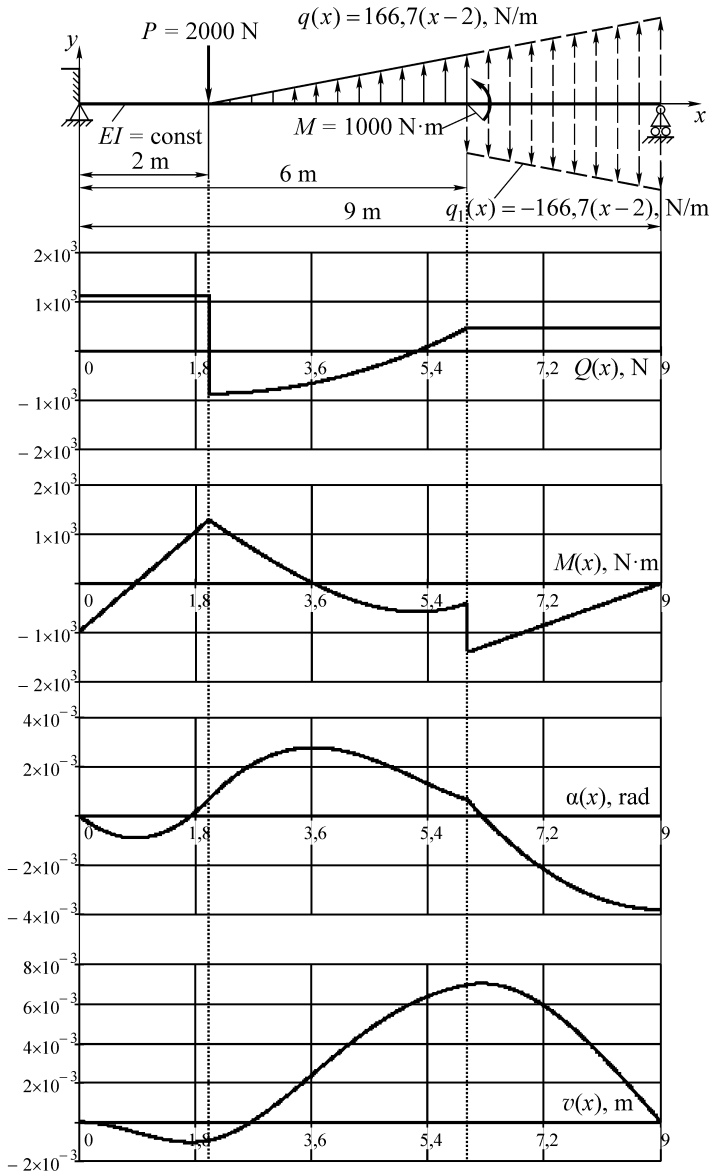


Fig. 19.2. The plots for elements of bending

Having laws of elements of bending we may estimate strength of the beam. To estimate strength of the beam it's necessary to operate with force parameters $Q(x)$, $M(x)$. It's necessary to find so called "the dangerous cross section" using $Q(x)$, $M(x)$. There are two ways of definition of the dangerous cross section: one of them (simpler) is using only the law of internal bending moments $M(x)$, but the second one (more complex) is using both $Q(x)$ and $M(x)$. Within our course we shall apply the first way for definition of dangerous cross section, when we use only bending moments $M(x)$.

Using the law of internal bending moments $M(x)$ (19.4) and plots of elements of bending presented on the fig. 19.2 we may find that the dangerous cross section (i.e. the section where $M(x)$ have the maximum value on modulus) is distanced from the left end for distance $x = 6,0$ m. The value of bending moment in the dangerous cross section $|M|_{\max} = 1400,0 \text{ N}\cdot\text{m}$. The values of force parameters, which will be used for definition of stresses in the dangerous cross section, are: $M = -1400,0 \text{ N}\cdot\text{m}$; $Q = 467,0 \text{ N}$.

To estimate the strength for the cross section it's necessary to create the law for equivalent stresses within height of the cross section $\sigma_{eq} = \sigma_{eq}(y)$ and to find out the maximum value of σ_{eq} . But for practical use they usually calculate equivalent stresses for several points within cross section height (see fig. 19.3).

Normal stresses σ , Pa and shear stresses τ , Pa can be calculated using next expressions (see lecture 6)

$$\sigma = -\frac{M \cdot y}{I}, \quad \tau = \frac{Q \cdot S^*}{I \cdot h}, \quad (19.13)$$

where M – is internal bending moment acting in given cross section, $[\text{N}\cdot\text{m}]$; y – is coordinate of given point relatively central principle axes (this is distance from neutral axis to given point in direction parallel to axis y considering sign), $[\text{m}]$; I – is central principle moment of inertia relatively principle axis y , $[\text{m}^4]$; Q – is shear force acting in given cross

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section, [N]; S^* – is static moment of cut area of the cross section relatively neutral axis, [m³]; h – is thickness of cross section at level of given point taken in cross section, [m].

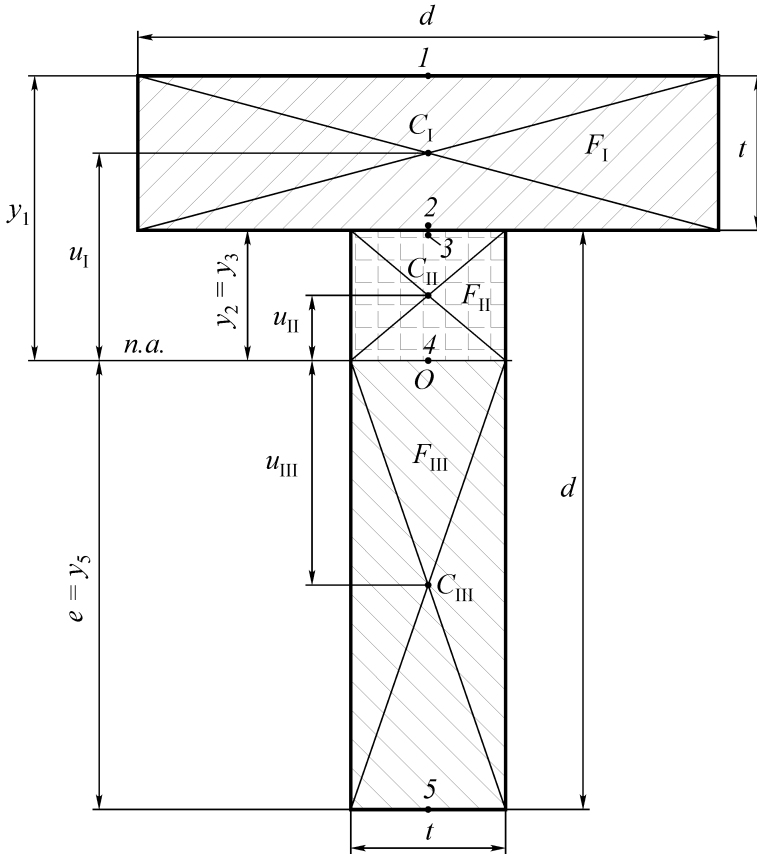


Fig. 19.3. The cross section

In the table 19.2 the calculation of the stresses σ and τ for points 1, 2, 3, 4, 5 is presented and plot of σ and τ is shown on the fig. 19.4. In the table 19.3 the calculation of the equivalent stresses is presented.

220 Table 19.2. Calculation of stresses in points of the cross section presented on the fig. 19.3 and the fig. 19.4

# of point	$y, \text{ m}$	$\sigma = -\frac{M \cdot y}{I}, \text{ MPa}$	$S^*, \text{ m}^3$	$h, \text{ m}$	$\tau = \frac{Q \cdot S^*}{I \cdot h}, \text{ MPa}$
1	$y_1 = d - e + t = 0,1 - 0,0775 + 0,01 = 0,0325$	$\sigma_1 = 19,3$	$S_1^* = 0$	$d = 0,1$	$\tau_1 = 0$
2	$y_2 = d - e = 0,1 - 0,0775 = 0,0225$	$\sigma_2 = 13,4$	$S_2^* = u_1 F_I =$ $= 0,0275 \cdot 0,001 = 2,75 \cdot 10^{-5};$ $u_1 = d - e + 0,5t =$ $= 0,1 - 0,0775 + 0,5 \cdot 0,01 = 0,0275;$ $F_I = t \cdot d = 0,01 \cdot 0,1 = 0,001$	$d = 0,1$	$\tau_2 = 0,055$
3	$y_3 = y_2 = 0,0225$	$\sigma_3 = \sigma_2 = 13,4$	$S_3^* = S_2^* = 2,75 \cdot 10^{-5}$	$t = 0,01$	$\tau_3 = 0,55$
4	$y_4 = 0$	$\sigma_4 = 0$	$S_2^* = u_1 F_I + u_{II} F_{II} \text{ or}$ $S_2^* = u_{III} F_{III} = 0,03875 \times$ $\times 7,75 \cdot 10^{-4} = 3,003 \cdot 10^{-5};$ $u_{III} = 0,5e = 0,5 \cdot 0,0775 = 0,03875;$ $F_{III} = e \cdot t = 0,0775 \cdot 0,01 = 7,75 \cdot 10^{-4}$	$t = 0,01$	$\tau_4 = 0,6$
5	$y_5 = -0,0775$	$\sigma_5 = -46,1$	$S_5^* = 0$	$t = 0,01$	$\tau_5 = 0$

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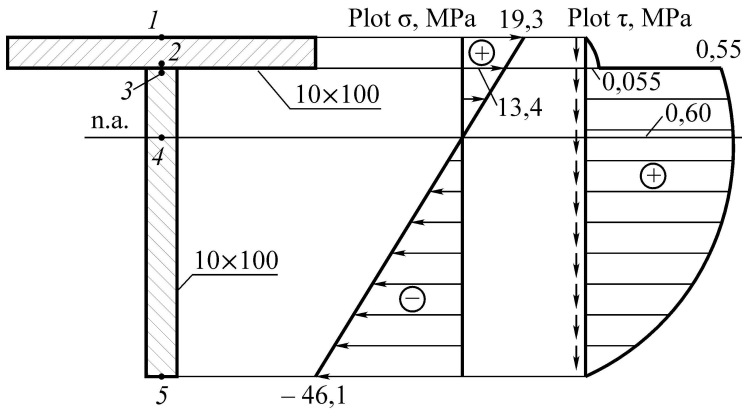


Fig. 19.4. Normal and shear stresses in the dangerous cross section

Table 19.3. Calculation of equivalent stresses in points of the cross section presented on the fig. 19.3 and the fig. 19.4

# of point	Normal stresses σ , MPa	Shear stresses τ , MPa	Equivalent stresses for the 3-rd theory $\sigma_{eq}^{III} = \sqrt{\sigma^2 + 4\tau^2}$, MPa	Equivalent stresses for the 4-rd theory $\sigma_{eq}^{IV} = \sqrt{\sigma^2 + 3\tau^2}$, MPa
1	$\sigma_1 = 19,3$	$\tau_1 = 0$	19,300	19,300
2	$\sigma_2 = 13,4$	$\tau_2 = 0,055$	13,400	13,400
3	$\sigma_3 = \sigma_2 = 13,4$	$\tau_3 = 0,55$	13,445	13,434
4	$\sigma_4 = 0$	$\tau_4 = 0,6$	1,200	1,039
5	$\sigma_5 = -46,1$	$\tau_5 = 0$	46,100	46,100

As we deal with plastic material further we shall operate with 4-th theory. The condition of providing of strength of beam can be written as

$$\sigma_{eq \max} \leq [\sigma], \quad (19.14)$$

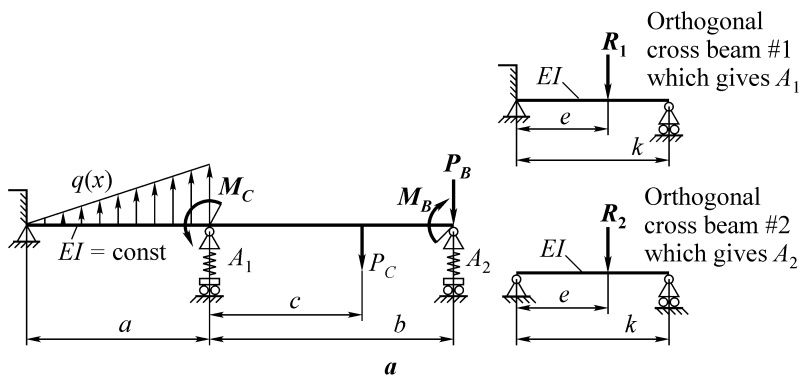
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where $\sigma_{eq\max} = 46,1 \text{ MPa}$ – is the maximum equivalent stresses for the cross section; $[\sigma] = \frac{\sigma_s}{n_s} = \frac{235 \cdot 10^6}{1,5} = 157 \text{ MPa}$ – are allowable stresses; $n_s = 1,5$ – is the accepted safety factor; $\sigma_s = 235 \text{ MPa}$ – is the yield limit.

We have that the strength condition (19.14) is provided, because $(\sigma_{eq\max} = 46,1 \text{ MPa}) \leq ([\sigma] = 157 \text{ MPa})$, that's why the strength of beam is provided.

Answer: the strength of beam for given external loadings and cross section is provided.

Task #4 (4 hours). Calculation of beams with many spans under bending for strength. Check the strength of beam standing on elastic hinges, shown on the fig. 20.1,*a*, using the method of five moments. The main beam stands on orthogonal cross beams, i.e. these orthogonal cross beams are modeled by elastic hinges. Places of joining of beams are shown by reactions R_1 and R_2 . The cross section is the same as for the task #3. There are no initial constructive deflections of elastic hinges.



