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Free Vibrations of Multilayered Orthotropic Ribbed Cylindrical Shells With Attached Solid Bodies

Authors
A.S. Kairov [1], **O. I. Vlasov** [2]
[1] Professor of Applied Mechanics and Engineering Technology Department, National University of Shipbuilding, Nikolaev, Ukraine
[2] Assistant of Applied Mechanics and Engineering Technology Department, National University of Shipbuilding, Nikolaev, Ukraine

ABSTRACT
It is considered the problem of investigation of free vibrations of elastic multilayer orthotropic cylindrical shells made of a composite material with reinforcing ribs and attached solid bodies. A more precise mathematical model of vibrations is developed in the linear equation, which takes into consideration the structural heterogeneity of the shell system. The solution is obtained by the Ritz variational method. The results of a numerical investigation of natural frequencies and free vibration forms of supported cylindrical shells are presented.

Keywords: *free vibrations, multilayer shell, structural heterogeneity, attached solids, natural frequencies and vibration modes, Ritz's variational method.*

1. INTRODUCTION

The heterogeneity of the material structure, the presence of reinforcing ribs, attached solids and their discrete arrangement all these create a local inertial heterogeneity of the shell structure and has a significant effect on its amplitude-frequency characteristics.

Vibrations of multilayer orthotropic composite shells are the subject of many studies, a review on which and the obtained results of solving specific problems are given in the works [1-7, 9-11, 13, 16]. However, the problems of vibrations of multilayer shells with reinforcing ribs and attached solid bodies have not been completely studied yet. Some tasks of this problem are mainly considered for isotropic homogeneous shells [8, 12, 14, 16]. Therefore, finding the solution of this problem has a great practical meaning and relevance.

The aim of the work is to develop a methodology for calculating the frequencies and forms of free vibrations of multilayer elastic reinforced shells with a heterogeneous structure of the material, taking into account the reinforcing ribs and attached solids.

2. FORMULATION OF THE PROBLEM

We consider free vibrations of a circular shell with constant thickness h, which consists of an arbitrary number e of rigidly connected orthotropic layers of thickness h_e $(e=\overline{1,N_0})$, reinforced with unidirectional high-strength fibers which are located at an angle ψ_e to the coordinate axes. The shell is reinforced by longitudinal and transverse ribs and carries on its surface is attached solid bodies at the points $\Theta_\upsilon(\alpha_1^{(\upsilon)},\alpha_2^{(\upsilon)})$. The mass of the attached bodies are M_υ $(\upsilon=\overline{1,Q})$. The surface of the shell is coincides with a curvilinear orthogonal coordinate system (α_1,α_2,η), where the axis α_1 is directed along the generatrix, the axis α_2 is in the peripheral direction.

The problem is considered on the basis of linear theory within the Kirghof-Lyav's hypotheses. The deformations of the constituent layers obey the generalized Hooke's law. The basic one, the middle surface of the shell is accepted, the displacement of its points is characterized by the displacement vector $u=\{u_1,u_2,u_3\}$. The deformed condition of the shell and ribs are described in the linear geometric equations [14].

Taking into account the accepted assumptions, the components of the displacement vector in the *e*-layer and the center of mass of the attached body, located at a distance η from the middle surface are expressed by the equations:

$$u_1^{(\eta)}(\alpha_1,\alpha_2,\eta) = u_1(\alpha_1,\alpha_2)+\eta\theta_1(\alpha_1,\alpha_2),$$
$$u_2^{(\eta)}(\alpha_1,\alpha_2,\eta) = u_2(\alpha_1,\alpha_2)+\eta\theta_2(\alpha_1,\alpha_2), \qquad (1)$$
$$u_3^{(\eta)}(\alpha_1,\alpha_2,\eta) = u_3(\alpha_1,\alpha_2).$$

