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# RESEARCH OF THE OPERATION OF THE SCREW PROPELLER ON THE BASIS OF THE NONSTATIONARY THEORY OF THE LIFTING SURFACE

## ДОСЛІДЖЕННЯ РОБОТИ ГРЕБНОГО ГВИНТА НА ОСНОВІ НЕСТАЦІОНАРНОЇ ТЕОРІЇ НЕСУЧОЇ ПОВЕРХНІ

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**Abstract.** A numerical model marine propeller at oblique flow, as based on unsteady lifting surface to applied Vortex Lattice Method (VLM) are discussed. The propeller's blades vortices and free trailing vortex sheet was replaced on vortex loops. Because the propeller's blades an oblique flow moves unsteady, then result thrust, torque and lateral force are varying in angle of rotation of the propeller. Lateral force has same direction as the lateral velocity component. In order to calculate, estimate degree of influence on thrust, torque and lateral force propeller at oblique flow. According to the proposed mathematical model (MM), has been studied numerically acting oblique flow at various angles flow. There is a noticeable increase propeller thrust coefficient and lateral force coefficient with increasing bevel angle, however, the change propeller torque coefficient is insignificant. The results calculation hydrodynamic forces for different angels flow at every instantaneous time are compared with works others authors. A mathematical model of a propeller operating in oblique flow, based on an unsteady lifting surface, allows one to estimate the magnitude of the variables per revolution of forces acting both on the propeller blade and on the screw as a whole. This MM can be used as a tool to study the hydrody-namics of propellers operating in oblique flow.

**Keywords:** marine propeller; oblique flow; unsteady lifting surface; Vortex Lattice Method; vortex loops; hydrodynamic forces.

**Анотація.** Наведена математична модель (ММ) гребного гвинта в косому потоці, яка побудована на основі нестаціонарної теорії несучої поверхні з використанням методу дискретних вихорів. Вихрова система гребного гвинта включає приєднані, замкнуті вихрові контури, які моделюють лопаті і розподіляються дискретно по гвинтових поверхнях з кроком, що враховує режим помірного навантаження, тілесність лопаті і поперечну компоненту швидкості. Пелена вільних вихорів заміщається замкнутими вихровими контурами (рамками), в кожен момент часу, що сходять услід, за гвинтом. При визначенні в кожному кутовому положенні лопаті циркуляцій нестаціонарних приєднаних вихрових рамок і вільних вихорів, виконувалися граничні умови на поверхні лопаті, дотримувалися, замкнутість вихрових систем і гіпотеза Чаплигіна-Жуковського в точках сходу вихрових завіс. Система лінійних алгебраїчних рівнянь (СЛАР), складена на основі, умови непроникності поверхні лопаті гребного гвинта, дозволила знайти циркуляції приєднаних вихорів в залежності від кутового положення лопаті або на кожному часовому кроці. За допомогою побудованої (ММ) було проведено чисельне вивчення дії косого потоку на роботу гребного гвинта для різних кутів течії. Гідродинамічні сили, що діють на гребний гвинт в косому потоці, визначалися у формі, безрозмірних коефіцієнтів, а саме, коефіцієнтів: упору, моменту і поперечної сили. За допомогою побудованої (ММ) було проведено чисельне вивчення дії косого потоку на роботу гребного гвинта при різних кутах натікання, які можуть виникати в умовах експлуатації суден, обладнаних азіподами. Подані результати розрахунків гідродинамічних сил на гвинті, у формі безрозмірних коефіцієнтів для різних кутових положень лопаті за оборот з урахуванням скоса потоку, а також порівняння їх з результатами робіт інших авторів.

**Ключові слова:** гребний гвинт; косий потік; нестаціонарна несуча поверхня; метод дискретних вихорів; вихрові рамки; гідродинамічні сили.

**Аннотация.** Приведена математическая модель (ММ) гребного винта в косом потоке, построенная на основе нестационарной теории несущей поверхности с использованием метода дискретных вихрей. Вихревая система гребного винта включает присоединенные, замкнутые вихревые контуры, которые моделируют лопасти и распределяются дискретно по винтовым поверхностям с шагом, учитывающим режим умеренной нагрузки, телесность лопасти и поперечную компоненту скорости. Пелена свободных вихрей замещается замкнутыми вихревыми контурами (рамками), в каждый момент времени, сходящими в след, за винтом. При определении в каждом угловом положении лопасти циркуляций нестационарных присоединенных вихревых рамок и свободных вихрей, выполнялись граничные условия на поверхности лопасти, соблюдались, замкнутость вихревых систем и гипотеза Чаплыгина-Жуковского в точках схода вихревых пелен. Система линейных алгебраических уравнений (СЛАУ), составленная на основе, условия непроницаемости поверхности лопасти гребного винта, позволила найти циркуляции присоединенных вихрей в зависимости от углового положения лопасти или на каждом временном шаге. Гидродинамические силы, действующие на гребной винт в косом потоке, определялись в форме, безразмерных коэффициентов: упора, момента и поперечной силы. С помощью построенной (ММ) было проведено численное изучение действия косого потока на работу гребного винта при различных углах натекания, которые могут возникать в условиях эксплуатации судов, оборудованных азиподами. Представлены результаты расчетов гидродинамических сил на винте, в форме безразмерных коэффициентов для различных угловых положений лопасти за оборот с учетом скоса потока, а также сравнение их с результатами работ других авторов.