Task #4

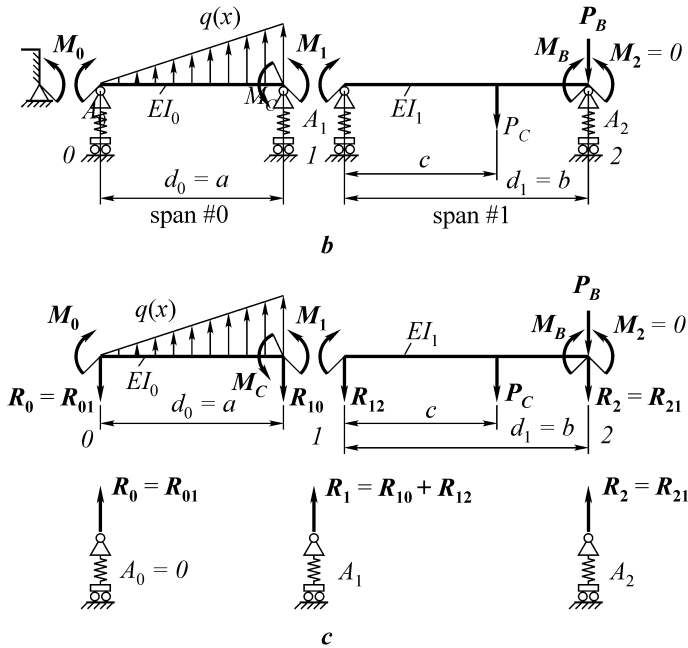


Fig. 20.1. The given beam on elastic hinges and appropriate calculation schemes for the method of five moments

Given: $a = 2,0$ m; $b = 4,0$ m; $c = 2,5$ m; $k = 1,2$ m; $e = 0,8$ m;
 $M_C = 4000,0$ N·m; $P_C = 3000,0$ N; $M_B = 2000,0$ N·m; $P_B = 1000,0$ N;
 $q(x) = 2000,0x$, N/m, relatively coordinate system which have begin at rigid restraint at left end; material is steel 3.

Solution

The calculation scheme is presented on the fig. 20.1, **b**.

First it's necessary to find suppleness factors A_0, A_1, A_2 . As we have rigid restraint at left end $A_0 = 0$. To find A_1 and A_2 it's necessary to find absolute values of deflections of orthogonal cross beams in points where reactions R_1 and R_2 are applied, caused by forces (reactions) $R_1 = 1,0$ and $R_2 = 1,0$. Using reference book [6], we can find

$$A_1 = \frac{e^3(k-e)}{12Elk} \left(1 + \frac{3(k-e)-e}{k} - \frac{e(k-e)}{k^2} \right),$$

$$A_2 = \frac{e^2(k-e)^2}{3Elk}. \quad (20.1)$$

Substituting numeral values into (20.1) we can find that:
 $A_1 = 3,357 \cdot 10^{-8} \text{ m/N}$; $A_1 = 6,042 \cdot 10^{-8} \text{ m/N}$.

There are two unknown values M_0 and M_1 which have to be found to estimate the strength of given beam.

Further it's necessary to compose compatibility equations for the given method. These compatibility equations are

$$\alpha_{01} = 0; \quad \alpha_{10} = \alpha_{12}. \quad (20.2)$$

Angles in (20.2) have to be expressed through external force factors and deflections on elastic supports using next relations

$$\left. \begin{aligned} \alpha_{01} &= \alpha_{01}(M_0) + \alpha_{01}(M_1) + \alpha_{01}(q) + \\ &\quad + \alpha_{01}(M_C) + \alpha_{01}(v_0, v_1) \\ \alpha_{10} &= \alpha_{10}(M_0) + \alpha_{10}(M_1) + \alpha_{10}(q) + \\ &\quad + \alpha_{10}(M_C) + \alpha_{10}(v_0, v_1) \\ \alpha_{12} &= \alpha_{12}(M_1) + \alpha_{12}(M_2) + \alpha_{12}(P_C) + \\ &\quad + \alpha_{12}(P_B) + \alpha_{12}(M_B) + \alpha_{12}(v_1, v_2) \end{aligned} \right\}. \quad (20.3)$$

To find rotation angles in (20.3) caused by external loadings (i.e. for example $\alpha_{01}(M_0)$, $\alpha_{01}(q)$, etc.) we consider that our separate beams for now are situated on rigid hinges.

Task #4

Using reference book [6] rotation angles in (20.3) can be written

$$\left. \begin{aligned} \alpha_{01} &= -\frac{M_0 d_0}{3EI_0} - \frac{M_1 d_0}{6EI_0} + \frac{7}{180} \frac{Q d_0^2}{EI_0} - \frac{M_C d_0}{6EI_0} + \frac{v_1 - v_0}{d_0} \\ \alpha_{10} &= \frac{M_0 d_0}{6EI_0} + \frac{M_1 d_0}{3EI_0} - \frac{2}{45} \frac{Q d_0^2}{EI_0} + \frac{M_C d_0}{3EI_0} + \frac{v_1 - v_0}{d_0} \\ \alpha_{12} &= -\frac{M_1 d_1}{3EI_1} - \frac{M_2 d_1}{6EI_1} - \frac{P_C c(b-c)}{6EI_1} \left(1 + \frac{b-c}{d_1} \right) + \\ &\quad + \frac{M_B d_1}{6EI_1} + \frac{v_2 - v_1}{d_1} \end{aligned} \right\} \quad (20.4)$$

Pay attention that there is no rotation angle caused by P_B , because we consider that our separate beams are situated on rigid hinges. Also $v_0 = 0$, because of rigid restraint, and $M_2 = 0$. The value Q can be found using expression

$$Q = \int_0^a q(x) dx = \int_0^{2,0} 2000x \, dx = 4000,0 \, \text{N}. \quad (20.5)$$

Deflections can be found using next expressions

$$v_1 = A_1 R_1, \quad v_2 = A_2 R_2. \quad (20.6)$$

Deflections v_1, v_2 have to be expressed through unknown reactive moments M_0, M_1 . To do this it's necessary to provide several intermediate steps. For the 1-st step it's necessary to throw away hinges (which are shown on the fig. 20.1,**b**) and to put reactions instead those hinges as it's shown on the fig. 20.1,**c**.

Composing equilibrium equations we may find reactions R_y . For the span #1 appropriate equilibrium equations and reactions are

$$\left. \begin{aligned} \Sigma \text{ Moments}_{z0} = 0 : & -M_0 + M_1 + M_C + \\ & + \int_0^a q(x) dx \cdot \frac{\int_0^a q(x)x dx}{\int_0^a q(x) dx} - R_{10}a = 0 \Rightarrow \\ \Rightarrow R_{10} = & -0,5M_0 + 0,5M_1 + 4666, (6). \end{aligned} \right\} \quad (20.7)$$

$$\left. \begin{aligned} \Sigma \text{ Moments}_{z1} = 0 : & -M_0 + M_1 + M_C - \\ & - \int_0^a q(x) dx \cdot \left(a - \frac{\int_0^a q(x)x dx}{\int_0^a q(x) dx} \right) + R_{01}a = 0 \Rightarrow \\ \Rightarrow R_{01} = & 0,5M_0 - 0,5M_1 - 666, (6). \end{aligned} \right\} \quad (20.8)$$

For the span#2 appropriate equilibrium equations and reactions are

$$\left. \begin{aligned} \Sigma \text{ Moments}_{z1} = 0 : & -M_1 + M_2 - \\ & - M_B - P_C c - P_B b - R_{21}b = 0 \Rightarrow \\ \Rightarrow R_{21} = & -0,25M_1 - 20250,0. \end{aligned} \right\} \quad (20.9)$$

$$\left. \begin{aligned} \Sigma \text{ Moments}_{z2} = 0 : & -M_1 + M_2 - \\ & - M_B + P_C(b-c) + R_{12}b = 0 \Rightarrow \\ \Rightarrow R_{12} = & 0,25M_1 - 10750,0. \end{aligned} \right\} \quad (20.10)$$

Task #4

Reactions R_0, R_1, R_2 can be found using next expressions

$$R_0 = R_{01}; \quad R_1 = R_{10} + R_{12}; \quad R_2 = R_{21}. \quad (20.11)$$

Substituting (20.7) ÷ (20.10) into (20.11) we shall have

$$\left. \begin{aligned} R_0 &= 0,5M_0 - 0,5M_1 - 666, (6); \\ R_1 &= -0,5M_0 + 0,75M_1 + 4041, (6); \\ R_2 &= -0,25M_1 - 3375,0. \end{aligned} \right\} \quad (20.12)$$

At last substituting (20.12) into (20.6), after substituting (20.6) into (20.4), also substituting all numeral values into (20.4), after substituting (20.4) into (20.2) we shall have the system of equations for definition of unknown values M_0, M_1 , which can be written

$$\left. \begin{aligned} -1,424 \cdot 10^{-6} M_0 - 6,954 \cdot 10^{-7} M_1 - 1,443 \cdot 10^{-3} &= 0; \\ 6,996 \cdot 10^{-7} M_0 + 1,429 \cdot 10^{-6} M_1 + 4,222 \cdot 10^{-3} &= \\ = 4,196 \cdot 10^{-9} M_0 - 2,842 \cdot 10^{-6} M_1 - 2,729 \cdot 10^{-3}. \end{aligned} \right\} \quad (20.13)$$

Solving the system (20.13) we can find the unknown reactive moments M_0, M_1 :

$$M_0 = -237,06 \text{ N} \cdot \text{m}; \quad M_1 = -1588,84 \text{ N} \cdot \text{m}. \quad (20.14)$$

Substituting (20.14) into (20.12) we have the reactions

$$R_0 = 9,22 \text{ N}; \quad R_1 = 2968,57 \text{ N}; \quad R_2 = -2977,79 \text{ N}. \quad (20.15)$$

Using cross section method we can show plots of shear (internal) forces Q and bending (internal) moments M , which are shown on the fig. 20.3.

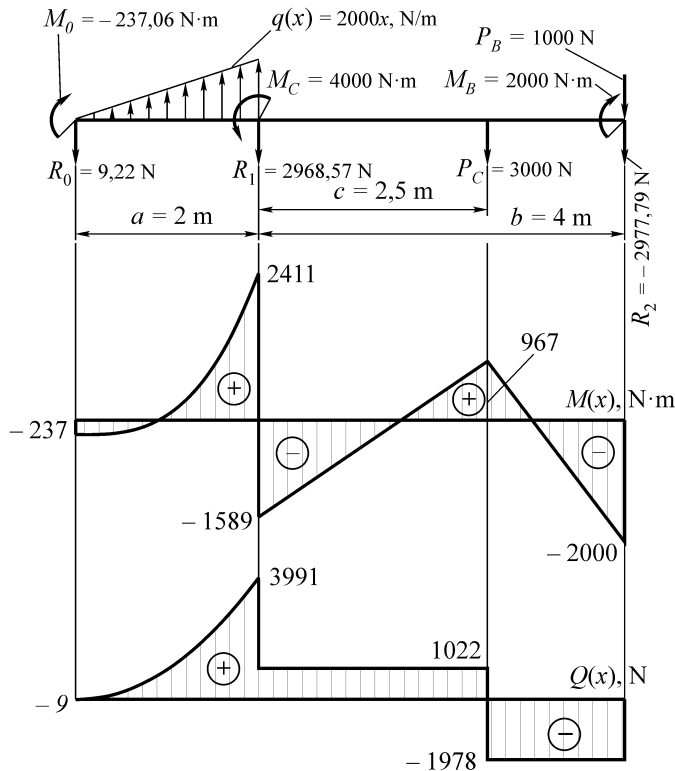


Fig. 20.3. Plots of internal shear forces Q and internal bending moments M

The dangerous cross section is section ($x = a = 2$ m), where $M_{\max} = 2411$ N·m, but accordant shear force $Q = 3991$ N. Having geometrical properties for our cross section we can estimate strength for two strength theories (for 3-rd and 4-th theories). Calculations of normal and shear stresses are presented in the table 20.1. Plots of normal and shear stresses are shown on the fig. 20.4.

Task #4

Table 20.1. Calculation of stresses in points of the cross section presented on the fig. 20.4

# of point	$y, \text{ m}$	$\sigma = -\frac{M \cdot y}{I},$ MPa	$S^*, \text{ m}^3$	$h, \text{ m}$	$\tau = \frac{Q \cdot S^*}{I \cdot h},$ MPa
1	0,0325	- 33,3	0,0	0,1	0,0
2	0,0225	- 23,0	$2,75 \cdot 10^{-5}$	0,1	0,5
3	0,0225	- 23,0	$2,75 \cdot 10^{-5}$	0,01	4,7
4	0,0	0,0	$3,003 \cdot 10^{-5}$	0,01	5,1
5	- 0,0775	79,4	0,0	0,01	0,0

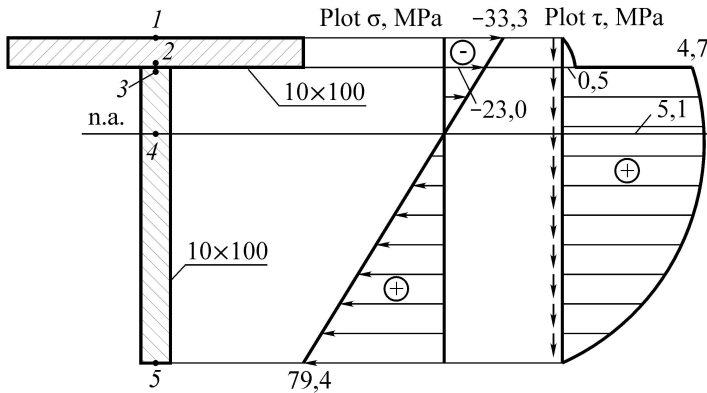


Fig. 20.4. Normal and shear stresses in the dangerous cross section on base of calculations in the table 20.1

Calculations of equivalent stresses for 3-rd and 4-th theories are presented in the table 20.2.