The displacement of the gravity centers of the cross-sections of the stringers and frames is represented through the components of the displacement vector of the middle surface of the shell:

$$u_{1i}(\alpha_1) = u_1(\alpha_1,\alpha_{2i})\pm\eta_{1i}\theta_{1i}(\alpha_1,\alpha_{2i}),$$
$$u_{2i}(\alpha_1) = u_2(\alpha_1,\alpha_{2i})\pm\eta_{1i}\theta_{2i}(\alpha_1,\alpha_{2i}),$$
$$u_{3i}(\alpha_1) = u_3(\alpha_1,\alpha_{2i}),$$
$$\theta_{1i}(\alpha_1) = \theta_1(\alpha_1,\alpha_{2i}), \quad \theta_{\text{кр}1i} = \theta_2(\alpha_1,\alpha_{2i}),$$
$$u_{1j}(\alpha_2) = u_1(\alpha_{1j},\alpha_2)\pm\eta_{2j}\theta_{1j}(\alpha_{1j},\alpha_2), \qquad (2)$$
$$u_{2j}(\alpha_2) = u_2(\alpha_{1j},\alpha_2)\pm\eta_{2j}\theta_{2j}(\alpha_{1j},\alpha_2),$$
$$u_{3j}(\alpha_2) = u_3(\alpha_{1j},\alpha_2),$$
$$\theta_{2j}(\alpha_2) = \theta_2(\alpha_{1j},\alpha_2), \quad \theta_{\text{кр}2j} = \theta_1(\alpha_{1j},\alpha_2),$$

where η_{1i}, η_{2j} – eccentricity of the i-th stringer and the j-th frame; $\theta_{\text{кр}1i}, \theta_{\text{кр}2j}, \theta_{1i}, \theta_{2j}$ – angles of torsion and rotation of ribs relative to the coordinate axes; α_{2i}, α_{1j} – coordinates of the centers of gravity of the cross-sections of the ribs which are directed along the axes α_1 and α_2; θ_1, θ_2 – angles of rotation of the normal to the median surface according to the tangential coordinate axes. Signs $\pm$ correspond to cases of external and internal reinforcement.

Taking into account the linear law of the distribution of displacements along the thickness, the deformations in the *e*-layer of the shell which is under consideration, at a distance η from the middle surface, we can represent as:

$$\varepsilon_1^{(\eta)} = \varepsilon_1(\alpha_1,\alpha_2)+\eta\varepsilon_4(\alpha_1,\alpha_2),$$
$$\varepsilon_2^{(\eta)} = \varepsilon_2(\alpha_1,\alpha_2)+\eta\varepsilon_5(\alpha_1,\alpha_2), \qquad (3)$$
$$\varepsilon_3^{(\eta)} = \varepsilon_3(\alpha_1,\alpha_2)+\eta\varepsilon_6(\alpha_1,\alpha_2)$$

The potential and kinetic energies of the shell system are defined as the sum of the potential energies of elastic deformations and the kinetic energies of its components:

$$\Pi = \Pi^{(0)}+\sum_{i=1}^{I}\Pi_i^{(1)}+\sum_{j=1}^{J}\Pi_j^{(2)}, \qquad (4)$$
$$K = K^{(0)}+\sum_{i=1}^{I}K_i^{(1)}+\sum_{j=1}^{J}K_j^{(2)}+\sum_{\upsilon=1}^{Q}K_\upsilon^{(3)},$$

where $K^{(0)}$, $K_i^{(1)}$, $K_j^{(2)}$, $K_\upsilon^{(3)}$, $\Pi^{(0)}$, $\Pi_i^{(1)}$, $\Pi_j^{(2)}$ are kinetic and potential energies of the shell, *i*-th and *j*-th of supporting ribs and attached bodies.

The potential energy of the shell deformation, taking into account the dependences (1) and (3), is defined as the sum of the potential energies of the elastic deformations of its constituent layers [1]:

$$\Pi^{(0)} = \frac{1}{2}\int_{\alpha'}^{\alpha''}\int_0^{2\pi}\Big\{C_{11}\varepsilon_1^2+2C_{12}\varepsilon_1\varepsilon_2+C_{22}\varepsilon_2^2+C_{66}\varepsilon_3^2+$$
$$+2\left[K_{11}\varepsilon_1\varepsilon_4+K_{12}(\varepsilon_1\varepsilon_5+\varepsilon_2\varepsilon_4)+K_{22}\varepsilon_2\varepsilon_5+2K_{66}\varepsilon_3\varepsilon_6\right]+$$
$$+D_{11}\varepsilon_4^2+2D_{12}\varepsilon_4\varepsilon_5+D_{22}\varepsilon_5^2+D_{66}\varepsilon_6^2\Big\}A_1A_2\,\partial\alpha_1\partial\alpha_2 \qquad (5)$$