**Ключевые слова:** гребной винт, косой поток, нестационарная несущая поверхность, метод дискретных вихрей, вихревые рамки, гидродинамические силы.

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**Problem setting.** In real conditions the operation of the screw propellers (SP) is possible in the nonuniform field of speed and in the oblique flow. It is typical for the ships which are equipped by the azipods. During study of (SP) operation in such conditions, it is appropriate to use the nonstationary theory of the lifting surface for the studying of these screws hydrodynamics. The airflow of the screw propeller blades by the steady oblique flow will be nonstationary and as the resultant support and the hydrodynamic moment are changing in accordance with the blade turning angle $\theta_л$, counted from the vertical position. The transverse force is appeared in this case, directed in the same direction like the side component of speed. The mathematical model of the screw propeller which operates in the oblique flow has been built on the base of the nonstationary theory of the lifting surface in the limits of the ideal liquid with the use of the method of the discrete vortexes. At the assessment of the efficiency of the screw propellers operation in the case of the oblique inleakage, the limitation of the experimental researches, including the use of CFD — sets shows the necessity of the use of the theoretical methods. The theoretical research of the screw propeller hydrodynamic forces on the base of the nonstationary theory of the lifting surface during operation in the oblique flow is of the scientific interest and it is actual.

**Analysis of the latest researches and publications.** The following works [9, 4, 13] should be noted among the works of such direction because the serious analysis of the marine screws nonstationary characteristics with use of the lifting surface in the oblique flow model (without the flow curvature). In works [14] the nonstationary nonlinear theory of the lifting surface has been used for the studying of the cavitating screw propellers hydrodynamics in the straight flow. As it was noted in [7], the theory of the main lift rotor in the oblique flow, introduced in the works [9], to use for the studying of the screw propellers in full volume is not possible. The active use of the sets CFD for the projecting of the screw propellers is observed in the last years.The analysis of the screw propeller hydrodynamic characteristics which operates in an oblique flow, with use of the medium OpenFOAM, has been given in work [15].

The theoretical research of the hydrodynamic forces on the screw propeller on the base of the nonstationary theory of the lifting surface during operation in the oblique flow is of the scientific interest. It is actual.

**Object, subject, methods.** In this article, ***the object of the research*** is the ship, equipped by the azipods or by the steering columns and also by the floating drilling platforms .

***The subject of the research*** is the screw propeller which can operate in the oblique flow on the ship. It gives the change of the hydrodynamic forces: the support and the moment, and also the appearance of the transverse force, acting on the screw. These moments are necessary to take into account for the ship safety during her manoeuvering or the positioning of the floating drilling platforms.

***The methods of research.*** The knowledge of the ship theory, the propulsive quality of ship, the wing theory and the vortex theory, the methods of the discrete vortexes and the methods of the nonstationary theory of the lifting surface are used. It allows to simulate efficiently the screw propeller operation in the oblique flow.

**THE ARTICLE AIM** — is to built the mathematical model of the screw propeller, which operates in the oblique flow on the base of the nonstationary theory of the lifting surface with use of the Discrete Vortex Models (DVM) in the regime of the moderate load and the simulation of the blade solid by the system of the resources-outflows. On the base of the mathematical model, the effect of the flow on the hydrodynamic forces (HF), is valued at the different angles of the bevel.

**Basic material.** The airflow of the screw propeller (SP) blades by the uniform oblique flow will be nonstationary, and as result, the resultant support and the hydrodynamic moment are changing in accordance with the blade turning angle $\theta_л$, which is counted from the vertical position. At the same time, the transverse force, directed into the same direction as the side component of speed will appeared.

The mathematical model (MM) of the screw propeller, operating in the oblique flow, has been built on the base of the nonstationary theory of the lifting surface, in the limits of the ideal liquid. In MM the materials of this author earliest works [4 – 7] have been used. The vortex scheme of the screw in the regime of the moderate load, the transverse component of speed and the blade solid have been take into account.

The vortex system of the screw propeller includes the jointed, closed vortex circuit, which model the blades and spread discretely on the screw surfaces with interval which is equal to $P(r) = 2\pi r \tan\beta_I$, where $r$ — is the radial coordinate on the blade vortex surface; $\beta_I$ — the angle of the inductive advance. In the trace will income the closed vortex circuit(frames) with the angle of the spiral line rise $\beta_I^{\theta}$ in each moment of time, which is in accordance with the blade turning into the angle $\Delta\theta$, and it is the difference between nonstationary and quasi-stationary model.

