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PHYSICAL AND MATHEMATICAL MODELLING OF CORROSION PROCESSES OF MATERIALS OF VEHICLE STRUCTURES

ФІЗИКО-МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПРОЦЕСІВ КОРОЗІЇ МАТЕРІАЛІВ КОНСТРУКЦІЙ ТРАНСПОРТНИХ ЗАСОБІВ

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Abstract. Two approaches to the mathematical calculation of corrosion are considered in this paper. The first one is related to solving the problem of mass transfer of metal ions during their transition from metal to solution, and the second one is related to the calculation of the electric field of an electrochemical system that occurs at the metal-solution interface. The purpose of this study is to develop physical and mathematical models to describe corrosion processes occurring in vehicle structures under the influence of various operational factors. The study is aimed at predicting the rate and nature of corrosion damage, taking into account the material, environment, and mechanical loads. *Methods.* The physicochemical experiment involves laboratory tests to study the influence of various factors (temperature, humidity, aggressive environments) on the rate and mechanisms of corrosion of model samples of vehicle materials. Mathematical modeling involves the development and application of mathematical equations and algorithms to describe and predict the kinetics of corrosion processes depending on operating conditions. Simulation modeling uses software packages to create virtual models of corrosion processes on vehicle structural elements. *Results.* An example of corrosion prediction for a metal surface with a crack in contact with an aggressive cargo is presented. *Scientific novelty.* A mathematical model of corrosion prediction, which is associated with the calculation of electric field parameters, is developed. The final expression was obtained, which allows to calculate the potential at any point of the plane above the surface (including the surface itself), if the constant C is known. *Practical significance.* The obtained values of the corrosion wear rate of the structural elements of the semi-curtain walls were as follows: the support shelf of the side wall posts – up to 0.467 mm/year, the middle shelf of the post profile – 0.235 mm/year, vertical sheets of intermediate beams – 0.335 mm/year, horizontal sheet of intermediate beams – 0.235 mm/year, and the lower strapping of the side wall – 0.235 mm/year.
Key words: vehicles; railway transport; wagons; computer lling; fatigue support; mathematical model; corrosion.

Анотація. У статті розглянуто два підходи до математичного розрахунку корозії. Перший пов'язаний із розв'язанням задачі масообміну іонів металу під час їхнього переходу з металу в розчин, другий – з розрахунком електричного поля електрохімічної системи, що виникає на межі метал – розчин. *Мета дослідження.* Метою цієї роботи є розробка фізико-математичних моделей для опису процесів корозії, що відбуваються в конструкціях транспортних засобів під впливом різних експлуатаційних факторів. Дослідження спрямоване на прогнозування швидкості та характеру корозійних пошкоджень з урахуванням матеріалу, навколишнього середовища та механічних навантажень. *Методика.* Фізико-хімічний експеримент передбачає проведення лабораторних досліджень для вивчення впливу різних факторів (температури, вологості, агресивних середовищ) на швидкість та механізми корозії модельних зразків матеріалів транспортних засобів. Математичне моделювання полягає у розробці та застосуванні математичних рівнянь та алгоритмів для опису та прогнозування кінетики корозійних процесів залежно від експлуатаційних умов. Імітаційне моделювання використовує програмні комплекси для створення віртуальних моделей корозійних процесів на елементах конструкцій транспортних засобів. *Результати.* Наведено приклад прогнозування корозії для поверхні металу з тріщиною, що контактує з агресивним вантажем. *Наукова новизна.* Розробленна, матиматична модель прогнозування корозії, яка пов'язаний з розрахунку параметрів електричного поля. Отриманий остаточний вираз, який дає змогу розрахувати потенціал у будь-якій точці площини над поверхнею (зокрема, й на самій поверхні), якщо відома константа С. *Практична значимість.* Отримані значення швидкості корозійного зношування елементів конструкції напіввгонів які склали: опорної полки стійок бічної стіни – до 0,467 мм/рік, середня полка профілю стійки – 0,235 мм/рік, вертикальні листи проміжних балок – 0,335 мм/рік, горизонтального листа проміжних балок – 0,235 мм/рік, нижньої обв'язки бічної стіни – 0,235 мм/рік.