Expressions for the potential energy of elastic deformation of the *i*-th longitudinal and the *j*-th transverse ribs with the equation given in (2) are as follows:

$$\Pi^{(1)} = \frac{1}{2}\sum_{i=1}^{I}\int_{\alpha'}^{\alpha''}\left[E_{1i}\left(S_{1i}\varepsilon_{1i}^2+I_{1i}\varepsilon_{4i}^2\right)+G_{1i}I_{\text{кр}1i}\varepsilon_{6i}^2\right]A_1\,d\alpha_1, \qquad (6)$$
$$\Pi^{(2)} = \frac{1}{2}\sum_{j=1}^{J}\int_0^{2\pi}\left[E_{2j}\left(S_{2j}\varepsilon_{2j}^2+I_{2j}\varepsilon_{5j}^2\right)+G_{2j}I_{\text{кр}2j}\varepsilon_{6j}^2\right]A_{2j}\,d\alpha_2$$

The kinetic energy of the shell, the ribs and attached bodies can be represented in the form:

$$K^{(0)} = \frac{1}{2}\sum_{e=1}^{N_0}\rho_e h_e\int_{\alpha'}^{\alpha''}\int_0^{2\pi}\sum_{k=1}^{3}\left(\frac{\partial u_{ke}}{\partial t}\right)^2 A_1A_2\,d\alpha_1 d\alpha_2, \qquad (7)$$
$$K^{(1)} = \frac{1}{2}\sum_{i=1}^{I}\left\{\rho_{1i}S_{1i}\int_{\alpha'}^{\alpha''}\left[\sum_{k=1}^{3}\left(\frac{\partial u_{ki}}{\partial t}\right)^2+\frac{I_{\text{кр}1i}}{S_{1i}}\left(\frac{\partial\theta_{\text{кр}1i}}{\partial t}\right)^2\right]\right\}A_1\,d\alpha_1,$$
$$K^{(2)} = \frac{1}{2}\sum_{j=1}^{J}\left\{\rho_{2j}S_{2j}\int_0^{2\pi}\left[\sum_{k=1}^{3}\left(\frac{\partial u_{kj}}{\partial t}\right)^2+\frac{I_{\text{кр}2j}}{S_{2j}}\left(\frac{\partial\theta_{\text{кр}2j}}{\partial t}\right)^2\right]\right\}A_2 d\alpha_2$$
$$K^{(3)} = \frac{1}{2}\sum_{\upsilon=1}^{Q}M_\upsilon\sum_{k=1}^{3}\left(\frac{\partial u_{k\upsilon}}{\partial t}\right)^2$$

In equations (5) – (7) the following symbols are marked: A_1, A_2 – coefficients of the first quadratic form; u_1, u_2, u_3 – meridional, peripheral and normal displacement of the points of the middle surface

$(\eta=0)$; $\varepsilon_1, \varepsilon_2, .., \varepsilon_6$ – components of deformations of the middle surface of the shell; C_{pq}, K_{pq}, D_{pq} – generalized elastic coefficients [1, 10]; u_{ke}, $u_{k\upsilon}$ – components of the displacements of the median surfaces of the constituent layers and the centers of mass of the attached bodies ($k=\overline{1,3}$); $\varepsilon_{1i}, \varepsilon_{4i}, \varepsilon_{6i}, \varepsilon_{2j}, \varepsilon_{5j}, \varepsilon_{6j}$ – components of deformations of stringers and frames [8]; h_e, ρ_e – thickness and density of the material of the *e*-coat layer; N, Q – number of constituent layers and attached bodies; E_{1i}, G_{1i}, ρ_{1i} – modulus of elasticity, shift modulus and density of stringer material; E_{2j}, G_{2j}, ρ_{2j} – similar physical and mechanical characteristics of the frames; S_{1i}, I_{1i}, $I_{\text{кр}1i}$, $\theta_{\text{кр}1i}$ – cross-sectional area of the stringer, their moments of inertia according to the circumferential coordinate, the moments of inertia in torsion, the angle of twisting; $S_{2j}, I_{2j}, I_{\text{кр}2j}, \theta_{\text{кр}2j}$ – similar characteristics of the frames; α', α'' – coordinates of the ends of the shell; M_υ – mass of attached bodies.