We introduce, as in [7], the right Cartesian coordinate system $Oxyz$, and the connected with the blade system of coordinate $Ox_1y_1z_1$, which rotates with the angle speed $\Omega$. The axis $Ox$ and $Ox_1$ and the beginning of the coordinate of these systems are the same, and we direct them on the axis of SP towards the leaked-in flow $V_A$ under angle of bevel $\alpha_{ск}$ (Fig. 1). The number of $Z$ blade — the blade screw is $n$ ($n = 0, 1, 2, \ldots, Z-1$).

As it was noted in [7], at the oblique inleakage into the screw depending on the angle position of the blade $\theta_л$, the angle of the inductive advance will be changed $\beta_I^{\theta_л}$, and the blade section angle of attack. We will have the following formula taking into account the leaked-in flow angle of bevel $\alpha_{ск}$, speed transverse component $\overline{V}_A \sin\alpha_{ск}$ and the blade angle position $\theta_л$:

$$\beta_I^{\theta_л} = a\tan\left(\frac{\overline{V}_A \cos\alpha_{ск} + \overline{w}_{x1}}{\overline{r} - \overline{w}_{\theta 1} - \overline{V}_A \sin\alpha_{ск} \cos\theta_л}\right),$$

where the received axial $\overline{w}_{x1}$ and circular velocity $\overline{w}_{\theta 1}$, in the screw disk, will be calculated to the first approximation in accordance with the theory of the lifting line, [3]; $\overline{V}_A = \dfrac{V_A}{\Omega R}$ — the relative speed of the flow, leaked-in on the screw under angle of bevel.

As we know, during the numerical solving of the nonstationary tasks, the continuous process of changing during the time of the boundary conditions and the hydrodynamic forces on the lifting surfaces will be changed by the gradation process. In this case, we think that, the jump changing of the boundary conditions and the forces on the blade of SP in some moments of time, that in accordance with the different angle positions of the blade $\theta_л = \theta_{л1}, \theta_{л2}, \ldots, \theta_{лts}$ ($ts = 0, 1, 2, \ldots, \tau S$). The specified above boundary conditions and the forces are the continuous in the intervals between these angle positions $\Delta\theta$. For the definition of the circulation of the nonstationary added frames in each blade angle position and free vortex it is necessary to execute the boundary conditions on the blade surface, to have the closure of the vortex systems and Chaplugin–Joukowski hypothesis in the points of the vortex cover escape. The vortex system of SP in the proposed mathematical model has been built as the following: the discrete added vortex circuit, which simulate the blade of SP and the flame of the free convergent vortex, changed as in [4, 9] by the closed vortex circuit (frames). During the turning of the blade on the angle $\Delta\theta$ ($\Delta\theta = \Omega \cdot \Delta t$, $\Omega$ – is the angle speed of SP), that it is in accordance with the discrete step by time $\Delta t$, and the angle positions are the following and sequentially: $\theta_{л1} = \Delta\theta$, $\theta_{л2} = 2\Delta\theta$, ..., $\theta_{лts} = ts\Delta\theta$ in the trace behind the screw the vortex circuit will be formed, received by the longitudinal and transverse vortex, induced by the boundary conditions change during some time (Fig. 2).

The blade $n$ is changing by the vortex lattice, compounded from the radial added and free vortex, creeping along the chord and forming the vortex frame in whole, (Fig. 2). These added vortex frames are placed in the limits of the cells, received by the division of the blade in accordance with the span (radius) on the panel — $p$, and in accordance with the chord — the division into compartments — $q$, [7]. In accordance with the supposition of

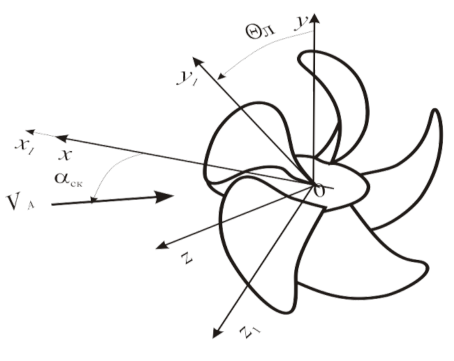


**Fig. 1.** Screw propeller coordinates systems at the oblique inleakage

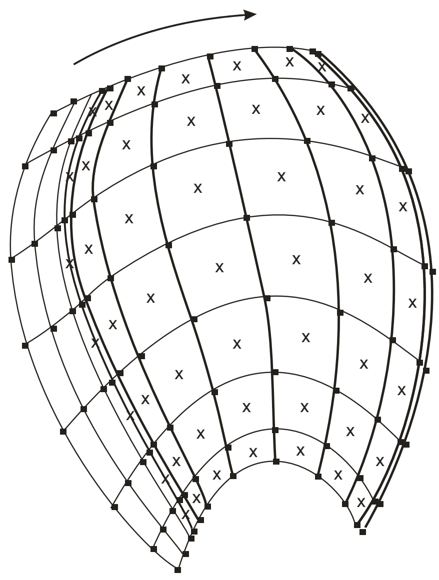

**Fig. 2.** The added blade vortex free frame in the trace behind the screw propeller

the linear theory, the boundary condition of the impermeability has been used on the close screw surface — the support surface, i.e. the variable step along the radius, in our case, from the blade surface. The control points, as the joints points, are places on the support surface with the variable step (in accordance with the rule «Cos»), as in accordance with the hob and in accordance with the blade chord [12].