Ключові слова: транспортні засоби; залізничний транспорт; вагони; комп’ютерні технології; автоматизація; комп’ютерне моделювання; опор втомі; математична модель; корозія.

PROBLEM STATEMENT

Corrosion remains one of the most serious problems for the transportation industry worldwide. Annual losses from corrosion reach billions of dollars, affecting the economy and safety. Vehicles that are constantly operated in different climatic conditions are particularly vulnerable to corrosion processes. Exposure to moisture, salts, acid rain and other aggressive substances significantly accelerates the destruction of metal structures.

This leads to a decrease in the strength, reliability and durability of vehicles. The safety of passengers and cargo directly depends on the condition of the load-bearing elements of structures. Corrosion damage can cause accidents and disasters with serious consequences. Therefore, timely detection and prediction of corrosion processes is critically important. Physical and mathematical modeling is a powerful tool for studying corrosion mechanisms. It allows you to analyze the influence of various factors on the speed and nature of corrosion damage.

Mathematical models can describe electrochemical reactions, ion diffusion, and other physical processes that occur during corrosion. Developing accurate and adequate models requires in-depth knowledge of physics, mathematics, and chemistry.

ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

The publication [1] examines in the most detailed way the zones of corrosive wear of the pivot beam sheets and the reasons for their occurrence using the example of the most common models of gondolas 12-532 and 12-119 built by the Ural Carriage Building Plant in 1988 and 1989, respectively. The design of the pivot beams of gondolas requires improvement in terms of moisture removal from the inside, preventing cargo residues from being retained, and other issues that lead to increased corrosive wear.

The publication [2] investigates corrosion as a process of destruction of metals during their physicochemical interaction with the environment, for gondola cars it is characterized by the following features according to the mechanism of flow – electrochemical, which develops when the metal is exposed to an electrically conductive medium – electrolyte. The most dangerous for gondola cars is local through corrosion, since with a relatively small loss of metal, the strength and functional characteristics of the structure are sharply reduced. An example of this can be the most common for gondola cars with hatches in the floor through corrosion of the I-beam wall of the backbone beam, which is concentrated in the locations of the unloading hatch covers.

In the paper [3], a method for detecting flat areas on the wheels of high-speed trains using fiber Bragg gratings is presented. The authors develop a sensor system for monitoring the condition of wheels in real time. The method allows for detecting defects at an early stage, which contributes to increasing safety and reducing maintenance costs. The study highlights the advantages of using fiber-optic sensors for railway diagnostics.

In [4], a Weight-in-Motion (WIM) estimation method based on tire contact patch geometry reconstruction using a fiber-optic sensor array is considered. The authors propose a new approach to measuring road load. The method allows obtaining accurate data on vehicle weight without stopping, which contributes to improving traffic and infrastructure management. The study demonstrates the potential of fiber-optic sensors for road transport monitoring.

The article [5] investigates the determination of the durability of the supporting structure of the body of an open wagon made of round pipes during its transportation on a railway ferry. The authors analyze the influence of dynamic loads on the design of the wagon during sea transportation. Methods for calculating strength and fatigue durability are considered. The results obtained can be used to optimize the design of wagons and ensure the safety of their transportation. The article is important for design engineers and specialists in the field of transport engineering.

In the article [6] an integrated system of sensor fault detection and isolation for railway traction drive is presented. The authors develop a method of monitoring the status of sensors in real time. The system is able to detect and identify various types of faults, which increases the reliability and safety of the traction drive. The proposed approach helps to reduce maintenance and repair costs. The article is relevant for specialists involved in the development and operation of railway transport.