3. METHOD OF SOLUTION

The problem is solved by the Ritz method. On the basis of the variational principle of Ostrogradskiy-Hamilton, the solution reduces to a variational equation $\delta Э(\alpha_1,\alpha_2)=0$, where $Э=K-\Pi$ is the Lagrange function; Π, K – potential and kinetic energy of deformation and vibrations of a heterogeneous shell system. The displacements of the middle surface of the shell are approximated by a system of basic functions

$$u_k=\sum_{m=1}^{M}\sum_{n=1}^{N}C_{mn}^{(k)}\varphi_{mn}^{(k)}(\alpha_1,\alpha_2) \qquad (k=\overline{1,3}), \tag{8}$$

where $\varphi_{mn}^{(k)}(\alpha_1,\alpha_2)=W_m^{(k)}(\alpha_1)\Psi_n^{(k)}(\alpha_2)\cos(\omega t)$ – linearly independent basic functions; $W_m^{(k)}(\alpha_1)$, $\Psi_n^{(k)}(\alpha_2)$ – coordinate functions of the arguments α_1 and α_2, satisfying the given boundary conditions on the shell contour and the periodicity conditions in the circumferential direction; M, N – number of members of the expansion in coordinates; $C_{mn}^{(k)}$ – coefficients of the eigen modes of the vibrations, which form the eigenvectors; ω is a circular frequency of natural vibrations; t – time.

The deformations of the shell and ribs are representable in the form:

$$\varepsilon_{p\upsilon}=\sum_{k=1}^{3}\sum_{m=1}^{M}\sum_{n=1}^{N}C_{mn}^{(k)}\Phi_{(\upsilon)pmn}^{(k)} \qquad (p=\overline{1,6};\ \upsilon=0,1,2) \tag{9}$$

The basic functions of deformations $\Phi_{(\upsilon)pmn}^{(k)}$ are determined by the dependences:

$$\Phi_{(0)1mn}^{(1)}=\frac{1}{A_1}\frac{\partial\varphi_{mn}^{(1)}}{d\alpha_1}, \qquad \Phi_{(0)2mn}^{(1)}=\frac{\varphi_{mn}^{(1)}}{A_1A_2}\frac{\partial A_2}{d\alpha_1},$$

$$\Phi_{(0)3mn}^{(1)}=\frac{1}{A_2}\frac{\partial\varphi_{mn}^{(1)}}{d\alpha_2}, \quad \Phi_{(0)4mn}^{(1)}=\frac{1}{A_1}\frac{\partial}{d\alpha_1}\left(\frac{\varphi_{mn}^{(1)}}{R_1}\right),$$

$$\Phi_{(0)5mn}^{(1)}=\frac{\varphi_{mn}^{(1)}}{A_1A_2R_1}\frac{\partial A_2}{d\alpha_1}, \quad \Phi_{(0)6mn}^{(1)}=\frac{1}{A_2R_1}\frac{\partial\varphi_{mn}^{(1)}}{d\alpha_2},$$

$$\Phi_{(0)1mn}^{(2)}=0, \quad \Phi_{(0)2mn}^{(2)}=\frac{1}{A_2}\frac{\partial\varphi_{mn}^{(2)}}{d\alpha_2},$$

$$\Phi_{(0)3mn}^{(2)}=\frac{A_2}{A_1}\frac{\partial}{d\alpha_1}\left(\frac{\varphi_{mn}^{(2)}}{A_2}\right), \quad \Phi_{(0)4mn}^{(2)}=0,$$

$$\Phi_{(0)5mn}^{(2)}=\frac{1}{A_2}\frac{\partial}{d\alpha_2}\left(\frac{\varphi_{mn}^{(2)}}{R_2}\right), \quad \Phi_{(0)6mn}^{(2)}=\frac{A_2}{A_1R_2}\frac{\partial}{d\alpha_1}\left(\frac{\varphi_{mn}^{(2)}}{A_2}\right),$$

