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MODAL ANALYSIS OF THE MODERN SINGLE SHAFT GAS TURBINE ENGINES FLAME TUBE

МОДАЛЬНИЙ АНАЛІЗ ЖАРОВОЇ ТРУБИ СУЧАСНОГО ГАЗОТУРБІННОГО ДВИГУНА

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Abstract. *Aim of the study*. The paper is devoted to the modal analysis of the modern single shaft gas turbine engines flame tubes. The main idea of the study is to find out the flame tube of the modern single shaft gas turbine engine flame tube natural vibration modes and frequencies spectrum. *Methodology*. The flame tubes are constructed as shells with variable surface curvature. For their correct approximation a refined mathematical model on the base of Lagrange variation principle and finite elements method has been developed. It can be used for different types of marine gas turbine engines flame tubes. The developed numerical calculation algorithm gives an opportunity to take into account the presence of different holes on the flame tube surface too. Thus the flame tube natural vibration frequency and modal spectra have been found. The mathematical model reliability is confirmed by the results of experimental studies. To carry out the experiment special laser equipment, based on the laser interferometry principle has been created. The developed calculation and experimental methodologies are simple in usage and can be widespread on other constructional elements of gas turbine engines. *Results*. The influence of the flame tube elements geometry on the natural vibration modes and frequency spectrum is shown. It has been determined that for all natural vibration modes of the shell the main are bending modes, that is caused by the shell walls thickness and the presence of holes. The shell natural vibration modes and frequencies have been found. The little divergence between the calculated and experimentally found vibration frequencies proves the high reliability of the developed mathematical model, that can be used in further studies. *Scientific novelty*. The mathematical model, giving an opportunity to take into account the sharp curvature of the flame tube shell elements, has been developed. Thus natural vibration of flame tubes has been studied and found out the most dangerous vibration modes. *Practical impact*. Calculated data can be used for the proper flame tube construction development and for the further studies, concerning the flame tube forced vibration and stress-strain state. Also obtained results can be considered as initial data for the flame tube fatigue stress-strain state parameters.
Key words: shell finite elements; flame tube; experimental methodology; numerical calculations; natural vibration.

Анотація. *Мета дослідження*. Дослідження присвячено модальному аналізу жарових труб сучасних одновальних газотурбінних двигунів. Основна ідея дослідження полягає у визначенні спектру власних коливань та частот жарової труби сучасного одновального газотурбінного двигуна. *Методологія*. Жарові труби мають форму оболонок зі змінною кривизною поверхні. Для їх коректної апроксимації розроблено уточнену математичну модель на основі варіаційного принципу Лагранжа та методу скінченних елементів. Вона може бути використана для різних типів жарових труб газотурбінних двигунів. Розроблений алгоритм числового розрахунку дає можливість враховувати також наявність різних отворів на поверхні жарової труби. Таким чином, було знайдено власні частоти та форми коливань жарової труби. Адекватність розробленої математичної моделі підтверджено результатами експериментальних досліджень. Для проведення експерименту було створено спеціальне лазерне обладнання, засноване на принципі лазерної інтерферометрії. Розроблені розрахункові та експериментальні методики прості у використанні та можуть бути використані при дослідженнях інших конструктивних елементів газотурбінних двигунів. *Результати*. Показано вплив геометрії елементів жарової труби на власні коливання та спектр частот. Визначено, що для всіх форм власних коливань оболонки основними є згинально-крутильні коливання, що зумовлено товщиною стінок оболонки та наявністю отворів. Визначено власні коливання оболонки та частоти. Незначна розбіжність між розрахунковими та експериментально знайденими частотами коливань доводить високу точність розробленої математичної моделі, яка може бути використана в подальших дослідженнях. *Наукова новизна*. Розроблено математичну модель, що дає можливість враховувати значну кривизну оболонкових елементів жарової труби. Таким чином, досліджено власні коливання жарових труб та виявлено найнебезпечніші форми коливань. *Практичне значення*. Розрахункові дані можуть бути використані для удосконалення конструкцій жарових труб сучасних газотурбінних двигунів та для подальших досліджень, що стосуються вимушених коливань жарових труб та їх напружено-деформованого стану. Також отримані результати можна розглядати як вихідні дані для дослідження параметрів втомної міцності жарової труби.

Ключові слова: оболонкові скінчені елементи; жарова труба; експериментальна методологія; чисельні розрахунки; власні коливання.