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Table 20.2. Calculation of equivalent stresses in points of the cross section presented on the fig. 20.4

# of point	Normal stresses σ , MPa	Shear stresses τ , MPa	Equivalent stresses for the 3-rd theory $\sigma_{eq}^{III} = \sqrt{\sigma^2 + 4\tau^2}$, MPa	Equivalent stresses for the 4-rd theory $\sigma_{eq}^{IV} = \sqrt{\sigma^2 + 3\tau^2}$, MPa
1	– 33,3	0,0	33,3	33,3
2	– 23,0	0,5	23,1	23,1
3	– 23,0	4,7	24,9	24,4
4	0,0	5,1	10,2	8,8
5	79,4	0,0	79,4	79,4

As we deal with plastic material further we shall operate with 4-th theory. The condition of providing of strength of beam can be written as

$$\sigma_{eq \max} \leq [\sigma], \quad (20.16)$$

where $\sigma_{eq \max} = 79,4$ MPa – is the maximum equivalent stresses for

the cross section; $[\sigma] = \frac{\sigma_s}{n_s} = \frac{235 \cdot 10^6}{1,5} = 157$ MPa – are allowable

stresses; $n_s = 1,5$ – is the accepted safety factor; $\sigma_s = 235$ MPa – is the yield limit.

We have that the strength condition (20.16) is provided, because $(\sigma_{eq \max} = 79,4 \text{ MPa}) < ([\sigma] = 157 \text{ MPa})$, that's why the strength of beam is provided.

Answer: the strength of beam on elastic hinges for given external loadings and cross section is provided.

Task #5

Task #5 (4 hours). Calculation of frames under bending for strength. Check the strength of ship middle frame presented on the fig. 21.1, with empty hold. The cross section of all rods is the same as for the task #3. The ship floats on still sea water with draft d .

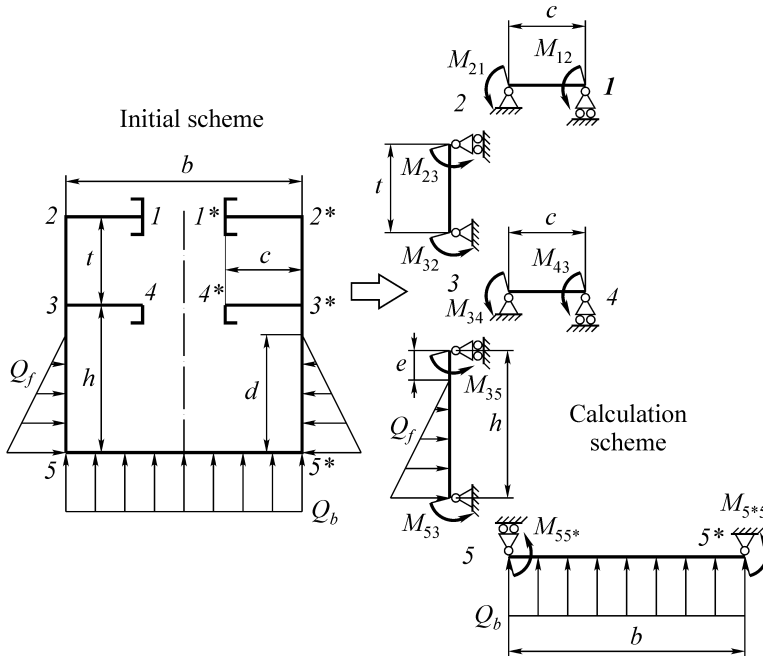


Fig. 21.1. The initial scheme and the calculation scheme

Given: $b = 8,0$ m; $h = 4,0$ m; $t = 3,0$ m; $c = 2,0$ m; $d = 3,0$ m; material is steel 3.

Solution

The calculation scheme is presented on the fig. 21.1 near to the initial scheme.

First we shall find the values of outer loadings Q_f and Q_b . The pressure of the water at bottom level is p_b is

$$p_b = \rho g d = 1025,0 \cdot 9,81 \cdot 3,0 = 30166 \text{ Pa}, \quad (21.1)$$

where $\rho = 1025 \text{ kg/m}^3$ – the density of sea water; $g = 9,81 \text{ m/sec}^2$ – gravity acceleration; $d = 3,0 \text{ m}$ – draft.

Total external loading for the frame Q_f can be found using expression

$$Q_f = 0,5 p_b a d = 0,5 \cdot 30166 \cdot 0,1 \cdot 3,0 = 4525 \text{ N}, \quad (21.2)$$

where $a = 0,1 \text{ m}$ – is width of the outer side of the beam.

Total external loading for the bottom Q_b can be found using expression

$$Q_b = p_b a b = 30166 \cdot 0,1 \cdot 8,0 = 24133 \text{ N}. \quad (21.3)$$

To solve this task we shall apply the method of angular displacements. For this method first it's necessary to compose equilibrium equations for every node. As we have symmetric construction and symmetric external loading it's enough to compose equilibrium equations only for one half. For our occasion these equilibrium equations can be written

$$\left. \begin{array}{l} \text{node \#1: } M_{12} = 0, \\ \text{node \#2: } M_{21} + M_{23} = 0, \\ \text{node \#3: } M_{32} + M_{34} + M_{35} = 0, \\ \text{node \#4: } M_{43} = 0, \\ \text{node \#5: } M_{53} + M_{55*} = 0. \end{array} \right\} \quad (21.4)$$

After, every equation in (21.4) has to be expressed through rotation angles and reactions in restraints, using general expression

$$M_{ij} = M_{ij}^r + \frac{2EI_{ij}}{d_{ij}}(2\alpha_i + \alpha_j), \quad (21.5)$$

Task #5

where M_{ij}^r – are reactions in restraints, considering that separate beams are rigidly constrained; these reactions can be found in [6]; d_{ij} – are distances of separate beam between nodes i -th and j -th.

If there is no outer loadings $M_{ij}^r = 0$. For our occasion

$$\left. \begin{aligned} M_{12}^r &= 0, & M_{21}^r &= 0, & M_{23}^r &= 0, & M_{32}^r &= 0, \\ M_{34}^r &= 0, & M_{43}^r &= 0, \\ M_{53}^r &= 1725,2 \text{ N} \cdot \text{m}; & M_{35}^r &= -933,3 \text{ N} \cdot \text{m}; \\ M_{55^*}^r &= -16088,0 \text{ N} \cdot \text{m}. \end{aligned} \right\} \quad (21.6)$$

Substituting numeral values of d_{ij} and I_{ij} for every separate beam into (21.5) we can write down expressions of M_{ij} , presented below

$$\left. \begin{aligned} M_{12} &= 941600,0\alpha_1 + 470800,0\alpha_2; \\ M_{21} &= 470800,0\alpha_1 + 941600,0\alpha_2; \\ M_{23} &= 627733,3\alpha_2 + 313866,7\alpha_3; \\ M_{32} &= 313866,7\alpha_2 + 627733,3\alpha_3; \\ M_{34} &= 941600,0\alpha_3 + 470800,0\alpha_4; \\ M_{35} &= 470800,0\alpha_3 + 235400,0\alpha_5 - 933,3; \\ M_{43} &= 470800,0\alpha_3 + 941600,0\alpha_4; \\ M_{53} &= 235400,0\alpha_3 + 470800,0\alpha_5 + 1725,2; \\ M_{55^*} &= 117700,0\alpha_5 - 16088,7. \end{aligned} \right\} \quad (21.7)$$

Substituting (21.7) into (21.4) we shall have five equations to find five unknown values $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$. These five equations are presented below

$$\left. \begin{aligned} 941600,0\alpha_1 + 470800,0\alpha_2 &= 0; \\ 627733,3\alpha_2 + 313866,7\alpha_3 + \\ &+ 470800,0\alpha_1 + 941600,0\alpha_2 = 0; \\ 313866,7\alpha_2 + 2040133,3\alpha_3 + \\ &+ 470800,0\alpha_4 + 235400,0\alpha_5 - 933,3 = 0; \\ 470800,0\alpha_3 + 941600,0\alpha_4 &= 0; \\ 235400,0\alpha_3 + 588500,0\alpha_5 - 14363,5 &= 0. \end{aligned} \right\} \quad (21.8)$$

Solving (21.8) we can find rotation angles of nodes, which are presented below

$$\left. \begin{aligned} \alpha_1 &= -3,459 \cdot 10^{-4}; \\ \alpha_2 &= 6,918 \cdot 10^{-4}; \\ \alpha_3 &= -2,940 \cdot 10^{-3}; \\ \alpha_4 &= 1,470 \cdot 10^{-3}; \\ \alpha_5 &= 2,558 \cdot 10^{-2}. \end{aligned} \right\} \quad (21.9)$$

Substituting (21.9) into (21.7) we may find values of node moments, which are presented below

$$\left. \begin{aligned} M_{12} &= 0,0; \quad M_{21} = 488,5; \\ M_{23} &= -488,5; \quad M_{32} = -1628,5; \\ M_{34} &= -2076,3; \quad M_{35} = 3704,8; \\ M_{43} &= 0,0; \quad M_{53} = 13077,5; \\ M_{55*} &= -13077,5. \end{aligned} \right\} \quad (21.10)$$

Task #5

On the fig. 21.2 plots of shear forces Q , and bending moments M are presented.

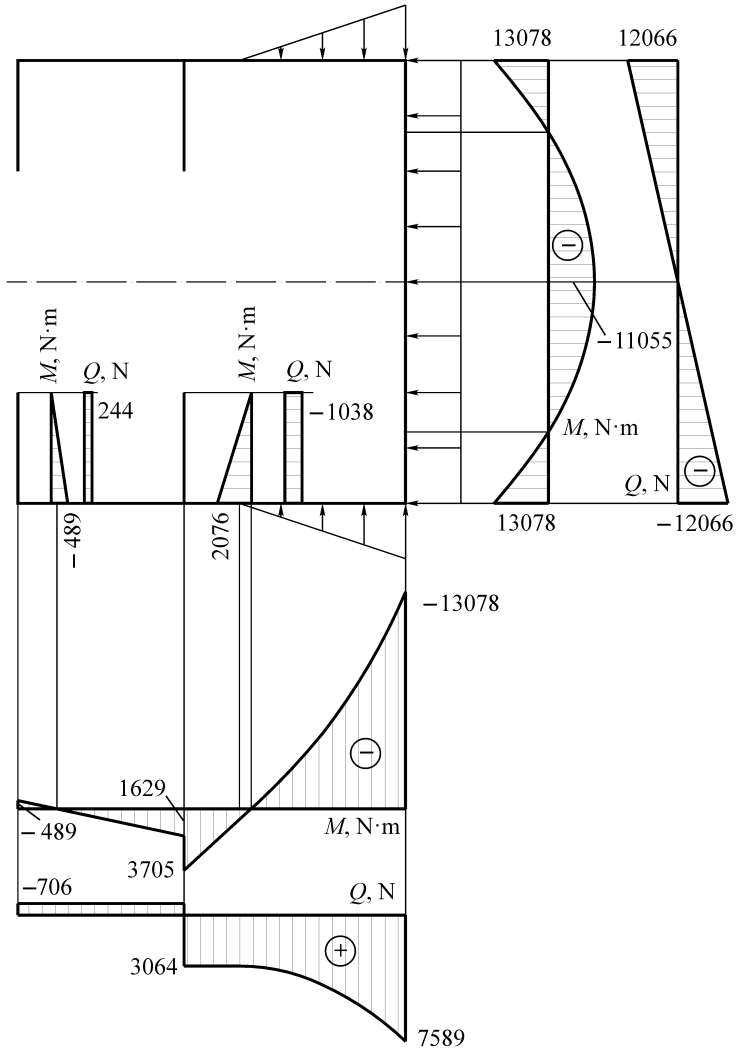


Fig. 21.2. The plots of shear forces Q and bending moments M

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The dangerous cross section is the section in node #5, where $M_{\max} = 13078 \text{ N}\cdot\text{m}$, but accordant shear force $Q = -12066 \text{ N}$. Having geometrical properties for our cross section we can estimate strength for two strength theories (for 3-rd and 4-th theories). Calculations of normal and shear stresses are presented in the table 21.1. Plots of normal and shear stresses are presented on the fig. 21.3.

Table 21.1. Calculation of stresses in points of the cross section presented

# of point	$y, \text{ m}$	$\sigma = -\frac{M \cdot y}{I}, \text{ MPa}$	$S^*, \text{ m}^3$	$h, \text{ m}$	$\tau = \frac{Q \cdot S^*}{I \cdot h}, \text{ MPa}$
1	0,0325	-180,5	0,0	0,1	0,0
2	0,0225	-125,0	$2,75 \cdot 10^{-5}$	0,1	-1,4
3	0,0225	-125,0	$2,75 \cdot 10^{-5}$	0,01	-14,1
4	0,0	0,0	$3,003 \cdot 10^{-5}$	0,01	-15,4
5	-0,0775	430,5	0,0	0,01	0,0

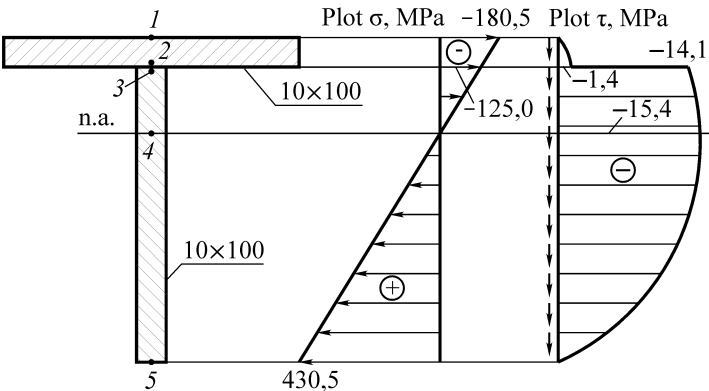


Fig. 21.3. Normal and shear stresses in the dangerous cross section on base of calculations in the table 21.1

Task #5

Calculations of equivalent stresses for 3-rd and 4-th theories are presented in the table 21.2.