In the specified below formulas the linear dimensions are relative to the screw radius $R$, the speed — to $\Omega R$, and the circulation — to $\Omega R^2$, where $\Omega$ — is the angle speed of the screw rotation.

The radial coordinates of the control (index is – $j$) and joint points (index is – $pq$) in accordance with the blade hob:

$$\bar r_j = \frac{r_j}{R} = \frac{1}{2}(1+\bar r_b) - \frac{1}{2}(1-\bar r_b)\cos\alpha_i,$$

$$\alpha_i = \frac{i+1}{M+1}\pi,\quad i = 0,1,2,\dots,M-1,$$

$$\bar r_{pq} = \frac{r_{pq}}{R} = \frac{1}{2}(1+\bar r_b) - \frac{1}{2}(1-\bar r_b)\cos\alpha_p,$$

$$\alpha_p = \frac{2p-1}{2(M+1)}\pi,\quad p = 0,1,2,\dots,M, \qquad (1)$$

where $\bar r_b$ — a relative radius of the screw propeller hob; $M$ — a quantity of the panels in accordance with the blade span.

The coordinates of the control (index is – $j$) and joint points (index is – $pq$) in accordance with the screw propeller blade chord:

$$s_{ji} = s_L - \frac{1}{2}\bar b_{jk}(\bar r_j)\cos\beta_{jk},\quad \beta_{jk} = \frac{q}{N}\pi,$$

$$k = 0,1,2,\dots,N-1;$$

$$s_{pq} = s_L - \frac{1}{2}\bar b_{pq}(\bar r_{pq})(1-\cos\beta_{pq}),\quad \beta_{pq} = \frac{(2(k+1)-1)}{2N}\pi,$$

$$q = 0,1,2,\dots,N,$$

where $\bar b_{j,pq}(\bar r_{j,pq}) = \dfrac{b_{j,pq}(r_{j,q})}{R}$ — a relative breadth of the screw propeller blade;

$S_L$ — a relative coordinate of the incoming edge;

$N$ — a quantity of the frames (compartments) in accordance with the chord.

Cartesian coordinates of the control and joint points, in accordance with the [7], but take into account the scheme of the moderate load screw, are the following:

$$\bar x_j = -\bar r_j\theta_j\tan\beta_I;\ \bar y_j = \bar r_j\cos\theta_j;\ \bar z_j = \bar r_j\sin\theta_j.$$

The coordinates of the lattice joints on the blade of SP:

$$\bar x_{pq} = -\bar r_{pq}\theta_{pq}\tan\beta_I;\ \bar y_{pq} = \bar r_{pq}\cos(\delta_n - \theta_{pq});$$
$$\bar z_{pq} = \bar r_{pq}\sin(\delta_n - \theta_{pq}), \qquad (2)$$

where $\tan\beta_I = \dfrac{\bar V_A\cos\alpha_{ск} + \bar w_{x1}}{\bar r - \bar w_{\theta 1} - \bar V_A\sin\alpha_{ск}\cos\theta_{л}}$ — it is in case of the oblique inleakage, are calculated in accordance with the scheme of the moderated load screw, and the axial $\bar w_{x1}$ and circular velocity $\bar w_{\theta 1}$, received speed in the screw disk, are calculated at the first approximation in accordance with the theory of the lifting line, [3];

$\bar V_A = \dfrac{V_A}{\Omega R}$ — the relative speed of the leaked — in flow on the screw under the bevel angle;

$\delta_n = n\dfrac{2\pi}{Z},\ n = 0,1,2,\dots,Z-1$ — the angular displacement of $n$-th blade.

The coordinates of the closed vortex cords joints will be on the free cover of SP. They are calculated in accordance with Eulerian equation of motion.