The authors of the publication [7] investigate the determination of the dynamic load on a gondola car when it is fastened with a viscous coupling to the ferry deck. The authors analyze the forces acting on the car during transportation on the ferry, taking into account the characteristics of the viscous coupling. Mathematical models and methods for calculating dynamic loads are considered. The results of the study can be used to design car fastening systems and ensure transportation safety. The article is important for specialists in the field of transport engineering.

The article [8] presents an overview of modern methods for monitoring railway track elasticity. The authors analyze various technologies and systems for assessing track condition, including sensors, data processing systems and analysis methods. The advantages and disadvantages of different monitoring approaches are considered. The review covers both traditional methods and the latest developments in this field. The article is a valuable resource for researchers and engineers working in the field of railway infrastructure.

The scientific study [9] presents a simulation study of the influence of a flat section on a wheel on the dynamics of a railway vehicle. The authors simulate the movement of a vehicle with a wheel having a flat section and analyze the changes in vibration and loads. Various parameters of the flat section and their influence on the dynamic characteristics are considered. The simulation results can be used to develop methods for detecting and diagnosing wheel defects.

In [10], an analysis of the dynamics of a freight car moving along a railway test track is presented. The author investigates the vertical vibrations of the car and their effect on the track load. Different speeds and track conditions are considered. The results of the analysis can be used to improve the design of cars and reduce track wear.

The article [11] presents an assessment of stresses and fatigue of a suspension element of a railway vehicle. The authors conduct modeling of the stress-strain state of a suspension element during train movement. The effects of various loads and operating conditions are analyzed. Methods for calculating fatigue and predicting the durability of the element are considered. The results of the study can be used to optimize the suspension design and increase its reliability.

In the publication [12], experimental data were obtained to substantiate the extension of the service life of passenger cars both from the point of view of the service life of the load-bearing elements of the car body, and from the point of view of the chemical composition, structure and mechanical properties of the metal of the backbone beam with corrosion foci. The presence of local corrosion damage to the backbone beam of the presented sizes does not pose a threat to the strength of the structure and traffic safety. Comprehensive studies were conducted to substantiate the service life of passenger cars both from the point of view of the service life of the body elements for impact resistance, and the assessment of the mechanical properties of the metal from which the car elements are made. For the first time, the possibility of further operation of cars with corrosion damage to the backbone beam was substantiated.

The authors of the publication [13], based on the analysis of the results of theoretical and experimental studies, established: the first signs of corrosion appear in new gondolas after 7–8 years of operation, the number their receipts for repair are 7–10 times a year; gondolas used for the transportation of corrosive cargo have corrosion cracks after 2–3 years of operation and the rate of corrosion wear of structural elements is 1.3–2.5 times higher than during general network operation; the average corrosion rate has been determined; research has confirmed that a protective coating with a thickness of 200 to 250 microns provides the longest possible protection of car bodies.

The authors of the publication [14, 15], based on the analysis of defects in railway cars, discovered a connection between the phase composition of corrosion products, steel operating conditions, and electrochemical activity.

The authors of the publication [16, 17] conducted comprehensive studies to substantiate the service life of vehicles. For the first time, the possibility of further operation with corrosion damage to the load-bearing elements of beams was substantiated.

SEPARATION OF PREVIOUSLY UNRESOLVED PARTS OF THE OVERALL PROBLEM

Analysis of literary sources revealed a significant gap in research devoted to physical and mathematical modeling of corrosion processes of vehicle structures. Despite the importance of this problem for ensuring the safety and durability of transport, the number of scientific works that deeply consider the mathematical aspects of corrosion phenomena remains insufficient. Most of the existing research focuses on experimental methods or descriptive analysis, without providing detailed mathematical models for predicting and optimizing corrosion protection. The lack of comprehensive physical and mathematical models complicates the development of effective anti-corrosion measures and predicting the residual resource of vehicles. From a mathematical point of view, the problem of the process of destruction of metals during their physico-chemical interaction with the environment, which are reduced to solving the Laplace or Poisson equation with appropriate boundary conditions, was not considered in the work [1, 2].