$$\Phi_{(0)1mn}^{(3)}=\frac{\varphi_{mn}^{(3)}}{R_1}, \quad \Phi_{(0)2mn}^{(3)}=\frac{\varphi_{mn}^{(3)}}{R_2}, \quad \Phi_{(0)3mn}^{(3)}=0,$$

$$\Phi_{(0)4mn}^{(3)}=-\frac{1}{A_1}\frac{\partial}{d\alpha_1}\left(\frac{1}{A_1}\frac{\partial\varphi_{mn}^{(3)}}{d\alpha_1}\right),$$

$$\Phi_{(0)5mn}^{(3)}=-\frac{1}{A_2}\left(\frac{1}{A_1^2}\frac{\partial A_2}{d\alpha_1}\frac{\partial\varphi_{mn}^{(3)}}{d\alpha_1}+\frac{1}{A_2}\frac{\partial^2\varphi_{mn}^{(3)}}{d\alpha_2^2}\right),$$

$$\Phi_{(0)6mn}^{(3)}=-\frac{1}{A_1}\frac{\partial}{d\alpha_1}\left(\frac{1}{A_2}\frac{\partial\varphi_{mn}^{(3)}}{d\alpha_2}\right), \qquad \Phi_{(1)1mn}^{(1)}=\frac{1}{A_1}\frac{\partial\varphi_{mn}^{(1)}}{d\alpha_1},$$

$$\Phi_{(1)4mn}^{(1)}=\Phi_{(1)6mn}^{(1)}=0, \quad \Phi_{(1)1mn}^{(2)}=\Phi_{(1)4mn}^{(2)}=0,$$

$$\Phi_{(1)6mn}^{(2)}=\frac{1}{A_1R_1}\frac{\partial\varphi_{mn}^{(2)}}{d\alpha_1}, \tag{10}$$

$$\Phi_{(1)1mn}^{(3)}=-\frac{\eta_{1i}}{A_1}\frac{\partial}{d\alpha_1}\left(\frac{1}{A_1}\frac{\partial\varphi_{mn}^{(3)}}{d\alpha_1}\right)+\frac{\varphi_{mn}^{(3)}}{R_1},$$

$$\Phi_{(1)4mn}^{(3)}=-\frac{1}{A_1}\frac{\partial}{d\alpha_1}\left(\frac{1}{A_1}\frac{\partial\varphi_{mn}^{(3)}}{d\alpha_1}\right)-\frac{\varphi_{mn}^{(3)}}{R_1^2},$$

$$\Phi_{(1)6mn}^{(3)}=-\frac{1}{A_1}\frac{\partial}{d\alpha_1}\left(\frac{1}{A_2}\frac{\partial\varphi_{mn}^{(3)}}{d\alpha_2}\right)\left(1+\frac{\eta_{1i}}{R_1}\right),$$

$$\Phi_{(2)2mn}^{(1)}=\Phi_{(2)5mn}^{(1)}=0, \qquad \Phi_{(2)6mn}^{(1)}=\frac{1}{A_2R_2}\frac{\partial\varphi_{mn}^{(1)}}{d\alpha_2},$$

$$\Phi_{(2)2mn}^{(2)}=\frac{1}{A_2}\frac{\partial\varphi_{mn}^{(2)}}{d\alpha_2}, \quad \Phi_{(2)5mn}^{(2)}=\Phi_{(2)6mn}^{(2)}=0,$$

$$\Phi^{(3)}_{(2)2mn} = -\frac{\eta_{2j}}{A_2^2}\frac{\partial^2\varphi^{(3)}_{mn}}{d\alpha_2^2} + \frac{\varphi^{(3)}_{mn}}{R_2},$$

$$\Phi^{(3)}_{(2)5mn} = -\frac{1}{A_2^2}\frac{\partial^2\varphi^{(3)}_{mn}}{d\alpha_2^2} - \frac{\varphi^{(3)}_{mn}}{R_2^2},$$

$$\Phi^{(3)}_{(2)6mn} = -\frac{1}{A_1A_2}\frac{\partial^2\varphi^{(3)}_{mn}}{d\alpha_1 d\alpha_2}\left(1+\frac{\eta_{2j}}{R_2}\right),$$

where A_1, A_2, R_1, R_2 are the coefficients of the first quadratic form and the principal radiuses of curvature of the middle surface of the shell.