We consider that the vortex added frames and they are in the trace, have been built from the rectilinear vortex segments, which connect four joint points on the blade and on the free cover. The inductive speed of this vortex are calculated by Biot-Savart law, in accordance with [9].It was recorded, taking into account the accepted coordinate system. They are in accordance with the following expressions:

$$\bar u_{X,jipq} = \frac{1}{4\pi}(\Delta\bar y_Q\Delta\bar z_P - \Delta\bar z_Q\Delta\bar y_P)I_S,$$
$$\bar u_{Y,jipq} = \frac{1}{4\pi}(\Delta\bar z_Q\Delta\bar x_P - \Delta\bar x_Q\Delta\bar z_P)I_S, \qquad (3)$$
$$\bar u_{Z,jipq} = \frac{1}{4\pi}(\Delta\bar x_Q\Delta\bar y_P - \Delta\bar y_Q\Delta\bar x_P)I_S,$$

where

$$\Delta\bar x_Q = \bar x_{p+1,q} - \bar x_{pq},\quad \Delta\bar y_Q = \bar y_{p+1,q} - \bar y_{pq},$$
$$\Delta\bar z_Q = \bar z_{p+1,q} - \bar z_{pq};$$
$$\Delta\bar x_P = \bar x_j - \bar x_{pq},\quad \Delta\bar y_P = \bar y_j - \bar y_{pq},\quad \Delta\bar z_P = \bar z_j - \bar z_{pq},$$
$$I_S = \frac{1}{ac-b^2}\left(\frac{a-b}{\sqrt{a-2b+c}} + \frac{b}{\sqrt{c}}\right),$$
$$a = \Delta\bar x_Q^2 + \Delta\bar y_Q^2 + \Delta\bar z_Q^2,\quad b = \Delta\bar x_Q\Delta\bar x_P + \Delta\bar y_Q\Delta\bar y_P + \Delta\bar z_Q\Delta\bar z_P,$$
$$c = \Delta\bar x_P^2 + \Delta\bar y_P^2 + \Delta\bar z_P^2.$$

Then the singular components of speed, induced by the vortex frame in the control points, on the blade will be the following expressions:

$$\bar u^{\oplus}_{X,jipq} = \bar u_{X,jiq} - \bar u_{X,ji,q+1} + \bar u_{X,jip} - \bar u_{X,jip+1},$$
$$\bar u^{\oplus}_{Y,jipq} = \bar u_{Y,jiq} - \bar u_{Y,ji,q+1} + \bar u_{Y,jip} - \bar u_{Y,jip+1}, \qquad (4)$$
$$\bar u^{\oplus}_{Z,jipq} = \bar u_{Z,jiq} - \bar u_{Z,ji,q+1} + \bar u_{Z,jip} - \bar u_{Z,jip+1},$$

The full speed (in the dimensionless form) for each control point $P_{ji}$ can be written in accordance with (12), like the sum of the inductive speeds from the blade vortex frames, the free vortex frames, convergent into the flow $\overline{w}_{ji}^{F}$, received by the speed from the sources $\overline{W}_{ji}^{S}$, the full relative speed of the external flow $\overline{V}_{ji}^{I}$:

$$\overline{V}_{ji} = \overline{W}_{ji}^{blade} + \overline{W}_{ji}^{F} + \overline{W}_{ji}^{S} + \overline{V}_{ji}^{I},\ j = 0,1,\dots,M-1,$$
$$i = 0,1,\dots,N-1. \tag{5}$$

The speed, induced in the control point $P_{ji}$ by the system of the added blade vortex in accordance with [12] can be written by the equation

$$\overline{W}_{ji}^{blade} = \frac{\pi}{2N}\sum_{n=0}^{Z-1}\sum_{p=0}^{M-1}\overline{b}(\overline{r})\sum_{q=0}^{N-1}\gamma_{pq}\overline{u}_{jipq}^{\oplus}\sin\beta_{jq}, \tag{6}$$

where $\overline{u}_{jipq}^{\oplus}$ — the speed from the vortex circuit of the singular circulation $\Gamma = 1$, which includes two added vortexes between joints $Q_{pq}$ and $Q_{p+1,q}$ and two free vortexes, convergent from these joints along the chord.

The normal speed, induced by $p$ th free vortex frame on $ts$-th interval in accordance with time in the control point $P_{ji}$:

$$\overline{W}_{jin}^{F} = \sum_{ts=1}^{\tau S}\sum_{p=0}^{M-1}\overline{u}_{pjin,}^{ts\oplus}\Gamma_{pji}^{ts}, \tag{7}$$

where $\overline{u}_{pjin}^{ts\oplus}$ — the singular normal speed (it is calculated in accordance with the equations (3, 4)), received by the convergent, on the $ts$-th interval in accordance with time by $p$-th free vortex frame in the control point, in accordance with its circulation $\Gamma_{pji}^{ts}$.