Table 21.2. Calculation of equivalent stresses in points of the cross section presented on the fig. 21.3

# of point	Normal stresses σ , MPa	Shear stresses τ , MPa	Equivalent stresses for the 3-rd theory $\sigma_{eq}^{III} = \sqrt{\sigma^2 + 4\tau^2}$, MPa	Equivalent stresses for the 4-rd theory $\sigma_{eq}^{IV} = \sqrt{\sigma^2 + 3\tau^2}$, MPa
1	-180,5	0,0	180,5	180,5
2	-125,0	-1,4	125,0	125,0
3	-125,0	-14,1	128,1	127,4
4	0,0	-15,4	30,8	26,7
5	430,5	0,0	430,5	430,5

As we deal with plastic material further we shall operate with 4-th theory. The condition of providing of strength of beam can be written as

$$\sigma_{eq \max} \leq [\sigma], \quad (21.11)$$

where $\sigma_{eq \max} = 430,5$ MPa – is the maximum equivalent stresses for

the cross section; $[\sigma] = \frac{\sigma_s}{n_s} = \frac{235 \cdot 10^6}{1,5} = 157$ MPa – are allowable

stresses; $n_s = 1,5$ – is the accepted safety factor; $\sigma_s = 235$ MPa – is the yield limit.

We have that the strength condition (21.11) is failed, because $(\sigma_{eq \max} = 430,5 \text{ MPa}) > ([\sigma] = 157 \text{ MPa})$, that's why the strength of frame beam is failed.

Answer: the strength of frame for given external loadings and cross sections is failed.

Task #6 (2 hours). Calculation of beams on elastic foundation under bending for strength. Check out the beam on elastic foundation presented on the fig. 22.1 for strength. The cross section for this beam is the same as in task #3.

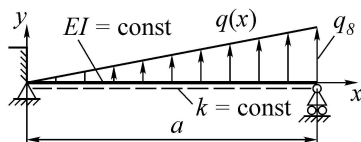


Fig. 22.1. The beam on elastic foundation

Given: $a = 4 \text{ m}$; $q_8 = 1500 \text{ N/m}$; $k = 200 \text{ N/m}^2$; material is steel 3.

Solution

To solve this task it's necessary to define the laws of distributions $q(x)$. Using the methodology of definition of the equation of the direct line using two points we can find the law of $q(x)$. The laws of $q(x)$ can be written

$$q(x) = \frac{q_8 x}{4}. \quad (22.1)$$

The differential equation for this beam can be written

$$EI v^{IV}(x) + kv(x) = q(x) = \frac{q_8 x}{4}. \quad (22.2)$$

The solution for this equation can be written

$$v(x) = D_0 V_0(\beta x) + D_1 V_1(\beta x) + D_2 V_2(\beta x) + D_3 V_3(\beta x) + \frac{q(x)}{k}, \quad (22.3)$$

where D_0, D_1, D_2, D_3 – are unknown factors, which have to be found providing boundary conditions (22.5); V_0, V_1, V_2, V_3 – are Puzirevsky functions;

Task #6

$$\beta = \sqrt[4]{\frac{k}{4EI}} = \sqrt[4]{\frac{200}{4 \cdot 2 \cdot 10^{11} \cdot 2,354 \cdot 10^{-6}}} = 0,10152 - \text{the factor.} \quad (22.4)$$

Boundary conditions for the beam presented on the fig. 22.1 can be written

$$\left. \begin{aligned} v(0) &= 0, & v(a) &= 0, \\ v'(0) &= 0, & EIv''(a) &= 0. \end{aligned} \right\} \quad (22.5)$$

Substituting (22.3) into (22.5) we shall have four equations for definition of four unknown factors D_0, D_1, D_2, D_3

$$\left. \begin{aligned} v(0) &= D_0V_0(\beta \cdot 0) + D_1V_1(\beta \cdot 0) + \\ &+ D_2V_2(\beta \cdot 0) + D_3V_3(\beta \cdot 0) + \frac{q_8 \cdot 0}{4k} = 0 \Rightarrow \\ \Rightarrow D_0 &= 0 \end{aligned} \right\} \quad (22.6)$$

$$\left. \begin{aligned} v'(0) &= \beta\sqrt{2}(-D_0V_3(\beta \cdot 0) + D_1V_0(\beta \cdot 0) + \\ &+ D_2V_1(\beta \cdot 0) + D_3V_2(\beta \cdot 0)) + \frac{q_8}{4k} = 0 \Rightarrow \\ \Rightarrow D_1 &= -\frac{q_8}{4\sqrt{2} \cdot k\beta} = -13,06029 \end{aligned} \right\} \quad (22.7)$$

$$\left. \begin{aligned}
 v(a) &= D_0 V_0(\beta \cdot a) + D_1 V_1(\beta \cdot a) + \\
 &+ D_2 V_2(\beta \cdot a) + D_3 V_3(\beta \cdot a) + \frac{q_8 \cdot a}{4k} = 0 \Rightarrow \\
 \Rightarrow \left| D_0 = 0; \quad D_1 &= -\frac{q_8}{4\sqrt{2} \cdot k\beta} \right| \Rightarrow \\
 -\frac{q_8}{4\sqrt{2} \cdot k\beta} \cdot 0,57374 + D_2 \cdot 0,16484 + \\
 &+ D_3 \cdot 0,03156 + \frac{q_8 \cdot a}{4k} = 0 \Rightarrow \\
 \Rightarrow D_2 \cdot 0,16484 + D_3 \cdot 0,03156 &= -6,7967 \cdot 10^{-3}
 \end{aligned} \right\} \quad (22.8)$$

$$\left. \begin{aligned}
 Elv''(a) &= 2\beta^2 (-D_0 V_2(\beta \cdot a) - D_1 V_3(\beta \cdot a) + \\
 &+ D_2 V_0(\beta \cdot a) + D_3 V_1(\beta \cdot a)) = 0 \Rightarrow \\
 \Rightarrow \left| D_0 = 0; \quad D_1 &= -\frac{q_8}{4\sqrt{2} \cdot k\beta} = -13,06029 \right| \Rightarrow \\
 \Rightarrow \frac{q_8}{4\sqrt{2} \cdot k\beta} 0,03156 + D_2 \cdot 0,99547 + \\
 &+ D_3 \cdot 0,57374 = 0 \Rightarrow \\
 D_2 \cdot 0,99547 + D_3 \cdot 0,57374 &= -0,41216
 \end{aligned} \right\} \quad (22.9)$$

Task #6

where

$$\left. \begin{aligned} V_0(\beta a) &= V_0(0,40606) = 0,99547, \\ V_2(\beta a) &= V_2(0,40606) = 0,16484, \\ V_1(\beta a) &= V_1(0,40606) = 0,57374, \\ V_3(\beta a) &= V_3(0,40606) = 0,03156. \end{aligned} \right\} \quad (22.10)$$

Solving (22.8) and (22.9) we find that values of missing factors D_2 and D_3 are next:

$$\left. \begin{aligned} D_2 &= 0,14421 \\ D_3 &= -0,96859 \end{aligned} \right\} \quad (22.11)$$

Knowing factors D_0, D_1, D_2, D_3 we can write down the expressions for elements of bending.

The expression for shear forces $Q(x)$, bending moments $M(x)$, rotation angles $\alpha(x)$ and deflections $v(x)$ can be written

$$\left. \begin{aligned} Q(x) &= E I v'''(x) = \\ &= 2\sqrt{2}\beta^3 [-D_0 V_1(\beta x) - D_1 V_2(\beta x) - D_2 V_3(\beta x) + D_3 V_0(\beta x)]; \\ M(x) &= E I v''(x) = \\ &= 2\beta^2 [-D_0 V_2(\beta x) - D_1 V_3(\beta x) + D_2 V_0(\beta x) + D_3 V_1(\beta x)]; \\ \alpha(x) &= v'(x) = \\ &= \beta\sqrt{2} [-D_0 V_3(\beta x) + D_1 V_0(\beta x) + D_2 V_1(\beta x) + D_3 V_2(\beta x)] + \frac{q_8}{4k}; \\ v(x) &= D_0 V_0(\beta x) + D_1 V_1(\beta x) + D_2 V_2(\beta x) + D_3 V_3(\beta x) + \frac{q_8 x}{4k}. \end{aligned} \right\} \quad (22.12)$$

Plots of elements of bending are presented on the fig. 22.2.

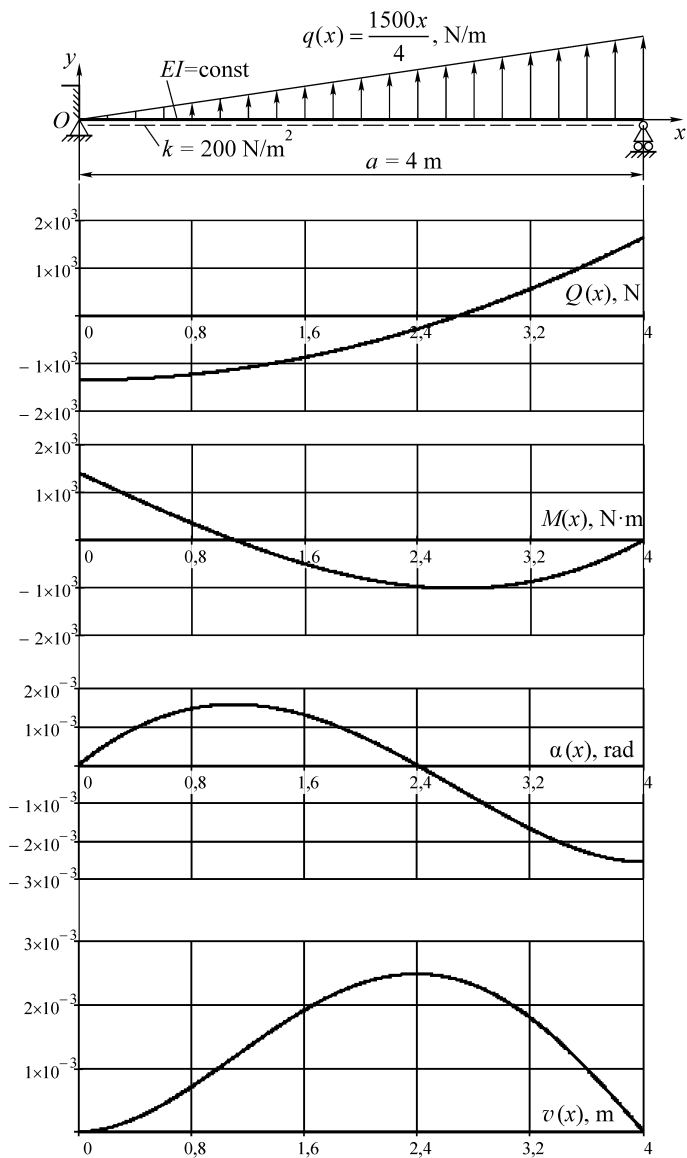


Fig. 22.2. Elements of bending for beam on elastic foundation

Task #6

The most dangerous cross section we shall define only for bending moments. The most dangerous cross section is in place where the value of bending moment on modulus has the largest value. For our case the dangerous cross section is for $x = 0$. For this cross section ($x = 0$) we have $M_{\max} = 1399 \text{ N} \cdot \text{m}$, but accordant shear force $Q = -1349 \text{ N}$.

Having geometrical properties for our cross section we can estimate strength for two strength theories (for 3-rd and 4-th theories). Calculations of normal and shear stresses are presented in the table 22.1. Plots of normal and shear stresses are presented on the fig. 22.3.

Table 22.1. Calculation of stresses in points of the cross section presented on the fig. 22.3

# of point	$y, \text{ m}$	$\sigma = -\frac{M \cdot y}{I},$ MPa	$S^*, \text{ m}^3$	$h, \text{ m}$	$\tau = \frac{Q \cdot S^*}{I \cdot h},$ MPa
1	0,0325	-19,3	0,0	0,1	0,0
2	0,0225	-13,4	$2,75 \cdot 10^{-5}$	0,1	-0,16
3	0,0225	-13,4	$2,75 \cdot 10^{-5}$	0,01	-1,58
4	0,0	0,0	$3,003 \cdot 10^{-5}$	0,01	-1,72
5	-0,0775	46,1	0,0	0,01	0,0

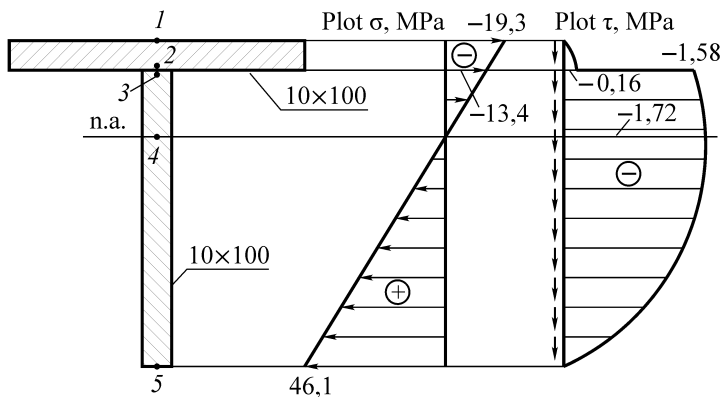


Fig. 22.3. Normal and shear stresses in the dangerous cross section

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Calculations of equivalent stresses for 3-rd and 4-th theories are presented in the table 22.2.

Table 22.2. Calculation of equivalent stresses in points of the cross section presented on the fig. 22.3

# of point	Normal stresses σ , MPa	Shear stresses τ , MPa	Equivalent stresses for the 3-rd theory $\sigma_{eq}^{III} = \sqrt{\sigma^2 + 4\tau^2}$, MPa	Equivalent stresses for the 4-rd theory $\sigma_{eq}^{IV} = \sqrt{\sigma^2 + 3\tau^2}$, MPa
1	-19,3	0,0	19,300	19,300
2	-13,4	-0,16	13,404	13,403
3	-13,4	-1,58	13,766	13,675
4	0,0	-1,72	3,442	2,981
5	46,1	0,0	46,100	46,100

As we deal with plastic material further we shall operate with 4-th theory. The condition of providing of strength of beam can be written as

$$\sigma_{eq \max} \leq [\sigma], \quad (22.13)$$

where $\sigma_{eq \max} = 46,1$ MPa – is the maximum equivalent stresses for

the cross section; $[\sigma] = \frac{\sigma_s}{n_s} = \frac{235 \cdot 10^6}{1,5} = 157$ MPa – are allowable

stresses; $n_s = 1,5$ – is the accepted safety factor; $\sigma_s = 235$ MPa – is the yield limit.