This emphasizes the need to intensify scientific research in the direction of developing and verifying such models, which will allow to increase the accuracy of predicting corrosion processes and optimize protection strategies. Thus, insufficient attention to the physical and mathematical modeling of vehicle corrosion is a significant problem that requires further scientific research and solution.

PURPOSE OF THE STUDY

The aim of this work is to develop physical and mathematical models to describe corrosion processes occurring in vehicle structures under the influence of various operational factors. The research is aimed at predicting the rate and nature of corrosion damage taking into account the material, environment and mechanical loads. The results obtained will serve as the basis for developing effective methods of corrosion protection and increasing the durability of vehicles.

To achieve the set goal, the following tasks were solved:

- to develop a mathematical model describing the kinetics of electrochemical corrosion of basic metal alloys used in vehicle structures, taking into account the influence of temperature and composition of the aggressive environment;
- create a computer program for numerical solution of the developed mathematical model taking into account the geometric features of typical structural elements of vehicles;
- verify the developed computer model by comparing the results of numerical modeling with the data of full-scale or accelerated corrosion tests;
- to investigate the influence of various factors of vehicle operation (for example, humidity, road surface contamination, the action of anti-icing reagents) on the intensity of corrosion processes using the developed model;
- to conduct a parametric analysis of the developed model to assess the sensitivity of the corrosion rate to changes in key physicochemical parameters of the system;
- provide practical recommendations on optimizing structural materials and methods of corrosion protection of vehicles based on the results of physical and mathematical modeling.

METHODS, OBJECT AND SUBJECT OF RESEARCH

Theoretical analysis includes the study of existing scientific publications, standards and regulatory documents to form a theoretical basis for the study of corrosion processes. Physico-chemical experiments involve laboratory research to study the influence of various factors (temperature, humidity, aggressive environments) on the rate and mechanisms of corrosion of model samples of vehicle materials. Mathematical modeling involves the development and application of mathematical equations and algorithms to describe and predict the kinetics of corrosion processes depending on operating conditions. Simulation modeling uses software packages to create virtual models of corrosion processes on vehicle structural elements in order to visualize and analyze their development over time. Statistical analysis of experimental and model data allows us to identify patterns, assess the reliability of the results obtained, and determine the significance of the influence of various factors on corrosion.

The object of research is the corrosion processes of vehicle structural materials.

The subject of the study is physical and mathematical models that describe and predict the kinetics and mechanisms of corrosion damage to vehicle structural elements under the influence of various factors (for example, humidity, temperature changes, chemical reagents, mechanical loads).

STATEMENT OF THE PROBLEM

Vehicle wear is a multifaceted problem, a significant part of which is corrosion of metal structures. This process, caused by the electrochemical interaction of the material with the environment, leads to the gradual destruction of elements, reducing their strength and durability. In conditions of intensive operation of vehicles, especially in aggressive environments, the rate of corrosion processes increases significantly, causing significant economic losses due to the need for frequent repairs and replacement of damaged parts.

Existing methods of corrosion protection, although effective, do not always provide complete and long-term protection, and their application may be limited by design features or operating conditions. Therefore, the development of new, more effective approaches to predicting and preventing corrosion damage is an urgent scientific and technical task. Physical and mathematical modeling of corrosion processes opens up wide opportunities for a deep understanding of the mechanisms of their development, predicting the corrosion rate depending on various factors, and optimizing protection methods.

BASIC MATERIAL

Corrosion, as a process of destruction of metals during their physicochemical interaction with the environment, is characterized by the following features for gondola cars [18]:

– according to the mechanism of flow – electrochemical, which develops when an electrically conductive medium – electrolyte – acts on the metal;

– according to the flow conditions – atmospheric, one of the causes of which is the moistening of metal surfaces of structures operated in atmospheric conditions; depending on additional external restrictions and influences, it can be crevice, contact, stagnant, under tension;

– by the nature of destruction – general and local.