INTRODUCTION

Gas turbine engines are used for the various types of ships and vessels as a part of their propulsive system. One of the main demands for such types of engines is the strong increase in their capacity. On the other hand, designers are forced to solve various problems to reach the appropriate level of engine's durability and reliability [1]. Thus, it is necessary to solve a number of problems in the area of mechanics of solid deformable body. One of the main problems in this area deals with the necessity of the turbine flame tubes vibration reduction, because the rotor impellers work under the influence of dynamic load [2].

Flame tubes of gas turbine engines (GTE) are complex thin-walled shell structures that are exposed to high temperatures, pressures and mechanical loads. As a result of these influences, the probability of vibration processes occurring increases, which can cause resonance phenomena and even premature failure of the structure. For ensuring the flame tubes vibration reliability and durability it is necessary to study their natural vibration taking into account different curvatures of their surface. The effect of the flame tube surface variable curvature remains poorly studied. On the other hand, modern numerical methods, in particular the finite element method (FEM), allow to make detailed modal analysis of solid body with complex geometry. Thus the appropriate and reliable mathematical model of the turbine flame tubes free vibration should be designed taking into account the influence of their geometry on the free vibration frequencies and modes spectra.

LITERATURE REVIEW

Since the advent of the finite element method (FEM), one of its areas of application has been strength calculations. The properties of finite element models are determined by the finite elements used. Depending on the complexity of the structures being studied, it becomes necessary to use existing or develop new finite elements. Flame tubes elements, that have various geometric shapes can be correctly described by curvilinear finite elements. Parts of this type are highly labor-intensive to manufacture, which, in turn, is due to the appearance of structural heterogeneity during their shaping. Therefore, issues related to the rational design of such structures based on the finite element method have been reflected in the works of a number of authors. Thus, in works [1, 2], the thin shells are studied using FEM, however, the disadvantage of the proposed methods is that the flame tube, and especially its cavity, cannot be correctly modeled by two-dimensional finite elements. This, in turn, reduces the reliability of the results obtained. Therefore, in a number of works [3 − 5] three-dimensional finite element models of flame tubes are used. Thus, in [6] the four-node shell-type finite elements are used, and for the shroud part of the flame tube – eight-node prismatic elements. When studying the tube conic part in work [7] eight-node prismatic elements were used for finite element approximation, and in [8] – four-node sector-type elements. Comprehensive studies of the thin shells used in turbomachines using FEM in a three-dimensional formulation were carried out in papers [9 - 12].

The influence of holes on the free vibrations of thin elastic shells, such as flame tubes, has been the subject of a relatively small number of theoretical and experimental studies, a review of which is given in [11, 13, 14]. Their analysis shows that traditional methods based on the use of variational principles [15] can only determine the lowest frequencies and vibration modes of shells with acceptable accuracy. In practice, this is insufficient, since for thin-walled structures, due to their increased sensitivity to vibrations and in connection with the increased dynamism of the acting loads, it is necessary to determine a wider part of the spectrum of natural frequencies and modes. In [3, 6, 9, 12] solutions to a number of problems of determining the fundamental vibration frequencies of smooth shells with an unsupported hole were obtained using various numerical methods. However, the nature of the influence of holes on the natural vibrations of structurally inhomogeneous shells is quite complex and has a number of characteristic features. As far as the authors know from published works, there are no analytical solutions to problems of free vibrations of shells with holes.

Taking into consideration all the above mentioned scientific works it is obvious that one of the main problems of the flame tubes vibration parameters determination is the flame tube constructional inhomogeneity and thus a necessity to use the shells of different geometry and curvature for the tube modeling.

RESEARCH METHODOLOGY

The research object is the flame tube of the modern marine gas turbine engine (fig. 1). The studied flame tube can be considered as an assembly of constructional elements that are thin shells of different geometry and curvature. Each shell has its vibration mode and frequency, caused by a gas flow, burning in the flame tube. Thus the main idea of the study is to make the modal analysis of the flame tube elements and find out their most dangerous vibration modes and frequencies. For this aim a refined mathematical model should be developed.