We have that the strength condition (22.13) is provided, because $(\sigma_{eq \max} = 46,1 \text{ MPa}) < ([\sigma] = 157 \text{ MPa})$, that's why the strength of beam is provided.

Answer: the strength of beam on elastic foundation for given external loadings and cross section is provided.

Task #7

Task #7 (4 hours). Calculation of grillages under bending for strength. Check the strength of bottom grillage presented on the fig. 23.1, with empty hold. The cross section of all rods is the same as for task #3. The ship floats on still sea water with draft dr .

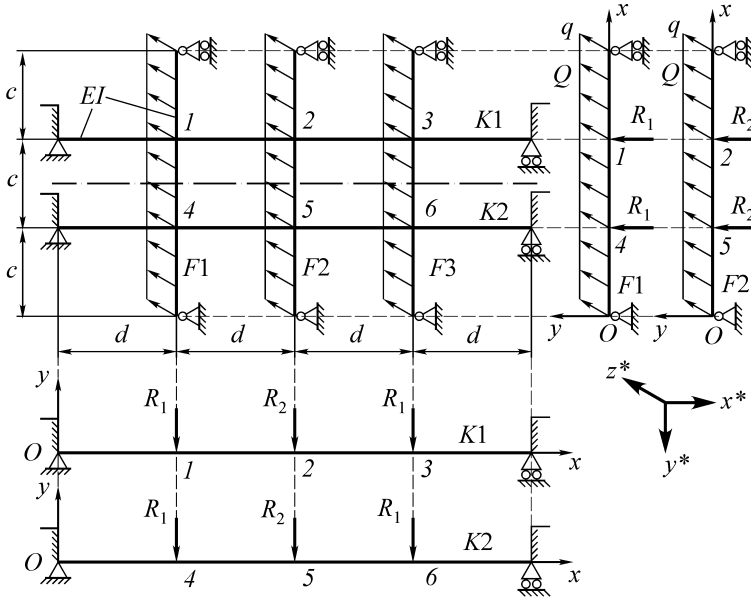


Fig. 23.1. The given bottom grillage and appropriate calculation scheme

Given: $c = 3,0$ m; $d = 4,0$ m; $dr = 3,0$ m; material is steel 3.

Solution

First we shall find the values of outer loadings Q . The pressure of the water at bottom level is p_b is

$$p_b = \rho g d = 1025,0 \cdot 9,81 \cdot 3,0 = 30166 \text{ Pa}, \quad (23.1)$$

where $\rho = 1025 \text{ kg/m}^3$ – the density of sea water; $g = 9,81 \text{ m/sec}^2$ – gravity acceleration; $dr = 3,0$ m – draft.

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Total external loading for the floor Q can be found using expression

$$Q = p_b 3ca = 30166 \cdot 3 \cdot 3,0 \cdot 0,1 = 27149 \text{ N}, \quad (23.2)$$

where $a = 0,1 \text{ m}$ – is width of the outer side of the beam.

The law for distributed external loading, acting for the floor, caused by the outer pressure can be found using next expression

$$q(x) = q = p_b a = 30166 \cdot 0,1 = 3016,6 \text{ N/m}. \quad (23.3)$$

Owing to the symmetry of the construction and external loadings we have next relations

$$\left. \begin{aligned} R_1 &= R_3 = R_4 = R_6, \\ R_2 &= R_5, \\ v_1 &= v_3 = v_4 = v_6, \\ v_2 &= v_5. \end{aligned} \right\} \quad (23.4)$$

Taking into account (23.4) the conditions of compatibility of deflections in nodes are next

$$v_1^{K1} = v_1^{F1}, \quad v_2^{K1} = v_2^{F2}. \quad (23.5)$$

Deflections in (23.5) have to be expressed through external loadings using next relations

$$\left. \begin{aligned} v_1^{K1} &= v_1^{K1}(R_1) + v_1^{K1}(R_2), \\ v_1^{F1} &= v_1^{F1}(R_1) + v_1^{F1}(Q), \\ v_2^{K1} &= v_2^{K1}(R_1) + v_2^{K1}(R_2), \\ v_2^{F2} &= v_2^{F2}(R_2) + v_2^{F2}(Q). \end{aligned} \right\} \quad (23.6)$$

Task #7

Substituting expressions for deflections caused by external loadings into (23.6) using reference book [6], after substituting the result we have got using (23.6) into (23.5) we shall have next compatibility conditions for deflections

$$\left. \begin{aligned} -\frac{5R_1d^3}{24EI} - \frac{R_2d^3}{6EI} &= \frac{5R_1c^3}{6EI} + \frac{11Qc^3}{36EI}, \\ -\frac{R_1d^3}{3EI} - \frac{R_2d^3}{3EI} &= \frac{5R_2c^3}{6EI} + \frac{11Qc^3}{36EI}. \end{aligned} \right\} \quad (23.7)$$

Solving the system (23.7) we have next numeral values for unknown reactions R_1, R_2 :

$$R_1 = -5530,8 \text{ N}; \quad R_2 = -2418,0 \text{ N}. \quad (23.8)$$

Having reactions we can find the elements of bending Q and M for all beams $K1, K2, F1, F2, F3$. For beams $F1, F2, F3$ we shall use the cross section method, but for beams $K1, K2$ we shall apply start parameters method. Also superposition method can be used using reference books.

Plots of internal shear forces Q and internal bending moments M are shown on the fig. 23.2.

In the most common occasion it is necessary to check strength of every separate beam of the grillage if these separate beams are different. But for our occasion it's enough to find one dangerous cross section, because all our beams are identical. The dangerous cross section is section in the middle of beam $F2$ where $M_{\max} = -23289 \text{ N}\cdot\text{m}$, but according shear force $Q = 0 \text{ N}$.

Having geometrical properties for our cross section we can estimate strength for two strength theories (for 3-rd and 4-th theories). Calculations of normal and shear stresses are presented in table 23.1. Plots of normal and shear stresses are presented on the fig. 23.3.

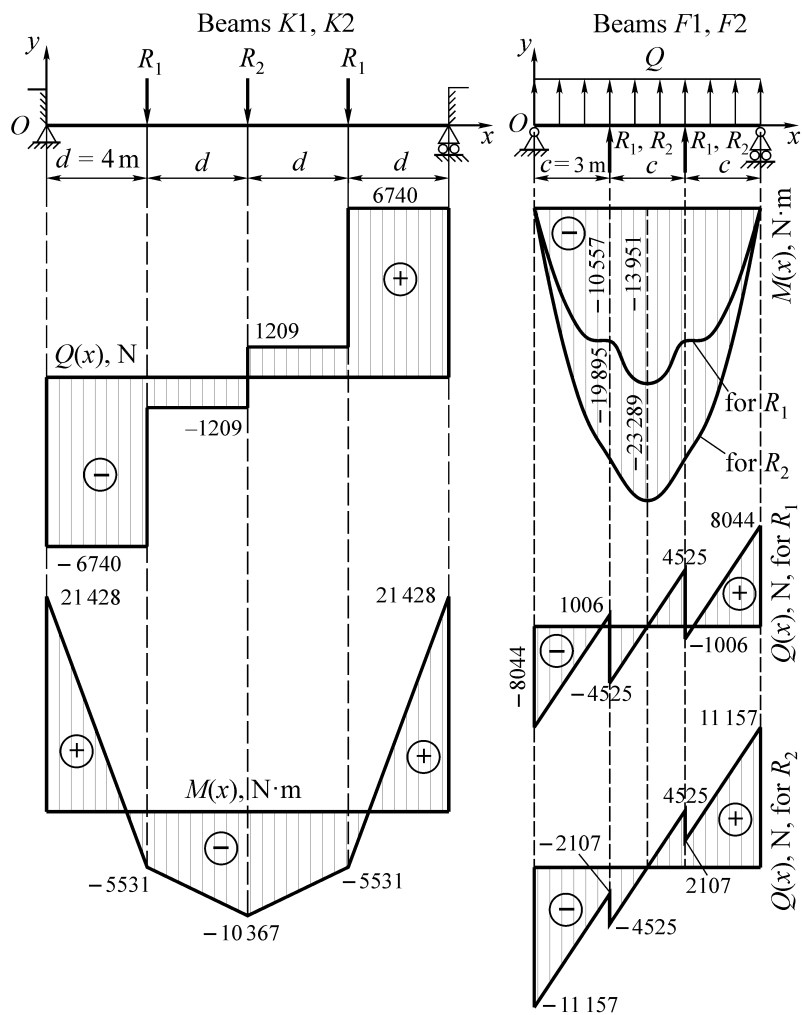


Fig. 23.2. Plots of internal forces Q and M for beams of grillage

Task #7

Table 23.1. Calculation of stresses in points of the cross section presented on the fig. 23.3

# of point	y, m	$\sigma = -\frac{M \cdot y}{I}, \text{ MPa}$	S^*, m^3	h, m	$\tau = \frac{Q \cdot S^*}{I \cdot h}, \text{ MPa}$
1	0,0325	312,5	0,0	0,1	0,0
2	0,0225	222,6	$2,75 \cdot 10^{-5}$	0,1	0,0
3	0,0225	222,6	$2,75 \cdot 10^{-5}$	0,01	0,0
4	0,0	0,0	$3,003 \cdot 10^{-5}$	0,01	0,0
5	-0,0775	-766,7	0,0	0,01	0,0

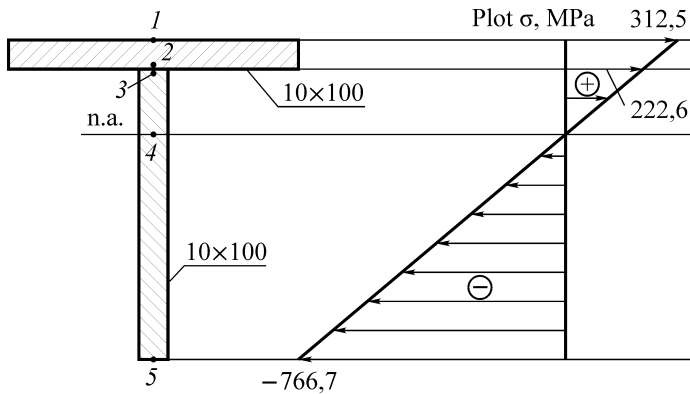


Fig. 23.3. Normal and shear stresses in the dangerous cross section on base of calculations in the table 23.1

Calculations of equivalent stresses for 3-rd and 4-th theories are presented in the table 23.2.

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Table 23.2. Calculation of equivalent stresses in points of the cross section presented on the fig. 23.3

# of point	Normal stresses σ , MPa	Shear stresses τ , MPa	Equivalent stresses for the 3-rd theory $\sigma_{eq}^{III} = \sqrt{\sigma^2 + 4\tau^2}$, MPa	Equivalent stresses for the 4-rd theory $\sigma_{eq}^{IV} = \sqrt{\sigma^2 + 3\tau^2}$, MPa
1	312,5	0,0	321,5	321,5
2	222,6	0,0	222,6	222,6
3	222,6	0,0	222,6	222,6
4	0,0	0,0	0,0	0,0
5	-766,7	0,0	766,7	766,7

As we deal with plastic material further we shall operate with 4-th theory. The condition of providing of strength of beam can be written as

$$\sigma_{eq \max} \leq [\sigma], \quad (23.9)$$

where $\sigma_{eq \max} = 766,7 \text{ MPa}$ – is the maximum equivalent stresses for

the cross section; $[\sigma] = \frac{\sigma_s}{n_s} = \frac{235 \cdot 10^6}{1,5} = 157 \text{ MPa}$ – are allowable

stresses; $n_s = 1,5$ – is the accepted safety factor; $\sigma_s = 235 \text{ MPa}$ – is the yield limit.

We have that the strength condition (23.9) is failed, because $(\sigma_{eq \max} = 766,7 \text{ MPa}) > ([\sigma] = 157 \text{ MPa})$, that's why the strength of grillage beam is failed.

Answer: the strength of the grillage for the given external loadings and cross section is failed.

Task #8

Task #8 (2 hours). Calculation of beams under bending with axial forces for strength. Check out the beam tensed by axial forces presented on the fig. 24.1 for strength. The cross section for this beam is the same as in the task #3. Also there is no initial constructive deflection.

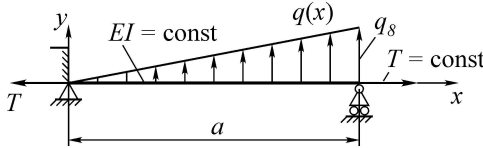


Fig. 24.1. The beam tensed by axial forces

Given: $a = 4$ m; $q_8 = 1500$ N/m; $T = 20000$ N; material is steel 3.