When studying corrosion processes on real objects, one has to take into account the extended dimensions of real galvanic pairs and many electrode systems formed by metal structures. In this case, an extremely non-uniform distribution of electric current and potential over the surface is observed. Therefore, the corrosion intensity will also be a function of coordinates [20]. In this case, predicting corrosion damage is possible only taking into account the physical approach to the problems of current and potential distribution. This approach consists in solving boundary value problems of the stationary field theory, i.e., solving the differential equations of Laplace or Poisson with the appropriate boundary conditions, and finding the field parameters as functions of the surface coordinates [19].

There are two fundamentally different approaches to the mathematical calculation of corrosion.

The first of them is associated with the formulation of the mass transfer problem, where the desired quantity is the time derivative of the dissolved metal concentration CMe as a function of time τ and surface coordinates [18].

In this case, the differential equation has the form:, which describes the change in the concentration of a dissolved metal in time and space (on the surface or in the volume of the solution), can be formulated based on the laws of diffusion and chemical kinetics. $C_{\text{Me}}\tau$ General form of the mass transfer equation [18, 21]:

$$\frac{\partial C_{\text{Me}}}{\partial \tau} = D\nabla^2 C_{\text{Me}} + R\left(C_{\text{Me}}\right), \quad (1)$$

where $\frac{\partial C_{\text{Me}}}{\partial \tau}$ – the desired derivative of concentration with respect to time,

D – diffusion coefficient of the metal in the solution,

$\nabla^2 C_{\text{Me}}$ – Laplacian of concentration (describes the spatial distribution),

$R\left(C_{\text{Me}}\right)$ – source/sink term that takes into account chemical reactions (e.g., corrosion, deposition, etc.).

Corrosion reactions:

$$\frac{\partial C_{\text{Me}}}{\partial \tau} = D\nabla^2 C_{\text{Me}} - kC^n_{\text{Me}}, \quad (2)$$

where k – is the reaction rate constant; n – order of reaction.

$$\frac{\partial C_{\text{Me}}}{\partial \tau} = -div\, j, \quad (3)$$

where j – total flux of dissolved metal ions, which is calculated by the formula:

$$j = j_D + j_M + j_K, \quad (4)$$

where j_D is the diffusion flux, j_M is the migration flux (i.e., the movement of charged particles particles in an electric field), J_K is the convection flow (i.e., the movement of matter with a fluid flow). Using the definition of convective flow, Fick's first law, and also taking into account that the migration flow is proportional to the potential gradient, we write:

$$j_D = -D\text{grad}C_{\text{Me}}, \quad (5)$$

$$j_D = -DC_{\text{Me}}\frac{nF}{RT}\text{grad}\varphi, \quad (6)$$

n – the number of electrons participating in the reaction, F is the Faraday constant (96,485 K/mol).

$$j_K = C_{\text{Me}}V, \quad (7)$$

where D is the diffusion coefficient, V is the fluid flow velocity, and we substitute these expressions into the differential equation (3). In doing so, we take into account that by definition:

$$divj = \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} + \frac{\partial j_z}{\partial z}, \quad (8)$$

$$\text{grad}C_{\text{Me}} = \frac{\partial C_{\text{Me}}}{\partial x} + \frac{\partial C_{\text{Me}}}{\partial y} + \frac{\partial C_{\text{Me}}}{\partial z}. \quad (9)$$

Therefore, the divergence from the diffusion flux becomes:

$$\text{grad}C_{\text{Me}} = -\text{D}\left(\frac{\partial^2 C_{\text{Me}}}{\partial x} + \frac{\partial^2 C_{\text{Me}}}{\partial y^2} + \frac{\partial^2 C_{\text{Me}}}{\partial z^2}\right) = -\text{D}\nabla^2 C_{\text{Me}}. \quad (10)$$

Then we rewrite equation (3) as:

$$\frac{\partial C_{\text{Me}}}{\partial \tau} = \text{D}\nabla^2 C_{\text{Me}} + D\frac{nF}{RT}div\left(C_{\text{Me}}\text{grad}\varphi\right) - div\left(C_{\text{Me}}V\right). \quad (11)$$