The convergence of the series (8) and (9) is understood in the sense of generalized functions of the final order. In the derivation of the basic functions (10) infinitesimal quantities are assumed to be zero. Substituting expressions (8) and (9) in the Lagrange equation and applying the Ritz procedure, from the stationarity condition of the functional $\partial Э / \partial C^{(k)}_{mn} = 0$ ($k = \overline{1,3}$; $m = \overline{1,M}$; $n = \overline{1,N}$) we obtain a resolving system of homogeneous linear algebraic equations for determining the approximate values of the eigen frequencies and vibration modes:

$$\sum_{k=1}^{3}\sum_{m=1}^{M}\sum_{n=1}^{N}\left(A^{(\zeta k)}_{mn} - \omega^2_{mn} B^{(\zeta k)}_{mn}\right)C^{(k)}_{mn} = 0 \quad (\zeta = \overline{1,3}), \qquad (11)$$

where $A^{(\zeta k)}_{mn}$, $B^{(\zeta k)}_{mn}$ – elements of stiffness matrixes and masses of the shell system [8]; $C^{(k)}_{mn}$ – elements of eigenvectors; ω_{mn} – circular frequency of natural vibrations.

In calculating the eigenvalues and vectors of the system of equations (11), the generalized Jacobi rotation method is used [15]. The algorithm for calculating the integrals during determining the elements $A^{(\zeta k)}_{mn}$ and $A^{(\zeta k)}_{mn}$ is performed numerically using Gaussian quadrature formulas.

4. NUMERICAL INVESTIGATION

Numerical studies of the free vibrations of the shell system are performed for the three-layer cylindrical shells of hybrid composites with relative geometric characteristics $L/R = 2.5$, $R/h = 125$, where R, L, h is the radius, the length and the thickness of the shell. The external layers have the same thickness, and the thickness of the middle layer is equal to the total thickness of the outer layers. The inner layer is made of fiberglass with mechanical characteristics: $E_1 = 38.6$ GPa; $E_2 = 10.1$ GPa; $G_{12} = 4.92$ GPa; $\nu_1 = 0.28$; $\nu_2 = 0.09$; $\rho_2 = 1.82\times10^3\,\mathrm{kg/m^3}$. The outer layers of the shell are made of carbon plastic with mechanical characteristics: $E_1 = 126$ GPa; $E_2 = 9.76$ GPa; $\nu_1 = 0.30$; $\nu_2 = 0.023$; $G_{12} = 5.42$ GPa; $\rho_1 = 1.56\times10^3\,\mathrm{kg/m^3}$, accordingly.

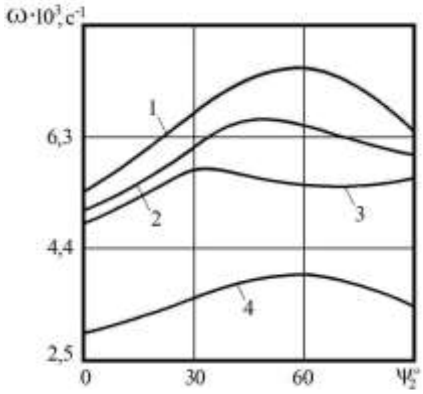

Fig. 1. Dependence of the lowest vibration frequency of a three-layer shell on the reinforcement angle ψ_2