Solution

To solve this task it's necessary to define the laws of distributions $q(x)$. Using the methodology of definition of the equation of the direct line using two points we can find the law of $q(x)$. The laws of $q(x)$ can be written

$$q(x) = \frac{q_8 x}{4}. \quad (24.1)$$

The differential equation for this beam can be written

$$EI v^{IV}(x) - T v''(x) = q(x) = \frac{q_8 x}{4}. \quad (24.2)$$

As we have the axial force cause tension the solution for this equation can be written

$$v(x) = A_0 + A_1 \beta x + A_2 \operatorname{ch} \beta x + A_3 \operatorname{sh} \beta x - \frac{q_8 x^3}{24T}, \quad (24.3)$$

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where A_0, A_1, A_2, A_3 – are unknown factors, which have to be found providing boundary conditions (24.5);

$$\beta = \sqrt{\frac{T}{EI}} = \sqrt{\frac{20000}{2 \cdot 10^{11} \cdot 2,354 \cdot 10^{-6}}} = 0,206109 - \text{the factor.} \quad (24.4)$$

Boundary conditions for the beam presented on the fig. 24.1 can be written

$$\left. \begin{aligned} v(0) = 0, \quad v(a) = 0, \\ v'(0) = 0, \quad EIv''(a) = 0. \end{aligned} \right\} \quad (24.5)$$

Substituting (24.3) into (24.5) we shall have four equations for definition of four unknown factors A_0, A_1, A_2, A_3

$$\left. \begin{aligned} v(0) = 0 &\Rightarrow A_0 + A_2 = 0, \\ v(a) = 0 &\Rightarrow A_1\beta + A_3\beta = 0, \\ v'(0) = 0 &\Rightarrow A_0 + A_1\beta a + \\ &+ A_2 \operatorname{ch} \beta a + A_3 \operatorname{sh} \beta a - \frac{q_8 a^3}{24T} = 0, \\ EIv''(a) &= EI \times \\ &\times \left(A_2\beta^2 \operatorname{ch} \beta a + A_3\beta^2 \operatorname{sh} \beta a - \frac{q_8 a}{4T} \right) = 0. \end{aligned} \right\} \quad (24.6)$$

Solving (24.6) we can find four unknown factors. Numeral values of these factors are

$$\left. \begin{aligned} A_0 &= -0,06821 \\ A_1 &= -1,81614 \\ A_2 &= 0,06821 \\ A_3 &= 1,81614. \end{aligned} \right\} \quad (24.7)$$

Task #8

The expression for shear forces $Q(x)$, bending moments $M(x)$, rotation angles $\alpha(x)$ and deflections $v(x)$ can be written

$$\left. \begin{aligned} Q(x) &= EIv'''(x) - Tv'(x) = \\ &= EI \left(A_2\beta^3 \operatorname{sh} \beta x + A_3\beta^3 \operatorname{ch} \beta x - \frac{q_8}{4T} \right) - \\ &\quad - T \left(A_1\beta + A_2\beta \operatorname{sh} \beta x + A_3\beta \operatorname{ch} \beta x - \frac{q_8 x^2}{8T} \right); \\ M(x) &= EIv''(x) = \\ &= EI \left(A_2\beta^2 \operatorname{ch} \beta x + A_3\beta^2 \operatorname{sh} \beta x - \frac{q_8 x}{4T} \right); \\ \alpha(x) &= v'(x) = A_1\beta + A_2\beta \operatorname{sh} \beta x + A_3\beta \operatorname{ch} \beta x - \frac{q_8 x^2}{8T}; \\ v(x) &= A_0 + A_1\beta x + A_2 \operatorname{ch} \beta x + A_3 \operatorname{sh} \beta x - \frac{q_8 x^3}{24T}. \end{aligned} \right\} \quad (24.8)$$

Plots of elements of bending defined by (24.8) are presented below on the fig. 24.2.

The most dangerous cross section we shall define only for bending moments. The most dangerous cross section is in place where the value of bending moment on modulus has the largest value. For our case the dangerous cross section is for $x = 0$. For this cross section ($x = 0$) we have $M_{\max} = 1364 \text{ N}\cdot\text{m}$, but accordant shear force $Q = -1341 \text{ N}$.

Having geometrical properties for our cross section we can estimate strength for two strength theories (for 3-rd and 4-th theories). Calculations of normal and shear stresses are presented in the table 24.1. Plots of normal and shear stresses are presented on the fig. 24.3.

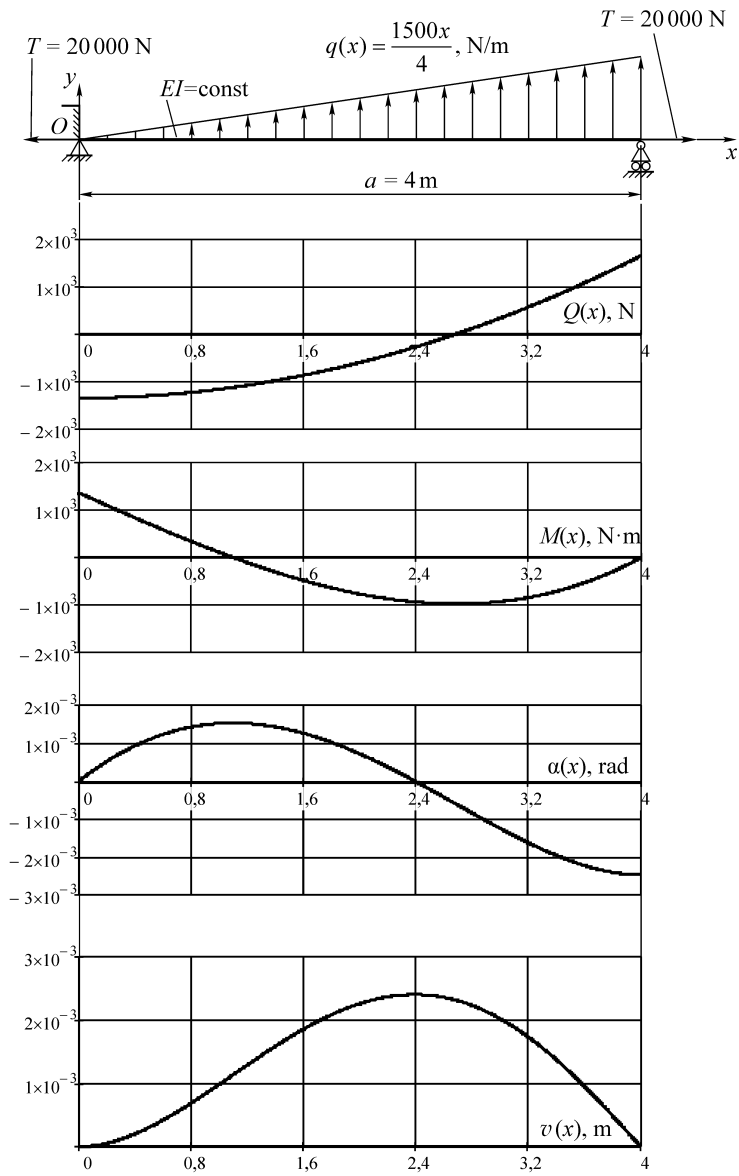


Fig. 24.2. Elements of bending

Task #8

Table 24.1. Calculation of stresses in points of the cross section presented on the fig. 24.3

# of point	$y, \text{ m}$	$\sigma = \frac{T}{F} - \frac{M \cdot y}{I},$ MPa	$S^*, \text{ m}^3$	$h, \text{ m}$	$\tau = \frac{Q \cdot S^*}{I \cdot h},$ MPa
1	0,0325	- 8,83	0,0	0,1	0,0
2	0,0225	- 3,04	$2,75 \cdot 10^{-5}$	0,1	- 0,16
3	0,0225	- 3,04	$2,75 \cdot 10^{-5}$	0,01	- 1,57
4	0,0	10,0	$3,003 \cdot 10^{-5}$	0,01	- 1,71
5	- 0,0775	54,9	0,0	0,01	0,0

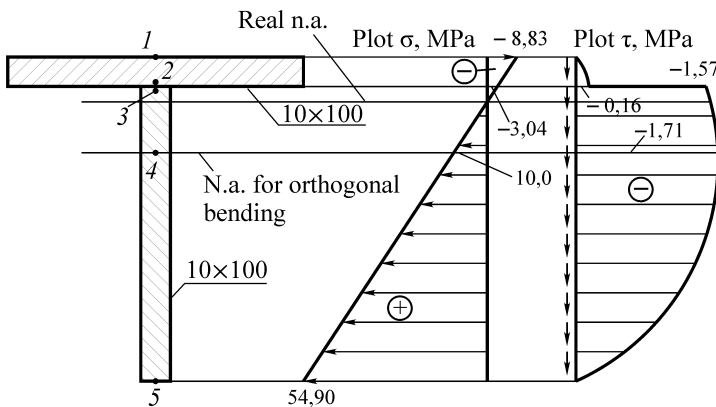


Fig. 24.3. Normal and shear stresses in the dangerous cross section on base of calculations in the table 24.1

Calculations of equivalent stresses for 3-rd and 4-th theories are presented in the table 24.2.

Table 24.2. Calculation of equivalent stresses in points of the cross section presented on the fig. 24.3

# of point	Normal stresses σ , MPa	Shear stresses τ , MPa	Equivalent stresses for the 3-rd theory $\sigma_{eq}^{III} = \sqrt{\sigma^2 + 4\tau^2}$, MPa	Equivalent stresses for the 4-rd theory $\sigma_{eq}^{IV} = \sqrt{\sigma^2 + 3\tau^2}$, MPa
1	- 8,83	0,0	8,830	8,830
2	-3,04	- 0,16	3,053	3,049
3	-3,04	-1,57	4,363	4,072
4	10,0	-1,71	10,569	10,430
5	54,9	0,0	54,903	54,903

As we deal with plastic material further we shall operate with 4-th theory. The condition of providing of strength of beam can be written as

$$\sigma_{eq \max} \leq [\sigma], \quad (24.9)$$

where $\sigma_{eq \max} = 54,9 \text{ MPa}$ – is the maximum equivalent stresses for

the cross section; $[\sigma] = \frac{\sigma_s}{n_s} = \frac{235 \cdot 10^6}{1,5} = 157 \text{ MPa}$ – are allowable

stresses; $n_s = 1,5$ – is the accepted safety factor; $\sigma_s = 235 \text{ MPa}$ – is the yield limit.

We have that the strength condition (24.9) is provided, because $(\sigma_{eq \max} = 54,9 \text{ MPa}) \leq ([\sigma] = 157 \text{ MPa})$, that's why the strength of beam is provided.

Answer: the strength of beam tensed by axial forces for given external loadings and cross section is provided.

Task #9

Task #9 (2 hours). Calculation of beam for stability. Find out the necessary sizes of the round cross section of the beam presented on the fig. 25.1 to provide strength and stability of it, for given compressive forces.

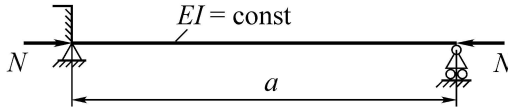


Fig. 25.1. The beam compressed by forces at its ends

Given: material is steel 3; $a = 3,0$ m; $N = 30000$ N.

Find: diameter d of the cross section of the rod.

Solution

First we shall find the necessary area of the cross section applying the strength condition. This scheme presented on the fig. 25.1 is very simple and it's obvious the necessary minimal area can be found using next expression

$$F \geq \frac{N}{[\sigma]} = \frac{30000}{168,0 \cdot 10^6} = 1,786 \cdot 10^{-4} \text{ m}^2, \quad (25.1)$$

where $[\sigma] = \frac{\sigma_s}{n_s} = \frac{235 \cdot 10^6}{1,4} = 168 \cdot 10^6 \text{ Pa}$ – are allowable stresses

for strength which also will be used as allowable stresses for stability; but for stability there may be another allowable stresses; $\sigma_s = 235 \text{ MPa}$ – is the yield limit of the steel 3; $n_s = 1,4$ – is the accepted safety factor.

Further it's necessary to define geometrical properties of the cross section to provide the stability of given rod, considering the deviation on the Hook law.

Eulerian collapse stresses can be defined using next expression

$$\sigma_{eu} = \frac{\pi^2 E}{\lambda^2}, \quad (25.2)$$

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where $E = 2 \cdot 10^{11}$ Pa – modulus of elasticity in tension; $\lambda = \frac{\mu a}{r}$ – is

flexibility; $r = \sqrt{\frac{I}{F}}$ – inertia radius; I – is minimal principal central moment of inertia (but not always); $\mu = 0,7$ – is special factor which depends on boundary conditions (BC) at ends, which can be found using reference book for simple BCs or/and solving the differential equation of the stability for any BCs.

The connection between Eulerian collapse stresses and critical collapse stresses can be written

$$\sigma_{cr} = f(\sigma_{eu}), \quad \sigma_{cr} = \varphi \sigma_{eu}, \quad (25.3)$$

where φ – is special reductive factor (see lectures).

There are a lot of relations like (25.3), which depends on material and other factors. We shall use one of the simplest relations $\sigma_{cr} = f(\sigma_{eu})$ taken in [10] and written below

$$\left. \begin{array}{l} \text{if } \frac{\sigma_{eu}}{\sigma_s} < 0,6 \quad \text{then} \quad \sigma_{cr} = \sigma_{eu} \\ \text{if } 0,6 \leq \frac{\sigma_{eu}}{\sigma_s} \quad \text{then} \quad \sigma_{cr} = \sigma_s \left(1,1 - \frac{0,3}{(\sigma_{eu}/\sigma_s)} \right) \\ \text{if } \sigma_{cr} \geq \sigma_s \quad \text{then} \quad \sigma_{cr} = \sigma_s \end{array} \right\} \quad (25.4)$$

Critical collapse stresses don't have to exceed the allowable stresses. I.e. calculating stability we have to provide next condition

$$\sigma_{cr} \leq [\sigma]. \quad (25.5)$$

In practice stability of construction is usually calculated relatively flexibility λ . It is necessary to have some function depending on λ .