We project the full speed (5) of the normal to the support surface in the control points and we receive the system of the linear algebra equations (SLAE), received on the base of the screw propeller blade impenetrability condition, the solution of this allows to find the circulation of the added vortexes $\Gamma_{pq}^{\theta_л}$ in accordance with the angular position of the blade $\theta_л$ or on each time interval $ts$:

$$\sum_{n=0}^{Z-1}\sum_{p=0}^{M-1}\sum_{q=0}^{N-1}\left[\left(\overline{u}_{X,jipq}^{\oplus B} - \overline{u}_{X,jipq}^{\oplus F}\right)n_{Ix} + \right.$$
$$\left. + \left(\overline{u}_{Y,jipq}^{\oplus B} - \overline{u}_{Y,jipq}^{\oplus F}\right)n_{Iy} + \left(\overline{u}_{Z,jipq}^{\oplus B} - \overline{u}_{Z,jipq}^{\oplus F}\right)n_{Iz}\right]\cdot\Gamma_{pq}^{\theta_л} =$$
$$= \left(\overline{V}_{Rj}(\varphi_j - \beta_I^{\theta_л}) + \overline{V}_{Rj}\frac{\partial\eta_c}{\partial\tilde{\xi}} - \overline{w}_{qnj}\right) -$$
$$- \sum_{ts=1}^{\tau S}\sum_{p=0}^{M-1}\overline{u}_{pjin}^{\oplus F}\Gamma_{pji}^{ts}, \tag{8}$$

where

$$n_{Ix} = \cos\beta_I/\Lambda_I;\ n_{Iy} = -(D_I\cos\beta_I\cos\theta_j - \sin\beta_I\sin\theta_j)/\Lambda_I;$$
$$n_{Iz} = -(D_I\cos\beta_I\sin\theta_j + \sin\beta_I\cos\theta_j)/\Lambda_I;$$
$$\Lambda_I = \left(1 + \cos^2\beta_I D_I^2\right)^{1/2};\ D_I = \theta_j\, d\lambda_I/d\overline{r};$$

$\lambda_I = \overline{r}\tan\beta_I^{\theta_л}$; the normal direction cosines to the support surface are written in accordance with [3] and [6];

$\beta_I^{\theta_л} = \arctan\left(\dfrac{\overline{V}_A\cos\alpha_{ск} + \overline{w}_{x1}}{\overline{r} - \overline{w}_{\theta 1} - \overline{V}_A\sin\alpha_{ск}\cos\theta_л}\right)$ — the angle of the blade section inductive advance, which has turned into angle $\theta_л$, at the resultant velocity of the leaked-in flow, taking into account the transverse speed component $\overline{V}_A\sin\alpha_{ск}$:

$$\overline{V}_{Rj} = \sqrt{(\overline{V}_A\cos\alpha_{ск} + \overline{w}_{x1})^2 + (\overline{r} - \overline{w}_{\theta 1} - \overline{V}_A\sin\alpha_{ск}\cos\theta_л)^2};$$

$\overline{u}_{X,Y,Z,jipq}^{\oplus F}$ — the inductive speeds from the free vortex frames of the singular circulation (the index is — $F$), which cover the outgoing blade edge and they are included in accordance with [10], in the right part of SLAE (8).

The finding of the normal component of the received speed $\overline{w}_{qnj}$ from the sources — the outflows and the derivative $\dfrac{\partial\eta_c}{\partial\tilde{\xi}}$ of the distribution function, the ordinates of the profile section medium line in accordance with the coordinate $\tilde{\xi} = 2\xi/b$, has been described in [5].

The solution of SLAE (8) in each time interval (in accordance with angle of the blade turning $\theta_л$), gives the circulations of the added vortexes $\Gamma_{pq}^{\theta_л}$, which allow to find the intensity of these vortexes $\gamma_{pq}^{\theta_л}$, and to find the inductive speeds in the points of the blade by use of (6), to find the inductive speeds from the free vortex frames (7) and calculate the forces, acting into the screw propeller.

During the finding of the nonstationary forces and the moment on the screw propeller, we use Kosh-Lagrang integral in the dimensionless form [1]:

$$\Delta\overline{p} = 2\left(\gamma w_{отн} - \frac{\partial\Gamma}{\partial\tau}\right).$$

The use of it to the blade lifting surface, we will receive the expression which allows to find the hydrodynamic load on the element of the surface, covered by the added vortex frame:

$$\Delta\overline{p} = p(t) - p_\infty = \pi^2\left(\Gamma_{pq}^{\theta_л}w_{отн} - \frac{\Delta\Gamma_{pq}^{\theta_л}}{\Delta\theta}\right). \tag{9}$$

Then the nonstationary hydrodynamic forces ($F_x$, $F_y$, $F_z$) and the moment $M_x$, acting into the screw propeller, are introduced by the following equations[13]:

$$\begin{aligned}
F_x(t) &= T(t) = \int_S (p(t) - p_\infty)n_x dS,\\
F_y(t) &= \int_S (p(t) - p_\infty)(n_y\cos(\Omega t) - n_z\sin(\Omega t))dS,\\
F_z(t) &= \int_S (p(t) - p_\infty)(n_z\cos(\Omega t) - n_y\sin(\Omega t))dS,\\
M_x(t) &= Q(t) = \int_S (p(t) - p_\infty)(n_y z - n_z y)dS,
\end{aligned} \tag{10}$$

where $\overline{n} = (n_x, n_y, n_z)$ — the normal to the blade support surface .