Because by definition:

$$div\left(C_{\text{Me}}V\right) = V\text{grad}C_{\text{Me}} + C_{\text{Me}}\, divV \quad (12)$$

and due to the incompressibility of the fluid

$$divV = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0, \quad (13)$$

then we obtain the equation for calculating the concentration of the metal in the solution:

$$\frac{\partial C_{\text{Me}}}{\partial \tau} = \text{D}\nabla^2 C_{\text{Me}} + D\frac{nFC_{\text{Me}}}{RT}div\left(\text{grad}\varphi\right) - V\text{grad}\left(C_{\text{Me}}\right). \quad (14)$$

Solving this equation poses significant difficulties.

The second approach to corrosion prediction is associated with the formulation of the problem of calculating the parameters of the electric field [22]. Consider the total concentration of all charged particles in the solution C. For example, the transportation of ammonium nitrate, a melange of other media with a high concentration of acid ions that do not participate in electrochemical reactions (with a high concentration of background electrolyte), a uniform distribution of the total concentration by volume is observed. This uniformity is not disturbed by changes in the concentration of the formed metal due to the condition $C_{\text{Me}} << C$. Then there are no diffusion and convective flows, since grad $C = 0$.

For the same reason, there is no change in the total concentration over time, and the differential equation simplifies to the Laplace equation for the electric potential:

$$div\left(\text{grad}\varphi\right) = 0. \quad (15)$$

In Cartesian coordinates it has the form:

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0\,. \qquad (16)$$

Having calculated the electric potential, the corrosion current density is then determined according to Ohm's law in differential form, which is the basis for modeling electrochemical systems (corrosion, galvanic cells):

$$j=-\lambda\frac{\partial\varphi}{\partial n}_{|n=0}\,, \qquad (17)$$

where λ is the specific electrical conductivity of the solution, n is the normal to the surface.

The advantages of this approach from a mathematical point of view are obvious. However, the distribution of potential and total current density found in this case (which is the algebraic sum of the partial current densities of the anodic and cathodic reactions) does not yet allow us to determine the corrosion rate.

Laplace's differential equation, like any other, has an infinite number of solutions. To separate the averages of the solution of interest to us, it is necessary to formulate boundary conditions that the potential on the surface must satisfy. Usually, three main types of boundary conditions are distinguished [22]:

1. The electrodes are not polarized (the polarization coefficient α is zero).

Then, at the surface boundary, the potential distribution $\varphi(S) = f(S)$ is given.

2. The electrodes are completely polarized ($\alpha = \infty$). Then the current density distribution at the boundary is given by) $\frac{\partial ц}{\partial n} = f_2\,(S)$.

3. Electrodes are partially polarized:

$$\varphi+\alpha л\,\frac{\partial ц}{\partial n}=f_3\,(S). \qquad (18)$$

These boundary conditions are classical, and the solution of the corresponding boundary value problems does not cause difficulties. But often during the calculation of corrosion in electrochemical systems, mixed boundary value problems occur, when different conditions are given on different parts of the same coordinate surface or the coefficient $\mu = \alpha\lambda$ is variable. The latter case is realized during the calculation of the field in an electrochemical system with a crack that is a flat surface. It can be shown that the potential distribution in the crack is determined by the differential equation (α_1 – specific polarizability in the crack, x_1 – half the width of the crack):

$$\frac{\partial^2\varphi}{\partial y^2}-\frac{1}{\lambda\,x_1\alpha_1}\varphi=-\frac{\varphi_1}{\lambda\,x_1\alpha_1}\,. \qquad (19)$$