Task #9

Substituting (25.2) into (25.4) we can create dependency between critical collapse stresses and flexibility presented below

$$\sigma_{cr}(\lambda) = \begin{cases} \text{if } \frac{\sigma_{eu}}{\sigma_s} < 0,6 & \text{then } \sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \text{ by (9.2)} \\ \text{if } 0,6 \leq \frac{\sigma_{eu}}{\sigma_s} & \text{then } \sigma_{cr} = \sigma_s \left(1,1 - \frac{0,3 \cdot \sigma_s \cdot \lambda^2}{\pi^2 E} \right) \\ \text{if } \sigma_{cr} \geq \sigma_s & \text{then } \sigma_{cr} = \sigma_s \end{cases} \quad (25.6)$$

Substituting (25.6) into (25.5) we can form the nonlinear equation relatively flexibility λ . This equation is written below

$$\begin{aligned} \sigma_{cr} \leq [\sigma] &\Rightarrow \sigma_{cr}(\lambda) - [\sigma] = 0 \\ \text{or } f(\lambda) &= \sigma_{cr}(\lambda) - [\sigma] = 0. \end{aligned} \quad (25.7)$$

Solving the nonlinear equation (25.7) we can find the necessary λ which provide stability for given level of allowable stresses $[\sigma]$.

For example we shall use the method of "halfdivision" to find the root of (25.7). The process of calculation of the root λ is presented in the table 25.1. We shall stop iteration when the difference between neighboring roots will be less than 0,5.

Table 25.1. Iterations for method of half divisions to find necessary flexibility

#	λ_A	λ_B	λ_i	$f(\lambda_i)$	$f(\lambda_A)$
1	50,00	150,00	100,00	6,568E+06	6,700E+07
2	100,00	150,00	125,00	-4,167E+07	6,568E+06
3	100,00	125,00	112,50	-1,573E+07	6,568E+06
4	100,00	112,50	106,25	-4,251E+06	6,568E+06
5	100,00	106,25	103,13	1,240E+06	6,568E+06
6	103,13	106,25	104,69	-1,485E+06	1,240E+06
7	103,13	104,69	103,91	-1,172E+05	1,240E+06
8	103,13	103,91	103,52	5,629E+05	1,240E+06

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So we have the flexibility which provide zero to (25.7) is $\lambda_0 = 103,52$. The Eulerian and critical collapse stresses for given λ_0 are

$$\sigma_{eu} = \frac{\pi^2 E}{\lambda^2} = \frac{3,14^2 \cdot 2 \cdot 10^{11}}{103,52^2} = 1,842 \cdot 10^8 \text{ Pa}; \quad (25.8)$$

$$\left. \begin{aligned} \frac{\sigma_{eu}}{\sigma_{cr}} &= \frac{1,842 \cdot 10^8}{235 \cdot 10^6} = 0,78 < 0,6 \quad \text{that's why} \\ \sigma_{cr} &= \sigma_s \left(1,1 - \frac{0,3}{(\sigma_{eu}/\sigma_s)} \right) = 235 \cdot 10^6 \times \\ &\times \left(1,1 - \frac{0,3}{(1,842 \cdot 10^8 / 235 \cdot 10^6)} \right) = 168,6 \text{ MPa.} \end{aligned} \right\} \quad (25.9)$$

We have the condition of stability (25.5) is provided (little excess of σ_{cr} not more than 5% is not horrible).

Using known value of flexibility λ_0 we can find the necessary geometrical properties. The necessary inertia radius can be found using relation

$$\lambda_0 = \frac{\mu a}{r} \Rightarrow r = \frac{\mu a}{\lambda_0} = \frac{0,7 \cdot 3,0}{103,52} = 2,0286 \cdot 10^{-2} \text{ m.} \quad (25.10)$$

The cross section area has to be not less than defined by (25.1). Using this condition we may define the minimum necessary moment of inertia

$$\begin{aligned} r &= \sqrt{\frac{I}{F}} \Rightarrow I = r^2 F = (2,0286 \cdot 10^{-2})^2 \times \\ &\times 1,786 \cdot 10^{-4} = 7,350 \cdot 10^{-8} \text{ m}^4. \end{aligned} \quad (25.11)$$

Task #10

The diameter of the cross section of the beam can be found using (25.11) applying next expression

$$\begin{aligned} I &= \frac{\pi d^4}{64} \Rightarrow d \geq \sqrt[4]{\frac{64 \cdot I}{\pi}} = \\ &= \sqrt[4]{\frac{64 \cdot 7,350 \cdot 10^{-8}}{3,14}} = 3,498 \cdot 10^{-2} \text{ m.} \end{aligned} \quad (25.12)$$

Having sizes of the cross section (i.e. diameter) it's necessary to find the area of the cross section again (from the conditions of the stability)

$$F = \frac{\pi d^2}{4} = \frac{3,14 \cdot (3,498 \cdot 10^{-2})^2}{4} = 9,610 \cdot 10^{-4} \text{ m}^2. \quad (25.13)$$

Comparing areas found applying conditions of strength (25.1) and stability (25.13) we have to choose the maximum value between these two ones. We have that cross section area defined by (25.13) is larger than defined by (25.1). That's why we accept the minimum necessary area for the cross section is $F = 9,610 \cdot 10^{-4} \text{ m}^2$. The appropriate diameter is defined by (25.12). The diameter defined by (25.12) also provides the necessary inertia moment (25.11).

Answer: to provide strength and stability of the rod considering the deviation on the Hook law the cross section area has to have the diameter not less than $d = 3,498 \cdot 10^{-2} \text{ m} \approx 35 \text{ mm}$.

Task #10 (2 hours). Calculation of rectangular plate for strength and stability. Find out the necessary minimal thickness of the bottom plate (fig. 26.1) of ship for known draft d from conditions of providing strength and stability of the bottom grillage. The system of arrangement of the bottom grillage is longitudinal.

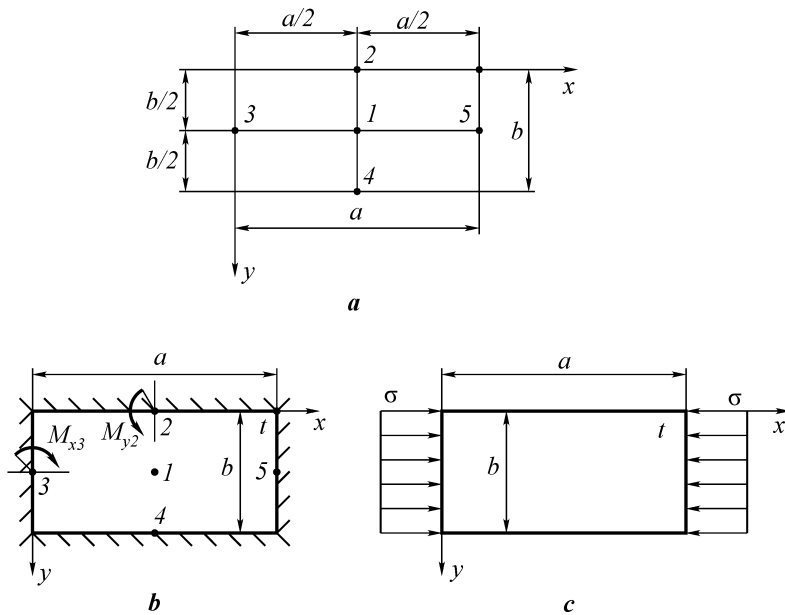


Fig. 26.1. Given plate to be calculated for strength and stability

Given: $a = 2,4$ m; $b = 0,8$ m; material is steel 3; $d = 8,0$ m; longitudinal ship hull arrangement.

Find: thickness t .

Solution

First we shall find the thickness applying strength condition. If ship plate is calculated for strength we consider that all edges have rigid restraints (see fig. 26.1,*b*), but the outer pressure is uniform.

The value of the outer pressure acting for the field of plate can be found using next expression

$$q = \rho g d = 1025,0 \cdot 9,81 \cdot 8,0 = 80442,0 \text{ Pa}, \quad (26.1)$$

where $\rho = 1025 \text{ kg/m}^3$ – density of sea water; $g = 9,81 \text{ m/sec}^2$ – gravity acceleration.

Task #10

Using reference book [7] we may find the calculation scheme for strength computations for uniform distributed loadings. Bending moments in the most dangerous points can be calculated using expressions

$$\left. \begin{aligned} M_{x3} &= v_{13}qb^2 = 0,0571 \cdot 80442 \cdot 0,8^2 = 2940 \text{ N}; \\ M_{y2} &= v_{22}qb^2 = 0,0832 \cdot 80442 \cdot 0,8^2 = 4283 \text{ N}. \end{aligned} \right\} \quad (26.2)$$

The most dangerous point is point 2, where the largest absolute value for bending moment takes place. So the maximum value of bending moment is

$$|M|_{\max} = M_{y2} = 4283 \text{ N}. \quad (26.3)$$

Estimating plate strength we shall deal only with bending moments as main external strength factor. The necessary minimal thickness can be found using next expression

$$t = \sqrt{\frac{6|M|_{\max}}{[\sigma]}} = \sqrt{\frac{6 \cdot 4283}{168 \cdot 10^6}} = 1,237 \cdot 10^{-2} \text{ m}, \quad (26.4)$$

where $[\sigma] = 168 \cdot 10^6 \text{ Pa}$ – are allowable stresses (which are taken from previous task).

So having the plate thickness $t = 1,237 \cdot 10^{-2} \text{ m} \approx 12,4 \text{ mm}$ the plate strength is provided.

Further it's necessary to find out the plate thickness from conditions of stability. The calculation scheme for stability calculation is presented on the fig. 26.1,c. If ship plate is calculated for stability we consider all edges to be on hinges constraints. Compressive uniform external loads are directed along long side for longitudinal arrangement.

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Eulerian collapse stresses for our plate can be found using next expression

$$\sigma_{eu} = 80 \left(\frac{100t}{b} \right)^2 \text{ MPa.} \quad (26.5)$$

Critical collapse stresses can be found using next expression

$$\left. \begin{array}{l} \text{if } \frac{\sigma_{eu}}{\sigma_s} < 0,6 \quad \text{then} \quad \sigma_{cr} = \sigma_{eu} \\ \text{if } 0,6 \leq \frac{\sigma_{eu}}{\sigma_s} \quad \text{then} \quad \sigma_{cr} = \sigma_s \left(1,63 - \frac{0,8}{\sqrt{\sigma_{eu}/\sigma_s}} \right) \\ \text{if } \sigma_{cr} \geq \sigma_s \quad \text{then} \quad \sigma_{cr} = \sigma_s. \end{array} \right\} \quad (26.6)$$

Critical collapse stresses don't have to exceed the allowable stresses. I.e. calculating stability we have to provide next condition

$$\sigma_{cr} \leq [\sigma]. \quad (26.7)$$

We shall calculate the stability of our plate relatively thickness t . It is necessary to have some function depending on t . Substituting (26.5) into (26.6) we can create dependency between critical collapse stresses and thickness presented below

$$\sigma_{cr}(t) = \left\{ \begin{array}{l} \text{if } \frac{\sigma_{eu}}{\sigma_s} < 0,6 \quad \text{then} \quad \sigma_{cr} = 80 \left(\frac{100t}{b} \right)^2 \quad \text{by (10.5)} \\ \text{if } 0,6 \leq \frac{\sigma_{eu}}{\sigma_s} \quad \text{then} \quad \sigma_{cr} = \sigma_s \left(1,63 - \frac{0,8\sqrt{\sigma_s}}{\sqrt{80 \left(\frac{100t}{b} \right)^2}} \right) \\ \text{if } \sigma_{cr} \geq \sigma_s \quad \text{then} \quad \sigma_{cr} = \sigma_s \end{array} \right. \quad (26.8)$$

Task #10

Substituting (26.8) into (26.7) we can form the nonlinear equation relatively thickness t . This equation is written below

$$\begin{aligned}\sigma_{cr} &\leq [\sigma] \Rightarrow \sigma_{cr}(t) - [\sigma] = 0 \\ \text{or } f(t) &= \sigma_{cr}(t) - [\sigma] = 0.\end{aligned}\quad (26.9)$$

Solving the nonlinear equation (26.9) we can find the necessary t which provide stability for given level of allowable stresses $[\sigma]$.

For example we shall use the method of "halfdivision" to find the root of (26.9). The process of calculation of the root t is presented in the table 26.1. We shall stop iteration when the difference between neighboring roots will be less than 0,5 mm.

Table 26.1. Iterations for method of half divisions to find necessary thickness from conditions of providing stability

#	t_A	t_B	t_i	$f(t_i)$	$f(t_A)$
1	0,0050	0,0150	0,0100	-43,0	-136,8
2	0,0100	0,0150	0,0125	8,8	-43,0
3	0,0100	0,0125	0,0113	-14,1	-43,0
4	0,0113	0,0125	0,0119	-2,0	-14,1
5	0,0119	0,0125	0,0122	3,5	-2,0

So we have the thickness which provide zero to (26.9) is $t_0 = 0,0122$ m. The Eulerian and critical collapse stresses for given t_0 are

$$\sigma_{eu} = 80 \left(\frac{100t}{b} \right)^2 = 80 \left(\frac{100 \cdot 0,0122}{0,8} \right)^2 = 186,1 \text{ MPa.} \quad (26.10)$$

$$\left. \begin{aligned} \frac{\sigma_{eu}}{\sigma_{cr}} &= \frac{186,1}{235} = 0,79 < 0,6 \quad \text{that's why} \\ \sigma_{cr} &= \sigma_s \left(1,63 - \frac{0,8}{\sqrt{\sigma_{eu}/\sigma_s}} \right) = \\ &= 235 \cdot \left(1,63 - \frac{0,8}{\sqrt{186,1/235}} \right) = 171,8 \text{ MPa} \end{aligned} \right\} \quad (26.11)$$

We have the condition of stability (26.9) is provided (little excess of σ_{cr} not more than 5 % is not horrible).

We have two values of thicknesses: the thickness found using strength conditions $t = 0,0124$ m and thickness found using stability conditions $t = 0,0122$ m. It's necessary to accept the largest value among these two thicknesses. For our occasion the largest thickness is $t = 0,0124$ m, which is found from conditions of strength.

Answer: to provide strength and stability of plate simultaneously the plate thickness have to be not less than $t = 0,0124$ m $\approx 12,4$ mm.

Task #11 (2 hours). Calculation of beam on elastic foundation under bending for strength using the finite difference method. Solve the task #6, but using the finite difference method (FDM). Compare numerical result got using finite difference method with accurate analytic results got in task #6.