The coefficient of the stop $K_T^{л}$ and the moment $K_Q^{л}$ of the screw propeller blade, turning in each moment of time into the angle $\theta_{л}$, during the operation in the oblique flow:

$$K_T^{Л} = \frac{T^{Л}(t)}{\rho n^2 D^4}; \quad K_Q^{Л} = \frac{Q^{Л}(t)}{\rho n^2 D^5}. \qquad (11)$$

In this work, the coefficient of the tangential force $K_Z^{Л}$ on the blade surface element (in the plane of the screw disk) at the turning into the angle $\theta_{л}$, has been calculated on the formula:

$$K_Z^{Л} = \frac{F_Z^{Л}(t)}{\rho n^2 D^4}, \qquad (12)$$

where ρ — the density of liquid; $n$ — the frequency of the screw propeller rotating; $D$ — the diameter of SP. The equation (12) allows to calculate the tangential force $F_z^{Л}$ on the blade at different angular positions.

During operation of SP in the oblique flow, the transverse component of speed $\overline{V}_A \sin\alpha_{ск}$, leads to the appearance of the side stabilizing force of the same direction.

In accordance with the built theoretical model, the preliminary calculations of the dimensionless hydrodynamic characteristics of SP have been executed such as the coefficients of the following terms: a stop — $K_T$, a moment — $K_Q$ and a tangential force — $K_Z$ (of the series Z4–85: Z = 4; $A_E/A_0$ = 0,85; $\overline{r}_b$ = 0,20; $e_0/D$ = 0,055) depending on the blade quantity — $Z$, the disk relation $A_E/A_0$, $P/D(r)$ at the specified value $P/D$ = 1,3 for $\overline{r}$ = 0,7, the relative conditional thickness — $e_0/D$ on the axis of SP, at the flow oblique angles $\alpha_{ск}$ = 15° and 30°. At the use of the quasistationary model, to receive the same distributions of the specified above coefficients, (Fig. 3, 4) , is not possible.

At first, $K_T$ and $K_Q$ of SP in the oblique flow have been calculated, at the advance $J$ = 0,30, for the comparison of them with diagram values [8], see table 1.

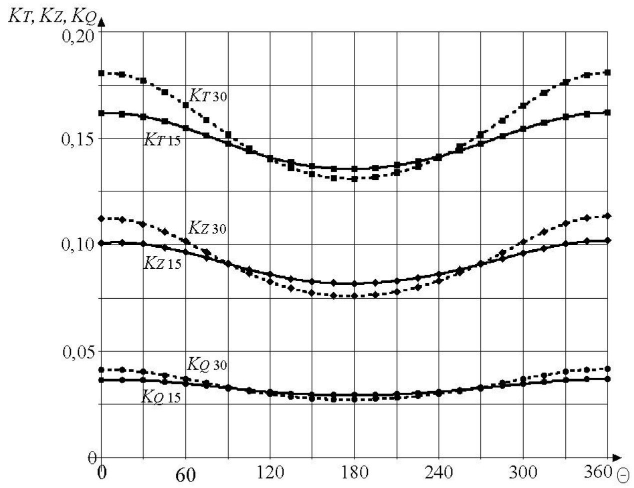


**Fig. 3.** The results of the following coefficients: the stop — $K^{Л}_T$, the moment — $K^{Л}_Q$ and the tangential force — $K^{Л}_Z$ per one blade turn in the oblique flow: $\alpha_{ск}$ = 15° and 30°.

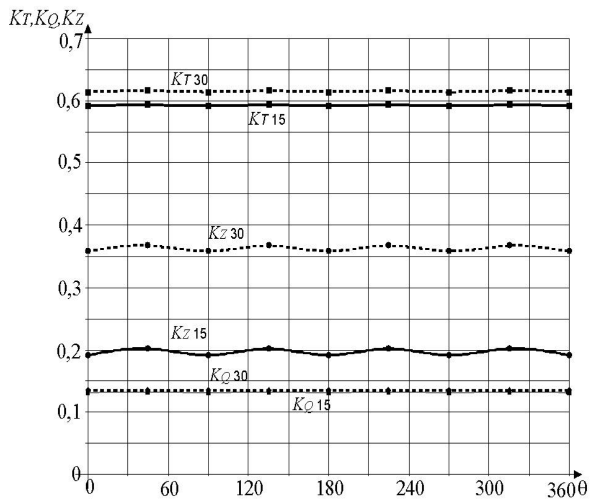


**Fig. 4.** The results of the following coefficients: the stop — $K_T$, the moment — $K_Q$ and the transverse force — $K_Z$ per one turn of the screw propeller in the oblique flow: $\alpha_{ск}$ = 15° and 30°.