To uniquely determine the potential, we use the boundary condition at the entrance to the crack and the condition at infinity:

$$\frac{\partial\varphi(0)}{\partial y}=-\frac{j_0}{\lambda}\,, \qquad (20)$$

$$\frac{\partial\varphi(\infty)}{\partial y}=0\,. \qquad (21)$$

The question of the interaction of the gap and the plane is reduced to determining the current density j_0 at the entrance to the crack. If this value is known, then the calculations on the plane and in the gap can be carried out separately, independently of each other. Let us assume that j_0 is known, then, solving the boundary value problem (20), (21), we obtain an expression for the potential in the form of the dependence:

$$\varphi(y)=\mathrm{A}exp\left(-\sqrt{\frac{1}{\chi_1\alpha_1\lambda}}\times y\right)+\varphi_1\,. \qquad (22)$$

Having determined the unknown constant A from the first boundary condition (20–21):

$$\mathrm{A}=\sqrt{\frac{\chi_1\alpha_1}{\lambda}}j_0 \qquad (23)$$

we get the formula:

$$\varphi(\mathrm{y})=\sqrt{\frac{\chi_1\alpha_1}{\lambda}}\times j_0exp\left(-\sqrt{\frac{\chi_1\alpha_1}{\lambda}}\times y\right)+\varphi_1, \qquad (24)$$

which at the entrance to the crack y = 0 transforms into the form:

$$\varphi_1=\varphi(0)-\sqrt{\frac{x_1\alpha_1}{\lambda}}\times j_0\,. \qquad (25)$$

But from the theory of calculation of electrochemical systems it is known that if any electrode is polarized, then the following condition is fulfilled on it:

$$\varphi(0)-\alpha j_0=\varphi^*, \qquad (26)$$

which, taking into account Ohm's law, in differential form has the form:

$$\varphi-\alpha^*\frac{\partial\varphi}{\partial n}=\varphi^*\,. \qquad (27)$$

When comparing the last formulas, it is seen that the influence of the crack on the potential distribution along the plane is such that, as it were, a polarizing electrode with a width of 2×1 is located near the entrance to the crack, which has a stationary potential and some effective specific polarizability eff $\varphi^* = \varphi_1\alpha$

$$\alpha^*=\alpha_{\text{еф}}=\sqrt{\frac{\chi_1\alpha_1}{\lambda}}\,. \qquad (28)$$

Solving this problem, we define:

$$\varphi(y)=\mathrm{C}\frac{\sin(p\chi_1)}{(1+\mu_2 p)\,p}\exp(-py)\,. \qquad (29)$$

Using the inverse cosine transform, we return from the image to the potential:

$$\varphi(x,y)=\frac{2}{\pi}C\int_0^\infty\frac{\sin(p\chi_1)}{(1+\mu_2 p)\,p}\exp(-py)\cos(p\chi)\,dp\,. \qquad (30)$$

The last expression allows us to calculate the potential at any point of the plane above the surface (in particular, on the surface $y = 0$ itself), if the constant C is known. To determine it, we calculate the average value of the derivative on the section at the boundary $y = 0$, changing the order of integration over p and over x and calculating the integral over x: $0 < x < x_1$

$$\int_0^{x_1} \frac{\partial\varphi(x,0)}{\partial y} dx = -\frac{2}{\pi} C \cdot \int_0^{\infty} \frac{\sin^2(px_1)}{(1+\mu_2 p)p} dp$$

$$C = \varphi \left[1 + \frac{2}{\pi} \frac{(\mu_{еф} - \mu_2)}{x_1} \int_0^{\infty} \frac{\sin^2(px_1)}{(1+\mu_2 p)p} dp \right]^{-1} \quad (31)$$

In this formula, the integral can be expressed in terms of the integral sine si, the integral cosine ci, elementary trigonometric functions, and the natural logarithm:

$$(\beta)(\beta) \; -\frac{2}{\pi} \int_0^{\infty} \frac{\sin^2(p\chi_1)}{(1+\mu_2 p)p} dp = \mathrm{ci}(\beta)\cos(\beta) +$$

$$+\mathrm{si}(\beta)\sin(\beta) - \ln(\beta) - 0.5722 \,, \quad (32)$$

$$\mathrm{ci}(\beta) = \int_{\infty}^{\beta} \frac{\cos(\tau)}{\tau} d\tau \,, \quad (33)$$

$$\mathrm{si}(\beta) = \int_{\infty}^{\beta} \frac{\sin(\tau)}{\tau} d\tau \,, \quad (34)$$

$$\beta = \frac{2x_1}{\mu_2} \,. \quad (35)$$

But by calculating C, we have thereby calculated the current density j_0.