Given: see task #6.

Solution. The initial scheme taken from the task #6 is presented on the fig. 27.1, a.

Task #11

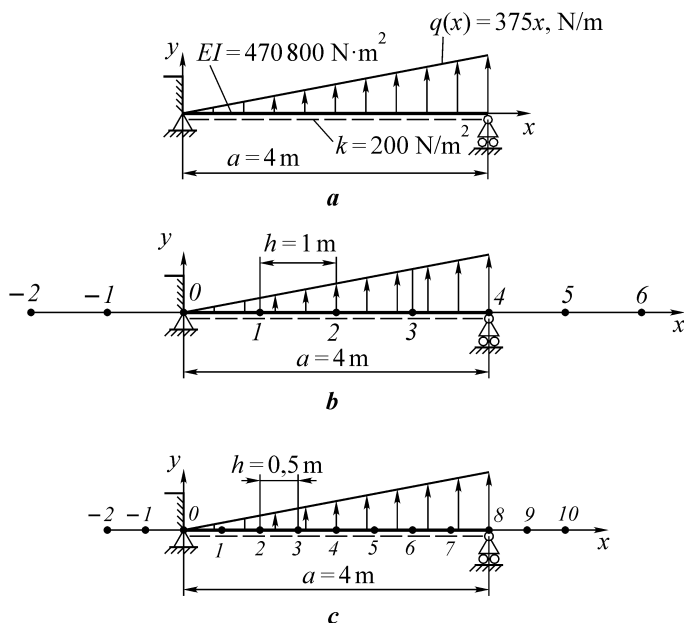


Fig. 27.1. The given beam to be calculated using FDM

Using results of the task #6 we can write down the differential equation

$$EIv^{IV}(x) + kv(x) = q(x) = \frac{q_8 x}{4}, \quad (27.1)$$

where $k = 200 \text{ N/m}^2$; $E = 2 \cdot 10^{11} \text{ Pa}$; $I = 2,354 \cdot 10^{-6} \text{ m}^4$; $q_8 = 1500 \text{ N/m}$.

Substituting numeral values in (27.1) we shall have next differential equation we shall deal with

$$470800v^{IV}(x) + 200v(x) = 375x. \quad (27.2)$$

It's necessary to create finite difference mesh. We accept this mesh to be regular (see fig. 27.1,*b*), because for regular mesh the simplest formulas can be applied. Points out of the beam: v_{-2} , v_{-1} , v_5 , v_6 are imagine but it have to be found for this method to work. The distance between points (so called step) $h = 1,0 \text{ m}$.

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We shall deal with the simplest central finite-difference formulas for derivatives written below

$$\left. \begin{aligned} f'(x_i) &= \frac{f(x_i + h) - f(x_i - h)}{2h}; \\ f''(x_i) &= \frac{f(x_i + h) - 2f(x_i) + f(x_i - h)}{h^2}; \\ f'''(x_i) &= \frac{f(x_i + 2h) - 2f(x_i + h) + 2f(x_i - h) - f(x_i - 2h)}{2h^3}; \\ f^{IV}(x_i) &= \frac{f(x_i + 2h) - 4f(x_i + h) + 6f(x_i) - 4f(x_i - h) + f(x_i - 2h)}{h^4}, \end{aligned} \right\} \quad (27.3)$$

where f is function depending on one variable; x_i – is fixed abscissa; h – is step.

As we deal with regular mesh, expressions for derivatives (27.3) can be written through neighboring function values

$$\left. \begin{aligned} v'_i &= \frac{1}{2h}(v_{i+1} - v_{i-1}); \\ v''_i &= \frac{1}{h^2}(v_{i+1} - 2v_i + v_{i-1}); \\ v'''_i &= \frac{1}{2h^3}(v_{i+2} - 2v_{i+1} + 2v_{i-1} - v_{i-2}); \\ v^{IV}_i &= \frac{1}{h^4}(v_{i+2} - 4v_{i+1} + 6v_i - 4v_{i-1} + v_{i-2}), \end{aligned} \right\} \quad (27.4)$$

where v_i – is the deflection in point i .

Using (27.4) we can write down the differential equation (27.2) for every i -th point, taking into account that $h = 1,0$ m. The algebraic system on base of (27.2) using (27.4) can be written

Task #11

$$\left. \begin{aligned} 470800(v_2 - 4v_1 + 6v_0 - 4v_{-1} + v_{-2}) + 200v_0 &= 375x_0 = 0; \\ 470800(v_3 - 4v_2 + 6v_1 - 4v_0 + v_{-1}) + 200v_1 &= 375x_1 = 375; \\ 470800(v_4 - 4v_3 + 6v_2 - 4v_1 + v_0) + 200v_2 &= 375x_2 = 750; \\ 470800(v_5 - 4v_4 + 6v_3 - 4v_2 + v_1) + 200v_3 &= 375x_3 = 1125; \\ 470800(v_6 - 4v_5 + 6v_4 - 4v_3 + v_2) + 200v_4 &= 375x_4 = 1500. \end{aligned} \right\} \quad (27.5)$$

Using (27.4) additionally boundary conditions have to be provided. Substitutions into boundary conditions are presented below

$$\left. \begin{aligned} v(0) = 0 &\Rightarrow v_0 = 0; \\ v'(0) = 0 &\Rightarrow v_1 - v_{-1} = 0; \\ EIv''(a) = 0 &\Rightarrow 470800(v_5 - 2v_4 + v_3) = 0; \\ v(a) = 0 &\Rightarrow v_4 = 0. \end{aligned} \right\} \quad (27.6)$$

Solving (27.5) and (27.6) together we can find 9 unknown deflections. These deflections are:

$$\left. \begin{aligned} v_{-2} &= 8,177 \cdot 10^{-3}; & v_1 &= 1,375 \cdot 10^{-3}; & v_4 &= 0,0; \\ v_{-1} &= 1,375 \cdot 10^{-3}; & v_2 &= 2,822 \cdot 10^{-3}; & v_5 &= -2,461 \cdot 10^{-3}; \\ v_0 &= 0,0; & v_3 &= 2,461 \cdot 10^{-3}; & v_6 &= 3,638 \cdot 10^{-4}. \end{aligned} \right\} \quad (27.7)$$

Rotation angles α , bending moments M and shear forces Q can be found using next expressions

$$\left. \begin{aligned} \alpha_i &= v'_i = \frac{1}{2h}(v_{i+1} - v_{i-1}); \\ M_i &= EIv''_i = EI \frac{1}{h^2}(v_{i+1} - 2v_i + v_{i-1}); \\ Q_i &= EIv'''_i = EI \frac{1}{2h^3}(v_{i+2} - 2v_{i+1} + 2v_{i-1} - v_{i-2}). \end{aligned} \right\} \quad (27.8)$$

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Using (27.7) and (27.8) we can write down parameters of bending for every i -th point which belong to beam and compare it with accurate analytical solutions. These bending parameters and comparisons are presented in the table 27.1.

Table 27.1. Comparisons of elements of bending for numerical and analytical solutions for mesh presented on the fig. 27.1,b

#	Elements of bending	Points within beam				
		#0	#1	#2	#3	#4
1	v_i by FDM (numerical solution)	0,0	1,375E-03	2,822E-03	2,461E-03	0,0
2	v_i by (22.12) (analytical solution)	0,0	1,015E-03	2,335E-03	2,090E-03	0,0
3	Relative difference ε_v , %	0,0	35,5	20,9	17,7	0,0
4	α_i by FDM	0,0	1,411E-03	5,428E-04	-1,411E-03	-2,461E-03
5	α_i by (22.12)	0,0	1,572E-03	7,431E-04	-1,294E-03	-2,548E-03
6	Relative difference ε_α , %	0,0	10,3	27,0	9,1	3,4
7	M_i by FDM	1294,6	34,1	-851,7	-988,1	0,0
8	M_i by (22.12)	1399,3	112,5	-799,6	-962,1	0,0
9	Relative difference ε_M , %	7,5	69,7	6,5	2,7	0,0
10	Q_i by FDM	-1260,5	-1073,2	-511,1	425,9	1738,1
11	Q_i by (22.12)	-1349,3	-1161,9	-599,8	337,3	1649,5
12	Relative difference ε_Q , %	6,6	7,6	14,8	26,3	5,4

Relative differences in the table 27.1 are estimated relatively accurate analytical solutions. Relative differences ε were found using next expressions

$$\left. \begin{aligned} \varepsilon_v &= \left| \frac{v^{\text{an}} - v^{\text{FDM}}}{v^{\text{an}}} \right| \cdot 100\%; & \varepsilon_\alpha &= \left| \frac{\alpha^{\text{an}} - \alpha^{\text{FDM}}}{\alpha^{\text{an}}} \right| \cdot 100\%; \\ \varepsilon_M &= \left| \frac{M^{\text{an}} - M^{\text{FDM}}}{M^{\text{an}}} \right| \cdot 100\%; & \varepsilon_Q &= \left| \frac{Q^{\text{an}} - Q^{\text{FDM}}}{Q^{\text{an}}} \right| \cdot 100\% \end{aligned} \right\} \quad (27.9)$$

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where $v^{\text{an}}, \alpha^{\text{an}}, M^{\text{an}}, Q^{\text{an}}$ – are analytical solutions defined by (22.12); $v^{\text{FDM}}, \alpha^{\text{FDM}}, M^{\text{FDM}}, Q^{\text{FDM}}$ – are numeric solutions found using finite difference method (FDM) explained within this task.

Analyzing results presented in the table 27.1 we may conclude that relative differences between analytic accurate solutions and numeric solutions are quite big in some points for some bending elements. This is because we took too little points. But if to take more points (see fig. 27.1,c) for regular finite difference mesh, the step of which is $h = 0,5 \text{ m}$, we can get more accurate results presented in tables 27.2 and 27.3 below. Denotations for tables 27.2 and 27.3 are the same as for the table 27.1.

Table 27.2. Comparisons of elements of bending for numerical and analytical solutions for more dense mesh for points #0 ÷ #4 (see fig. 27.1,c)

#	Elements of bending	Points within beam				
		#0	#1	#2	#3	#4
1	v_i by FDM (numerical solution)	0,0	3,645E-04	1,106E-03	1,896E-03	2,458E-03
2	v_i by (22.12) (analytical solution)	0,0	3,120E-04	1,015E-03	1,782E-03	2,335E-03
3	Relative difference ε_v , %	0,0	16,8	8,9	6,4	5,2
4	α_i by FDM	0,0	1,106E-03	1,531E-03	1,352E-03	6,920E-04
5	α_i by (22.12)	0,0	1,130E-03	1,572E-03	1,402E-03	7,431E-04
6	relative difference ε_α , %	0,0	2,1	2,6	3,6	6,9
7	M_i by FDM	1372,8	709,3	92,6	-430,3	-812,8
8	M_i by (22.12)	1399,3	732,5	112,5	-413,8	-799,6
9	Relative difference ε_M , %	1,9	3,2	17,6	4,0	1,6
10	Q_i by FDM	-1327,1	-1280,2	-1139,7	-905,4	-577,5
11	Q_i by (22.12)	-1349,3	-1302,5	-1161,9	-927,7	-599,8
12	Relative difference ε_Q , %	1,7	1,7	1,9	2,4	3,7

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Table 27.3. Comparisons of elements of bending for numerical and analytical solutions for more dense mesh for points #5 ÷ #8 (see fig. 27.1,c)

#	Elements of bending	Points within beam			
		#5	#6	#7	#8
1	v_i by FDM (numerical solution)	2,588E-03	2,183E-03	1,264E-03	0,0
2	v_i by (22.12) (analytical solution)	2,472E-03	2,090E-03	1,209E-03	0,0
3	Relative difference ε_v , %	4,7	4,5	4,5	0,0
4	α_i by FDM	-2,748E-04	-1,324E-03	-2,183E-03	-2,527E-03
5	α_i by (22.12)	-2,301E-04	-1,294E-03	-2,174E-03	-2,548E-03
6	relative difference ε_{α} , %	19,4	2,4	0,4	0,8
7	M_i by FDM	-1007,9	-968,7	-648,4	0,0
8	M_i by (22.12)	-998,0	-962,1	-645,1	0,0
9	Relative difference ε_M , %	1,0	0,7	0,5	0,0
10	Q_i by FDM	-155,9	359,5	968,7	1671,8
11	Q_i by (22.12)	-178,1	337,3	946,5	1649,5
12	Relative difference ε_Q , %	12,5	6,6	2,3	1,3

Analyzing results presented in tables 27.2, 27.3 for more dense mesh we may see some improvements for it and approaching of numerical results to accurate analytical ones.

So there is general conclusion: the denser mesh we operate with the more accurate solutions we have. The finite difference method is one of the easiest universal numeric methods which can be applied for all differential equations. It's easy for algorithm composing and

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realization on computer. That's why it was widely used in past. But also FDM has lacks: impossibility to approximate complex geometry of massive bodies, plates, shells, and curvilinear rods; we can know the function value only in node point; very huge number of solution equations, etc.

CONCLUSION

In these lectures only brief overview of the courses "The Theory of Elasticity" and "Structural Mechanics" is presented for initial acquaintance with these disciplines and simple practical tasks are shown. But these knowledge and conceptions can be used to understand more complex ideas in the structural mechanics and the theory of elasticity.

Moreover it's possibly to solve nonlinear tasks of the theory of elasticity using the linear theory of elasticity as the "superstructure". Within these work only checkout calculations for strength are presented for rod systems. But checkout calculation is only one approach for projective calculation. For projective calculations we define unknown geometrical parameters. Using these tasks as specimens and lectures it's possibly to do projective calculations for strength, rigidity etc.

More information about problems discussed here can be found in [1]–[11], particularly in textbooks [9, 2, 3].

Other important parts of the structural mechanics such as fatigue, vibration, shock, dynamic loads without which it's impossibility to design modern construction including ships are now reviewed here completely.

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