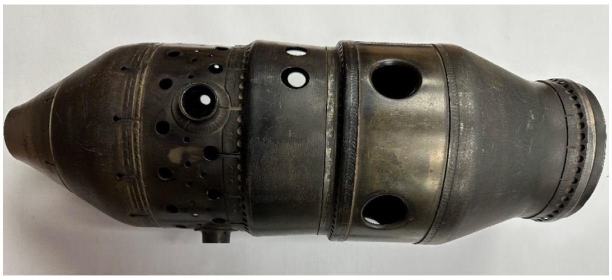


Fig. 1. The modern gas turbine engine flame tube

For the appropriate solution of the given problem several main tasks should be done:

– Develop the mathematical model of the flame tube elements vibration;

– Experimentally verify the developed mathematical model;

– Make the flame tube elements modal analysis.

Determination of the flame tube elements modes and frequencies. Taking into consideration the flame tube construction for its modal analysis the equations of the whole tube or its elements free vibration motion can be written as follows:

$$[M]\left\{\frac{d^2\delta}{dt^2}\right\}+[C]\left\{\frac{d\delta}{dt}\right\}+[K]\{\delta\}=0, \quad (1)$$

where M – mass matrix of the section; C – matrix of damping; K – stiffness matrix; δ – generalized displacement vector of the section finite elements model.

Let's do some transformations. Represent the generalized displacement vector $\{\delta\}$ as a decomposition into a Fourier row (2):

$$\delta_i = a_1 \cos k\omega t + a_2 \sin k\omega t, \quad (2)$$
$$(i = 1, 2, \ldots, n)$$

where i – is the number of the finite elements model nodes; n – is an amount of nodes; ω – is the finite element free vibration frequency.

Using the dependences (1, 2), it is possible to receive next matrix equation (3), according to which the amplitudes of the flame tube could be found:

$$[S]\{a\} = 0 \quad (3)$$

where S – is the global matrix.

From the matrix equation (3) the set of the flame tube finite elements model free vibration amplitudes could be found as follows [15]:

$$\left[K_{ij}\right]a_i - \omega^2\left[M_{ij}\right]a_i = 0. \quad (4)$$

Equation (4) represents the so called generalized proper value problem and gives us an opportunity to find the flame tube free vibration modes and frequency spectrum. In general its solution could be obtained as follows:

$$[A]x - \lambda[B]x = 0. \quad (5)$$

where A and B – are symmetric matrixes.

To solve the matrix equation (5) and we need to use the positive decomposition of the Kholetskyi numerical algorithm [13]. Thus:

$$[B]=[L][M]\left[L^{-1}\right]^T; \quad [A]=[K]=[L][L]^T, \quad (6)$$

where L – the positive triangle matrix.

Experimental scheme used for modal analysis. To verify the calculated data, experimental studies of the flame tube dynamic parameters were carried out using the method of digital electronic speckle interferometry [16]. The optical scheme of the interferometer and the block diagram of the hardware of the experimental setup are shown in Fig. 2. On the interferometric table, shell *1* is supported by soft silicone supports that practically do not affect the shapes and frequencies of resonant oscillations. Therefore, the boundary conditions of the studied model can be considered as *F–F*.

The excitation of oscillations and the tuning of resonant frequencies are set by the signal of the sound generator GZ, which is amplified in voltage by a low-frequency amplifier PNF and activates a miniature bimorph piezoelectric element BP, rigidly attached to the surface of the shell. Its influence on the rigidity and inertia of the structure can also be neglected. In the optical scheme of the interferometer proposed in [17], next to the object of study 1, there are a plane-parallel