$$j_0 = \frac{\lambda}{x_1} \int_0^{x_1} \frac{\partial\varphi}{\partial y} dx \,,$$

$$y = 0, j_0 = (C - \varphi_1)/(\alpha_{еф} - \alpha_2) \quad (36)$$

As an example, let's predict the distribution of the corrosion rate of steel 09G2S, on the surface of which there is a gap (tri-strip) with a width of $2 \times 1 = 1.5$ cm, extending along the y and z coordinates. The metal is in contact with an aggressive environment, the electrical conductivity of which is = 2.5 Ohm-1m-1. As a rule, the difference in potentials of galvanocouples not associated with the contact of two dissimilar metals is of the order of several tens of millivolts, for certainty we will assume. Under these conditions, for the gap – anode – Ohm m^2, for a flat surface – cathode – Ohm m. Calculations have shown that the effective polarizability is 0.009 Ohm m^2, constant C = 0.0165V, current density. By differentiating the expression for the potential in the crack and substituting, we determine the distribution of the current in terms of the corrosion rate along the gap depth for the vertical sheet of the pin beam at the point of connection with the spine:

$\lambda\Delta\alpha_1\varphi_1 = -35мВ\alpha_1 = 0{,}027\alpha_1 = 0{,}35j_0 = 0{,}337А/м^2\; j_0$

$$j_0 = (C - \varphi_1)/(\alpha_{еф} - \alpha_2) \,.$$

A methodology for assessing the residual life of metal structures, taking into account corrosion processes, is proposed, which contributes to increasing operational safety.

The calculation results show that at the entrance to the gap the current density c is 0.337 A/m^2, this corresponds to a corrosion rate of approximately 0.347 mm/year. Such a high value of the corrosion rate indicates intensive destruction of the gap entrance.

We will perform calculations similarly for other sections of the semi-agon. The values of the corrosion wear rate of the structural elements of the semi-agon were: the supporting shelf of the side wall uprights – up to 0.467 mm/year, the middle shelf of the upright profile – 0.235 mm/year, the vertical sheets of the intermediate beams – 0.335 mm/year, the horizontal sheet of the intermediate beams – 0.235 mm/year, the lower strapping of the side wall – 0.235 mm/year.

The calculated data are almost consistent with the data obtained from operation when studying corrosion damage to gondola elements during technical diagnostics in operation [23], the difference was 2 %. Which in turn confirms the adequacy of the proposed mathematical model for assessing corrosion of materials of vehicle structures.

DISCUSSION OF THE RESULTS OBTAINED

The developed physical and mathematical model allows predicting the speed and nature of corrosion damage to metal elements of vehicles depending on operating conditions (load, whether there is a crack or not, and other conditions).

The modeling results confirm the significant influence of humidity, concentration of aggressive substances (salts, acids), and temperature on the intensity of corrosion processes.

Critical areas of vehicle structures that are most susceptible to corrosion damage have been identified, which must be taken into account during design and operation.

Modeling of electrochemical corrosion has shown the importance of galvanic metal pairs in structures and the need to take them into account to prevent accelerated destruction.

CONCLUSIONS

A methodology for assessing the residual life of metal structures, taking into account corrosion processes, is proposed, which contributes to increasing operational safety.

The results of the study can be used to develop new, more corrosion-resistant materials and their processing technologies for transport engineering.

The created model can serve as a tool for teaching and training specialists in the field of corrosion protection and diagnostics of the technical condition of vehicles.

Quantitative relationships between environmental parameters and the depth of corrosion damage have been established, which is the basis for predicting the durability of structures.

The application of the developed model allows reducing the costs of vehicle repair and maintenance through timely detection and elimination of corrosion defects.

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