**Table 1.** The comparison of $K_T$ and $K_Q$ ГВ of SP in the oblique flow with the diagram values

| Hydrodynamic characteristics | $K_T$ | $K_Q$ |
|---|---|---|
| Calculation | 0,529 | 0,1098 |
| Diagram values, [8] | 0,528 | 0,0987 |

Then $K_T^Л$ and $K_Q^Л$ of SP blade have been calculated at different angular positions $\theta_л$ and the flow bevel angles $\alpha_{ск}$ = 15° and 30°, Fig. 3. In accordance with the calculation results of the tangential force $K_Z^Л$, coefficients, the screw transversal stabilizing force coefficients have been calculated. For the comparison of the calculation results , the formula in accordance the material [5], has been used, which has been received the dimensionless form in [7]:

$$K_{Zп} \approx 2{,}14\, K_{Qшв}\left[1-\left(J/J_Q\right)\right] J \sin\alpha_{ск}; \qquad (13)$$

where $K_{Zп} = Y_E/\rho n^2 D^4$ — the coefficient of the transversal stabilizing force; $K_{Qшв}$ — the coefficient of the moment at the operation of SP at the mooring regime; $J_Q$ — the advance of the zero moment.

The values $K_Z^Л$, which are showed at the Fig. 3, 4, allow to calculate the mean values per one screw turn transversal force coefficient $K_{Zп}$ ($\alpha_{ск}$ = 15° and 30°) and compare them with the values, which have been calculated in accordance with the formula (13), see the Table 2.

In this work in accordance with the proposed MM, the coefficient of the stop change $q_T$ and the coefficient of the moment $q_Q$ of SP in the oblique flow have been calculated . The comparison of them with the results, which have been received in accordance with the information [2] and [7], see the Table 3, taking into account that $q_T = K_T^*/K_T$; $q_Q = K_Q^*/K_Q$. The mean values of SP turn values of the coefficients: the stop $K_T^*$ and the moment $K_Q^*$ in the oblique flow at $\alpha_{ск}$ = 15° and 30°, have been calculated in accordance with the results, see the Fig. 3. The values $K_T$ and $K_Q$ in the oblique flow (the calculation ) have been used from the Table 1. The comparison results are in the Table 3.

**Table 2.** The comparison of $K_{Zп}$ calculation with the values, received by use of [2] and [7]

| $K_{Zп}$ | $\alpha_{ск}$, degrees | |
|---|---|---|
| | 15 | 30 |
| calculation | 0,01276 | 0,0351 |
| calculation in accordance with [7] | 0,01321 | 0,02635 |
| by use of [2] | 0,0155 | 0,0276 |

**Table 3.** The coefficients $q_T$ and $q_Q$ calculation comparison results

| | $\alpha_{ск}$, degrees | |
|---|---|---|
| | 15 | 30 |
| $q_T$, calculation | 1,0340 | 1,0927 |
| $q_T$, calculation by use of [7] | 1,0317 | 1,0765 |
| $q_T$, by use of [2] | 1,0315 | 1,0750 |
| $q_Q$, calculation | 1,0564 | 1,102 |
| $q_Q$, by use of [7] | 1,0498 | 1,0765 |
| $q_Q$, by use of [2] | 1,0255 | 1,0825 |

**Discussion of the received results.** It is clear that the accessing of the results which are better for SP operation in the real conditions , leads to the serious complication of the mathematical model ,or to the necessity of the modern tools of the calculative hydrodynamics use — CFD sets, which are based on the numerical solution of Navier-Stokes equation by RANS method. CFD sets is the expensive license commercial product, which we must have . In this article, the use of the tool in the form of the mathematical model, MM, of the screw propeller in the oblique flow, has allowed to receive the acceptable results of the hydrodynamic forces on the engine calculation. The proposed tool, has the limited possibilities comparing with the modern CFD sets, but the calculation time in accordance with this model is more less.

**CONCLUSIONS.** The mathematical model of SP, operating in the oblique flow, on the base of the nonstationary theory of the lifting surface. MM takes into account the real geometry of the blade section profile, its thickness and the change of the interval in accordance with SP blade radius. MM can be compared with the quasistationary model [7], for more exact assessment of the variables of the forces per one turn which influence on the blade of the screw and on the screw in whole. The programme complex is realized on the programming language C# with use of the object-oriented way in the area of the development Visual Studio 2005. The comparing of the results of SP calculation at the loading coefficient $C_T$ = 13,9 with use of the theory of the nonstationary surface, has showed the acceptable matching with the experimental characteristics from the information [8], like the transversal stabilizing force and in the calculation of SP coefficients of the stop and the moment change calculation. The proposed MM can be used like the method of the research at the first approximation of the screw propellers hydrodynamics which operate in the oblique flow.

The further improvement of SP model can be directed into taking into account the influence of the gondola streamline coat of azipod, that it is in real conditions.

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Статтю рекомендує до друку
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