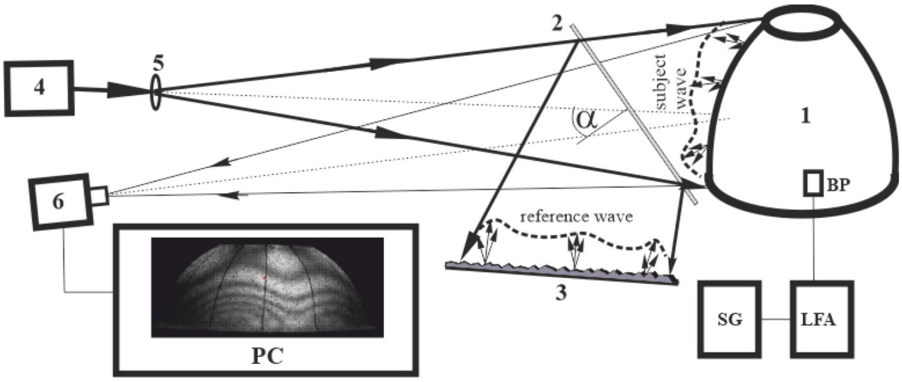


Fig. 2. Optical scheme and block diagram of the experimental stand hardware

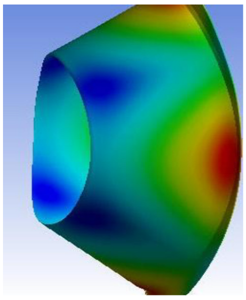
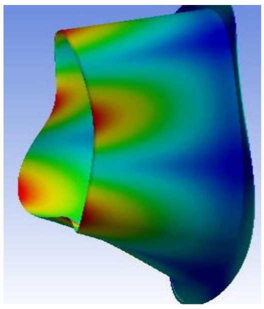
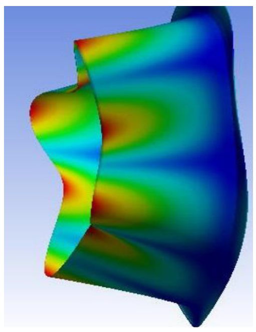
transparent plate 2, which serves as a beam splitter, and a reflective diffuser 3. Their overall parameters correspond to the dimensions of the shell. The source of coherent radiation is a single-mode DPSS laser 4 with a power of 50 mW and a wavelength of $\lambda = 532$ nm. The laser beam is expanded by the lens 5 and divided into two beams by the plate 2. The light that has passed through the divider is scattered by the object and forms an object wave, which passes through the plate 2 to the video camera 6, creating a set of light grains – object speckles on the video matrix. The laser beam reflected by the divider is scattered by the diffuser 3, forming a reference beam, is repeatedly reflected by the plate 2 in the direction of the video camera, and creates a reference speckle field on the video matrix. The intensity of the reference wave is optimized by selecting the angle α. As a result of the interference of the object and reference speckles on the video matrix, a resulting speckle field appears. Its image – a speckogram – is transmitted to the computer PC, but information about the amplitudes of oscillations of the object points is obscured by the random distribution of the phases of the light waves in the speckles. The procedure for decoding specklegrams and synthesizing speckle-interferograms of oscillatory forms was carried out by the proposed method for determining the contrast of the dynamic speckle pattern. The method involves creating the effect of «flickering» of speckles, which is realized by small low-frequency oscillations of the diffuser. The computer program analyzes the range of the brightness change of each i-th pixel of the specklegram and forms an interferogram.

RESULTS AND DISCUSSION

In order to study the practical suitability of the developed mathematical model, calculations of the flame tube elliptical shell part free vibration modes and frequencies have been held. The studied shell material is the heat-resistant alloy (material density $\rho = 8100$ kg/m^3, Young's modulus E = 1.79x106 MPa, Poisson's ratio $\upsilon = 0.3$). The diameter of the shell $D = 375$ mm.

The shell vibration modes and frequencies are given in table 1. Calculated data is experimentally verified.

Table 1. The flame tube elliptical part free vibration modes and frequencies

Free vibration mode	Free vibration frequency, Hz	
	Calculated	Experimental
$m = 0$ $n = 4$		
	534,89	508
$m = 1$ $n = 6$		
	1520,85	1490
$m = 1$ $n = 8$		
	2998,62	2886

Analysis of the data given in table 1 allows us to make a conclusion that the flame tube shell elements during their free vibration have the prevailing bending forms. The vibration frequency range is rather high and can change from two hundreds to nearly two thousands hertz for the lowest vibration forms. The reason for such significant gap is the flame tube walls thickness. But on the other hand, it can also be caused the heat exchange between the gas flow and the solid body of the flame tube.

CONCLUSIONS

In this paper the problem of the modern single shaft gas turbine engine flame tube free vibration has been studied. The problem solution is based on the specially developed refined mathematical model. The usage of this model gives the designers an opportunity to take into account the inhomogeneity of the flame tube geometric form, that can be considered as an assembly of several shells of different shape. The developed model has been experimentally verified using the specially developed experimental equipment on the base of speclinterferometry method.

It has been determined that for all free vibration modes of the shell the main are bending modes, that is caused by the shell walls thickness and the presence of holes. The shell free vibration modes and frequencies have been found. The little divergence between the calculated and experimentally found vibration frequencies proves the high reliability of the developed mathematical model, that can be used in further studies.

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