Dependence of the lowest vibration frequency ω of the shell on the angle of reinforcement of the outer layers is shown in Fig.1. The angle of reinforcement of the middle layer is $\psi_1 = 0^\circ$. Curves 1 – 4 correspond to the boundary conditions at the ends of the shell: rigid fixing, joint rigid fixing and hinge support, hinge support and console fixation, accordingly. As it can be seen from the ratio $\omega = f(\psi_2)$ for different boundary conditions, there is an angle of reinforcement ψ_2 at which the lowest frequency of vibrations reaches its highest and lowest indicators. In this case, the corresponding forms of oscillations are also changed. With $0 \le \psi_2 \le 45^\circ$ for boundary conditions 1 to 3 the number of waves in the peripheral direction is $n = 6$, and for the console shell is $n = 5$. With $45^\circ \le \psi_2 \le 70^\circ$ for boundary conditions 2 and 3 there is a number $n = 5$; with increasing ψ_2 to 90° is $n = 4$. For boundary conditions 1 and 4, for a number $45^\circ \le \psi_2 \le 90^\circ$ $n = 5$ and 4, accordingly to boundary conditions. In this case, the number of semi-waves in the longitudinal direction is $m = 1$.

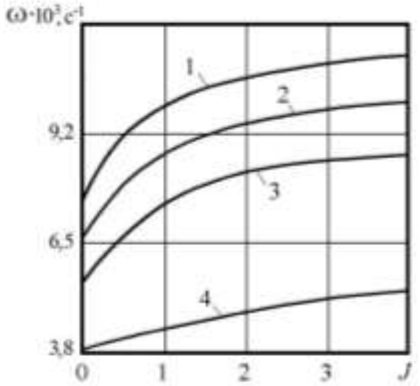

Fig. 2. Dependence of the lowest frequency of vibrations of the shell on the number of external frames

It shows the dependence of the lowest frequency of vibrations on the number of frames J for the initial shell in Fig. 2. Curves 1 – 4 correspond to the boundary conditions at the ends of the shell. The shell is reinforced from the outside by equidistant frames of 6×1 mm rectangular profile, which are made of the material of the external layers. The peripheral wave number decreases to $n=4$, but for a console fixed shell is $n=3$.

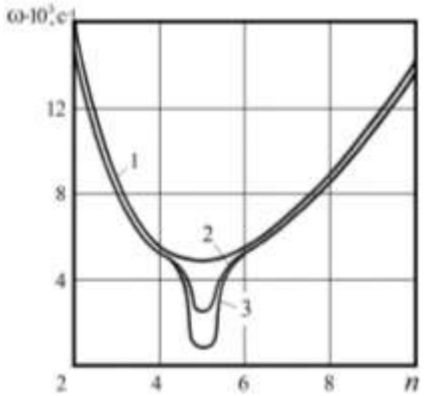

Fig. 3. Dependence of the vibrations frequency on the wave numbers n and on the mass of the attached body

In Fig. 3, for a hinge reinforced shell, the dependence of the lower frequency spectrum of vibrations on the peripheral wave numbers n for different values of the mass of the attached body is given, which is characterized by the ratio M_1/M_0, where M_1 is the mass of the body, M_0 is the mass of the shell. The attached solid is located on the external surface at the point with the coordinates $\alpha_1 = L/2$ and $\alpha_2 = 0$. Curve 1 corresponds to the shell without attached solids ($M_1/M_0 = 0$), and curves 2 and 3 correspond to the ratio $M_1/M_0 = 0.2$ and 0.4, accordinargly to the curves. The attached body exerts the greatest influence on the lower frequency of vibrations, decreasing it, which corresponds to a curved form with one semi-wave in the longitudinal direction ($m=1$). A common feature of their own forms is the localization of deformations of the bearing surface in the area of the location of the attached body and their attenuation with distance from it.

Curve 1 corresponds to the unloaded shell (M_1/M_0 = 0), and curves 2 and 3 correspond to the ratio M_1/M_0 = 0.2 and 0.4, accordinargly to the curves

5. CONCLUSIONS

The method of Ritz solved a problem of the determination of frequencies and forms of free vibrations of multilayer circular structurally heterogeneous composite cylindrical shells with attached solid bodies. The calculated mathematical model is based on the linear theory of thin elastic anisotropic shells within the Kirghof-Lyav's hypotheses.

The results of a numerical analysis of dependence of amplitude-frequency characteristics of the considered shell system for various composites and boundary conditions taking into account the structural heterogeneity have been received. It has been revealed the essential influence of the attached solid bodies, the supporting ribs and orientation of reinforcing fibers in orthotropic layers of material on values of the lowest frequencies and a form of the free vibrations.

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